Extra Problems

1. For which values of a and b does the following system has no solution exactly one or infinitely many solutions:

$$\begin{cases} ax + z = 2 \\ ax + ay + 4z = -4 \\ (a-1)y + 2z = b \end{cases}$$

2. For which value(s) of a is the set $S = \{ax^3, x^3 + (2a-1)x^2, a^2, x^2 + (a-4)x\}$ linearly independent.

3. a- Find matrices A, B, C such that $AC = BC$ with $A \neq B$.
b-Determine all 2×2 matrices A such that $A^2 = 0$.

4. For which value(s) of λ the reduced row echelon form of B is I.

$$B = \begin{pmatrix} 1 & 1 & \lambda & 2 \\ 1 & \lambda & 1 & -1 \\ \lambda & 1 & 1 & 0 \\ 2\lambda - 1 & 2 - \lambda & 1 & 0 \end{pmatrix}$$

5. If $(I + (2A)^{-1})^{-1} = \begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix}$. Find A.

6. Let A be a square matrix such that : $A^3 + 4AA^T - 2A - 7I = 0$. Find an expression for A^{-1}.

7. Let $J = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Find J^2 and J^3. Deduce J^n.

8. Let J be the $n \times n$ matrix each of which entries is 1. Show that if $n > 1$ then:

$$(I - J)^{-1} = I - \frac{1}{n-1} J$$

9. True/False (prove or explain if true, give a counter-example if false)
a- If $A^T A$ is invertible so is A.
b- If $A^4 = 0$ then $I - A$ is invertible.
c- If A is invertible then $Ax = x$ has exactly one solution.
d. If $\{v_1, v_2, \dots v_r\}$ is a linearly dependent set then $\{v_1, v_2, \dots v_r, v_{r+1} \dots v_n\}$ is also linearly dependent.