Math 233

HAIGAZIAN UNIVERSITY
Mathematics Department

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Mat 233 (probability & Statistics) Quiz 2 NOV 13/2010 Time : 60 mins

Name:--------------------------------

Part I

Do this part on the answer booklet .

(30%) 1. The lifetime of an electric component (in years) is a random variable X with the (p.d.f)

$$f(x) = \begin{cases} 1/5 \operatorname{Exp}(-x/5), & x>0 \\ 0, & \text{otherwise} \end{cases}$$

a) Calculate the probability that a component lasts less than 3years.
b) Calculate the probability that a battery lasts over 6 years.
c) Calculate the mean lifetime of the component.
d) Find the (c.d.f) formula.
e) Deduce the median value of the lifetime "m" ($p(X\le m) = 0.5$).

(30%)2. X and Y have the joint p.d.f $f(x;y) = \begin{cases} 1/2, & 0<x<y<2 \\ 0, & \text{otherwise} \end{cases}$

a) Find μ_X and μ_Y.
b) Find σ_X and σ_Y.
c) Find $p(XY<1)$.
d) Find $E(X/Y=1)$ and $E(Y/X=1)$.
e) Find the variance of $Z = 2X - Y$.

(10%) 3. A box contains 9 balls numbered 1 through 9. Two balls are to be selected at random without replacement. Let X be the smallest of the two numbers, find a formula for the (p.m.f) of X.

$\frac{9-x}{36}$

PART 2

Please circle the correct answer.

1. Let X and Y be the numbers obtained on throwing a pair of fair dice. What is the value of $E(X+Y)$?

a) 3.5 b) 10 c)6 d) 7 e) none

2. X has a mean of 3 and variance of 7. What is $E(X^2 + X + 4)$?

a) 21 b)20 c)23 d)25 e)none

3.Which of the following s not necessarily true?

a) The (c.d.f) of a continuous variable is continuous.
b) The (p.d.f) ≥ 0.
c) The (p.d.f) of a continuous variable is continuous.
d) If X is continuous then $p(X= a) = 0$.
e) None of the above.

4.Let X be any random variable with a given mean and standard deviation. Let

$Y = -3X + 2$. The correlation between X and Y is

a) -3   b) 3   c) 1   d) -1   e) none

5. A player bets on the event "A" with $p(A) = 0.4$. He / she wins \$3 if A occurs. How much should the fee to play the game be if the expected net income of the player is zero?

