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8
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6
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Find the **derivative** of the function $f(x,y) = 2xy + 3y^2$ at $P_0(3,-1)$ in the direction of $A = 4i - 3j$

switched signs

$$\vec{\nabla} f\Big|_{P_0} = \frac{\partial f}{\partial x}\Big|_{P_0}\vec{i} + \frac{\partial f}{\partial y}\Big|_{P_0}\vec{j}$$

$$= 2y\Big|_{P_0}\vec{i} + (2x + 6y)\Big|_{P_0}\vec{j}$$

$$= -2\vec{i} +$$

$$|\vec{A}| = \sqrt{4^2 + 3^2} = 5$$

unit vector $\vec{u} = \frac{4}{5}\vec{i} - \frac{3}{5}\vec{j}$

$$D_{\vec{u}} f = \left(\frac{4}{5}i - \frac{3}{5}j\right) \cdot (-2i)$$

$$\boxed{D_{\vec{u}} f = -\frac{8}{5}}$$

✓

Find an equation for the **tangent plane** to $x^2 - y^2 + z^2 = 1$ at the point $(1,2,2)$

$f(x,y,z) = x^2 - y^2 + z^2 - 1$

✓

$\left.\frac{\partial f}{\partial x}\right|_{P_0} = 2x = 2$

$\left.\frac{\partial f}{\partial y}\right|_{P_0} = -2y = -4$

$\left.\frac{\partial f}{\partial z}\right|_{P_0} = 2z = 4$

Tagent plane: $2(x-1) - 4(y-2) + 4(z-2) = 0$

$2x - 2 - 4y \cancel{+8} + 4z \cancel{-8} = 0$

$2x - 4y + 4z - 2 = 0$

$\boxed{x - 2y + 2z - 1 = 0}$ ✓

normal line:

$x = 1 + 2t$

$y = 2 - 4t$

$z = 2 + 4t$

Given $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$, use the **Chain Rule** to express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ as functions of u and v

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$= (y+z)\cdot 1 + (x+z)\cdot 1 + (y+x)\cdot v$$

$$= u \cancel{- v} + uv + u \cancel{+ v} + uv + (u \cancel{- v} + u \cancel{+ v})\, v$$

$$\cancel{\frac{\partial w}{\partial u} = 2u + 4uv}$$

$$\boxed{\frac{\partial w}{\partial u} = 2u + 4uv}$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

$$= (y+z)\cdot 1 + (x+z)\cdot(-1) + (y+x)\cdot u$$

$$= u - v + uv + (u + v + uv)(-1) + (2u)\, u$$

$$= \cancel{u} - v \cancel{+ uv} \cancel{- u} - v \cancel{- uv} + 2u^2$$

$$\boxed{\frac{\partial w}{\partial v} = -2v + 2u^2}$$

Use the method of **Lagrange Multipliers** to find the maximum and minimum values of $f(x,y)=x^3+y^2$ subject to the constraint $x^2+y^2=1$

$g(x,y) = x^2+y^2-1$

~~$\vec{\nabla} f = 3x^2$~~

$\vec{\nabla} f = 3x^2\vec{i} + 2y\vec{j}$

$\vec{\nabla} g = 2x\vec{i} + 2y\vec{j}$

$\begin{cases} \vec{\nabla} f = \lambda \vec{\nabla} g \\ g(x,y) = 0 \end{cases} \longrightarrow \begin{cases} 3x^2 = \lambda\, 2x \longrightarrow \cancel{\lambda = \frac{3}{2}x} \\ 2y = \lambda\, 2y \;(2) \longrightarrow \lambda = 1 \\ g(x,y) = 0 \end{cases}$

$x = 0$

$\lambda = 1 \Longrightarrow x = \frac{2}{3}$

$x^2+y^2-1=0$

$\frac{4}{9}+y^2-1=0$

$y^2 = \frac{5}{9}$

$y = \pm\frac{\sqrt{5}}{3}$

$f\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right) = \frac{8}{27} + \frac{5}{9} = \frac{23}{27}$

$f\left(\frac{2}{3}, -\frac{\sqrt{5}}{3}\right) = \frac{8}{27} + \frac{5}{9} = \frac{23}{27}$

$\boxed{f(-1, 0) = -1 \quad \text{~~max~~}}$

$\boxed{f(1, 0) = 1 \quad \text{max}}$

~~$\lambda = \frac{3}{2}x$~~

$2y(\lambda - 1) = 0$

(2) $2y = 0$ or $\lambda - 1 = 0$

$y = 0$ $\quad\quad \lambda = 1$

$y = 0 \Longrightarrow x = \pm 1$

Minimum value: $f(-1,0) = -1$

Maximum value: $f(1,0) = 1$

Use the **Second Derivative Test** to find all local maxima, minima, and saddle points of the function $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$

$$\frac{\partial f}{\partial x} = 3x^2 + 6x = 0 \longrightarrow x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = 0 \quad \text{or} \quad x = -3$$

$$\frac{\partial f}{\partial y} = 3y^2 - 6y = 0 \longrightarrow y^2 - 2y = 0$$

$$y(y-6) = 0$$

$$y = 0 \quad \text{or} \quad \cancel{y = 6}$$

$$y = 2$$

Critical points

$(0,0)$

$(0,6)$

$(-3,0)$

$(-3,6)$

$$f_{xx} = 6x + 6$$

$$f_{yy} = 6y - 6$$

$$f_{xy} = 6$$

$$f_{xx}f_{yy} - f_{xy}^2 = (6x+6)(6y-6) - 36$$

$$= 36xy - 36x + 36y - 36 - 36$$

$$= 36(xy - x + y - 2)$$

$(0,0) \longrightarrow -72 < 0$ saddle point

$(0,6) \longrightarrow 144 > 0$ & $f_{xx(0,6)} = 6 > 0 \Rightarrow$ local min

$(-3,0) \longrightarrow 36 > 0$ & $f_{xx(-3,0)} = -12 < 0 \Rightarrow$ local max

$(-3,6) \longrightarrow -396 < 0$ saddle point

$(0,0)$ saddle point

$(0,6)$ local minimum

$(-3,0)$ local maximum

$(-3,6)$ saddle point