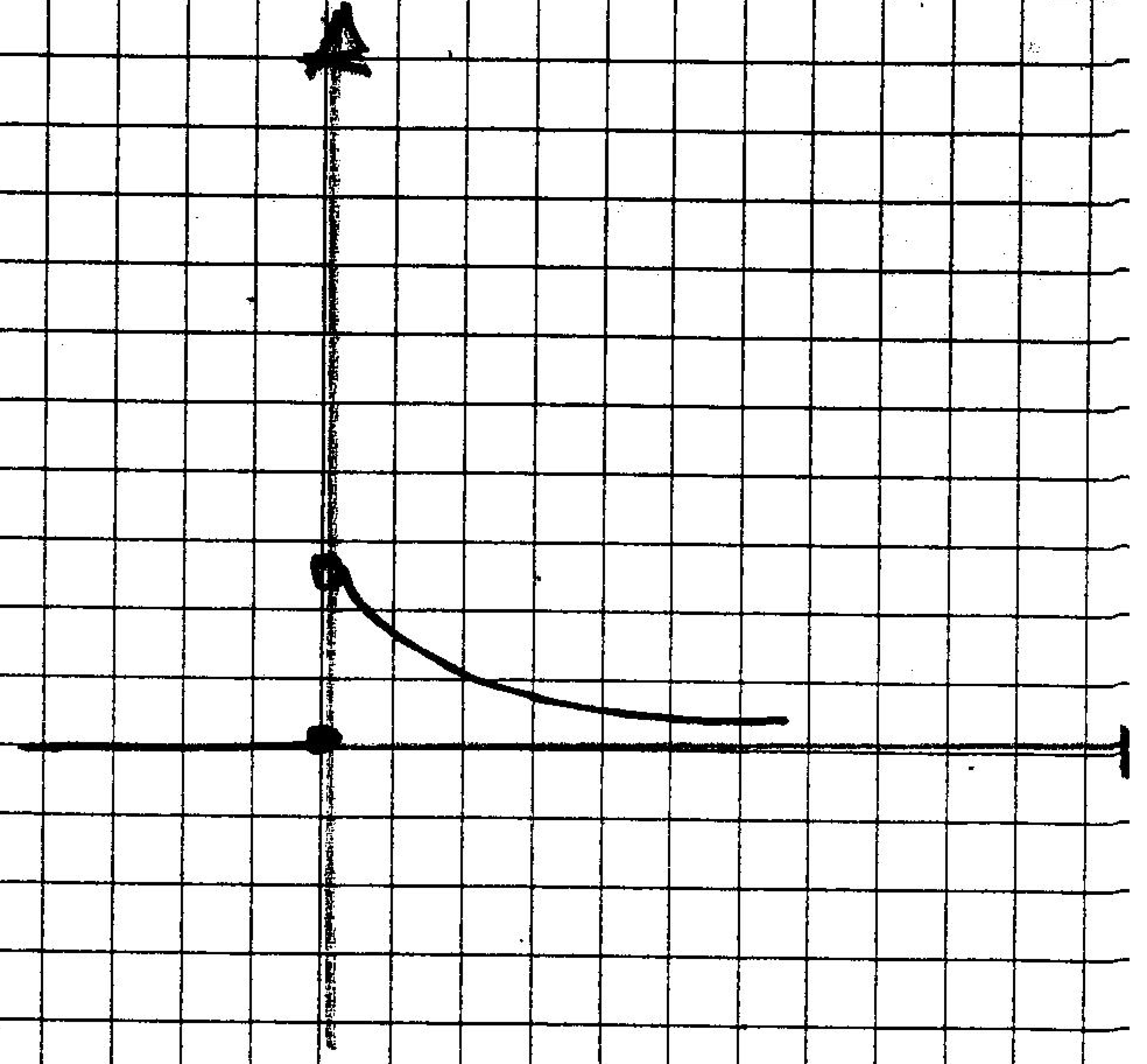
a) \$ 1.4   b) \$ 1.2   c) \$1.6   d) \$1.1   e) none

GOOD LUCK\$\$\$\$\$

1.

a) $p(X<3) = \int_0^3 \frac{e^{-\frac{x}{5}}}{5}\,dx = -e^{-\frac{x}{5}}\Big|_0^3$

$= 1 - e^{-0.6}$



b) $p(X>6) = \int_6^\infty \frac{e^{-\frac{x}{5}}}{5}\,dx = e^{-1.2}$

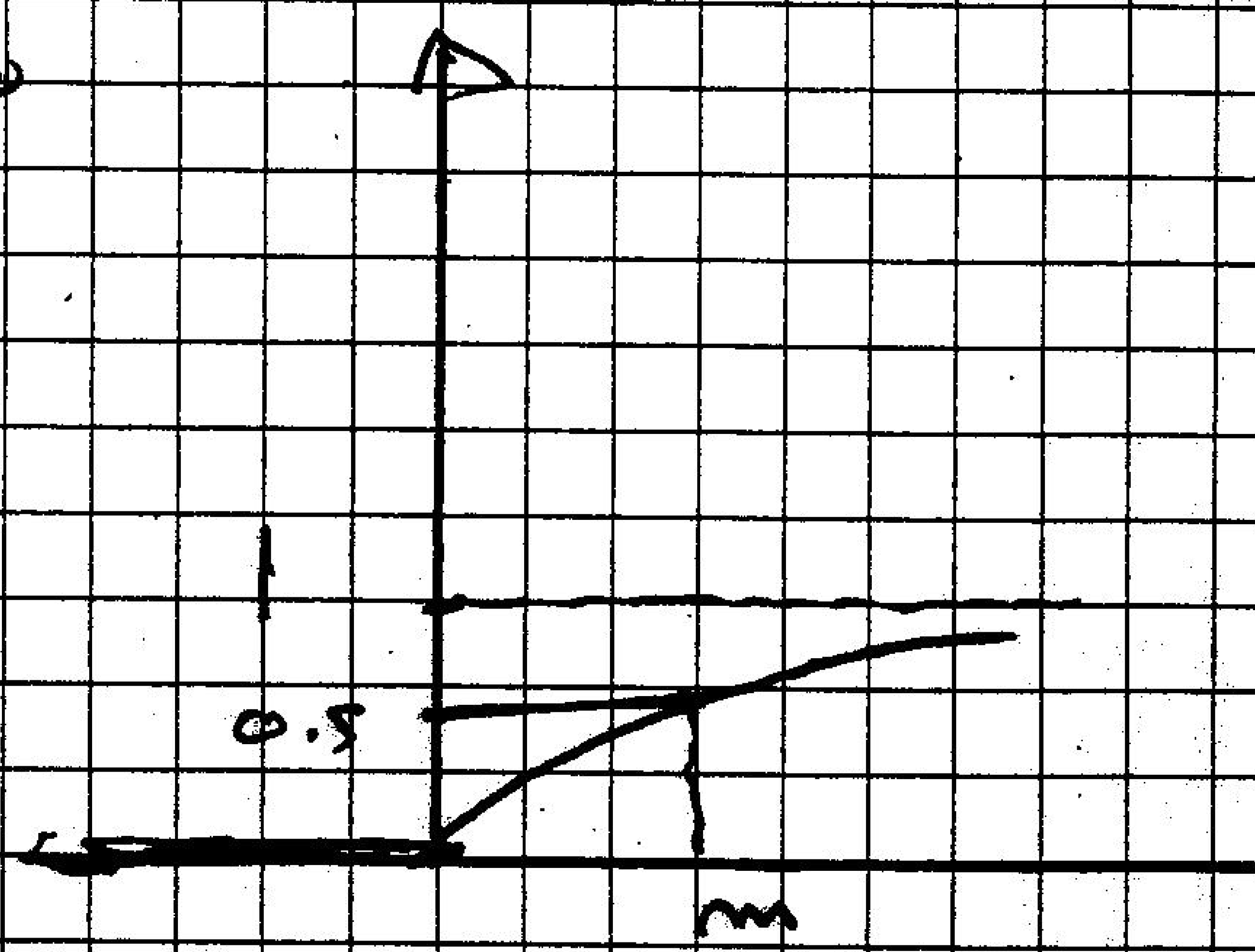
c) $\mu = E(X) = \int_0^\infty \frac{x e^{-\frac{x}{5}}}{5}\,dx$

$= -x e^{-\frac{x}{5}} - 5 e^{-\frac{x}{5}}\Big|_0^\infty = \boxed{5 \text{ years}}$

d) $F(x) = \begin{cases} 0, & x \le 0 \\ C - e^{-\frac{x}{5}}, & x > 0 \end{cases}$

since $\lim_{x\to\infty} F(x) = 1 \Rightarrow C = 1$

$\Rightarrow F(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-\frac{x}{5}}, & x > 0 \end{cases}$

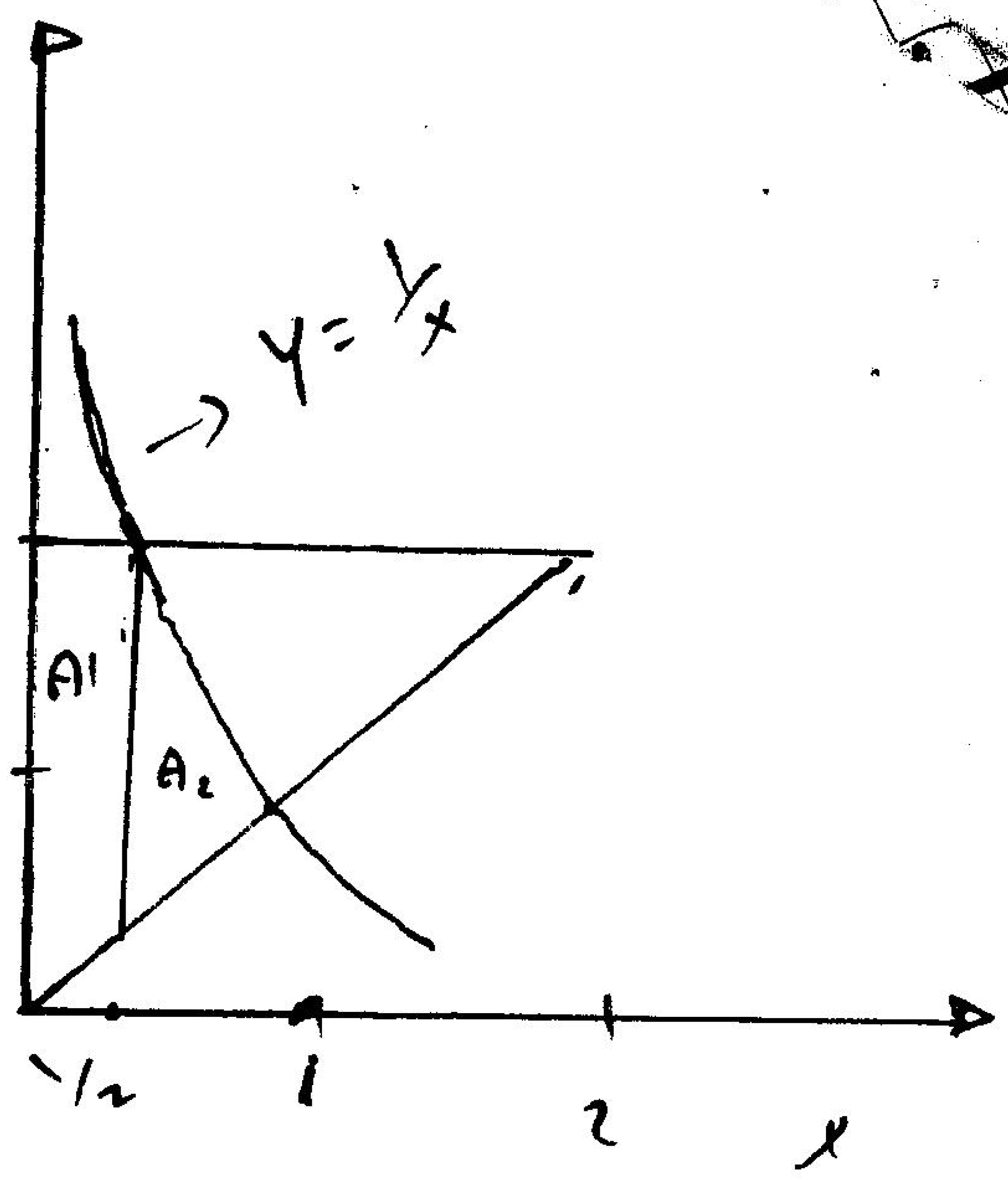


e) $F(m) = 0.5$

$\Rightarrow 1 - e^{-\frac{m}{5}} = 0.5$

$\Rightarrow m = (5 \ln 2)$ years

) $M_X = \int_0^2 \int_x^2 \frac{x}{2}\, dy\, dx$



$= \int_0^2 \frac{x}{2}(2-x)\, dx = \boxed{\frac{2}{3}}$

$M_Y = \int_0^2 \int_x^2 \frac{y}{2}\, dy\, dx$

$= \frac{1}{2} \int_0^2 \left(\left. \frac{y^2}{2} \right|_x^2 \right) dx = \frac{1}{4} \int_0^2 (4 - x^2)\, dx$

$= \boxed{\frac{4}{3}}$

ii) $E(X^2) = \frac{1}{2} \int_0^2 \int_x^2 x^2\, dy\, dx = \boxed{\frac{2}{3}}$

$E(Y^2) = \frac{1}{2} \int_0^2 \int_x^2 y^2\, dy\, dx = \boxed{2}$

$\sigma_X^2 = \frac{2}{3} - \left(\frac{2}{3}\right)^2 = \frac{2}{9} \Rightarrow \sigma_X = \boxed{\frac{\sqrt{2}}{3}}$

$\sigma_Y^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9} \Rightarrow \sigma_Y = \boxed{\frac{\sqrt{2}}{3}}$

) $p(XY<1) = p(Y < \frac{1}{X}) = \iint_{A_1} + \iint_{A_2}$

Now: $\iint_{A_1} f\, dy\, dx = \frac{1}{2} \left[\frac{(2 + \frac{3}{2}) \frac{1}{2}}{2} \right]$

and $\iint_{A_2} f\, dy\, dx = \frac{1}{2} \int_{1/2}^{1} \int_x^{1/x} dy\, dx$

$= \frac{1}{2} \int_{1/2}^{1} \left(\frac{1}{x} - x \right) dx = \frac{1}{2} \left(\ln x - \frac{x^2}{2} \right|_{1/2}^{1}$

$$\Rightarrow P(XY<1) = \boxed{\frac{1}{4} + \frac{\ln 2}{2}}$$

d) $\cancel{f(x \mid Y=1)}$

$$h(x) = \frac{1}{2}\int_x^2 dy = \frac{2-x}{2}$$

$$g(y) = \frac{1}{2}\int_0^y dx = \frac{y}{2}$$

$$f(x \mid Y=1) = \frac{\frac{1}{2}}{\frac{1}{2}} = 1 \ , \quad 0<x<1$$

$$f(y \mid X=1) = \frac{1/2}{1/2} = 1 \ , \quad 0<y<1$$

$$E(X \mid Y=1) = \int_0^1 x\,dx = \boxed{\frac{1}{2}}$$

$$E(Y \mid X=1) = \int_0^1 y\,dy = \boxed{\frac{1}{2}}$$

e) $\sigma_Z^2 = 4\sigma_X^2 + \sigma_Y^2 - 4\sigma_{X,Y}$

$$\sigma_{X,Y} = E(XY) - \mu_X\mu_Y$$

$$E(XY) = \frac{1}{2}\int_0^2\int_x^2 xy\,dy\,dx = \frac{1}{2}\int_0^2 \left(\left(\frac{y^2}{2}\Big|_x^2\right)x\,dx\right)$$

$$= \frac{1}{4}\int_0^2 (4 - x^2)x\,dx = \frac{1}{4}\int_0^2 (4x - x^3)\,dx$$

$$= \frac{1}{4}\left(2x^2 - \frac{x^4}{4}\right)\Big|_0^2 = \frac{1}{4}(8-4) = 1$$

$$\Rightarrow \sigma_{X,Y} = 1 - \left(\frac{2}{3}\right)\left(\frac{4}{3}\right) = 1 - \frac{8}{9}$$

c) ~~Gxp~~

$$\sigma^2_Z = \frac{8}{9} + \frac{2}{9} - 4\left(1 - \frac{8}{9}\right)$$

$$= \frac{8}{9} + \frac{2}{9} - \frac{4}{9} = \frac{6}{9} = \boxed{\frac{2}{3}}$$

3.) $p(x) = \begin{cases} \dfrac{9-x}{36}, & x = 1, \ldots, 8 \\ 0, & \text{otherwise} \end{cases}$
