LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Business

Sample Final Exam

QBA 201

Managerial Statistics

1. (10 Points) Grades of students in Managerial Statistics course are normally distributed with mean of 80 points and standard deviation of 16 points.
   a) One student is selected at random. What is the probability that this student scored between 40 and 100 points?
   b) What is the grade G above which 93.7% of the students score?

2. (10 Points) The average annual salary in Lebanon was $12200. Assume that the salaries were normally distributed with standard deviation of $2500.
   a) Find the probability that, for a randomly selected sample of 25 individuals, the mean salary was less than $13000.
   b) Find the probability that, for a randomly selected sample of 16 individuals, the total salaries was more than $180000.

3. (10 Points) A survey found that out of 200 workers, 170 said they were interrupted three or more times an hour by phone messages. Find the 80% confidence interval of the population proportion of workers who are interrupted three or more times an hour.

4. (15 Points) The number of cars sold annually by used car salespeople is normally distributed. A random sample of 10 salespeople was taken, and the number of cars each sold is listed here. Find 99% confidence interval estimate of the population mean.

   79 43 58 66 101 63 79 33 58 70

5. (15 Points) A study of 25 statistical professors showed that they spent, on average, 9.5 minutes correcting a student's test.
   a) Find the point estimate of the mean
   b) Find 98% confidence interval of the mean time if the population standard deviation is 2.6 minutes. Interpret.

6. (10 Points) A researcher is interested in estimating the average salary of teachers in a large school. She wants to be 95% confident that her estimate is correct. If the standard deviation is \$1000, how large a sample is needed to be accurate within \$200?

7. (15 Points) A researcher claims that the coffee drinker consumes an average of 3.1 cups per day. A sample of 9 drinkers revealed they consumed the following amounts of coffee, reported in cups.

   3.1 3.3 3.5 2.6 4.3 3.7 3.8 3.1 4.1

   If the population standard deviation is 0.54, test the researcher's claim using a 0.01 significance level.

8. (15 Points) A restaurant chain claims that the mean waiting time of customers is less than 3 minutes. While examining the waiting time, the manager found, in a random sample of 41 customers, that the sample average is 2.7 minutes and the sample standard deviation is 1.2 minute. At the 0.1 significance level, test the claim.

Key Formulas

Population mean: $= \frac{\sum X}{N}$; Sample mean: $\bar{X} = \frac{\sum X}{n}$

Population standard deviation: $\sigma = \sqrt{\frac{\sum(X-\mu)^2}{N}}$

Sample standard deviation: $s = \sqrt{\frac{\sum(X-\bar{X})^2}{(n-1)}}$

Or use equivalently:

$$s = \sqrt{\frac{n\sum X^2 - (\sum X)^2}{n(n-1)}}$$

Standard normal value: $Z = \frac{X-\mu}{\sigma}$

Z-value of $\bar{X}$, when μ and σ known: $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

Confidence interval for μ, with σ known:

$$\bar{X} - z\,\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z\,\frac{\sigma}{\sqrt{n}}$$

Confidence interval for μ, with σ unknown:

$$\bar{X} - t\,\frac{s}{\sqrt{n}} \le \mu \le \bar{X} + t\,\frac{s}{\sqrt{n}}$$

Sample proportion: $p = \frac{X}{n}$

Confidence interval for proportion:

$$p - z\sqrt{\frac{p(1-p)}{n}} \le p \le p + z\sqrt{\frac{p(1-p)}{n}}$$

Sample size for estimating mean: $n = \left(\frac{z\sigma}{E}\right)^2$

Sample size for proportion: $n = p(1-p)\left(\frac{z}{E}\right)^2$

Testing a mean, σ known: $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

Testing a mean, σ unknown:

$$t = \frac{\bar{X}-\mu}{s/\sqrt{n}}$$

Variance of the distribution of difference in means:

$$\sigma^2_{\bar{x}_1-\bar{x}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Two-sample test of means, known σ: $Z = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$

Coefficient of correlation: $r = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{(n-1)S_x S_y}$,

or use equivalently:

$$r = \frac{n\left(\sum xy\right)-\left(\sum x\right)\left(\sum y\right)}{\left[\sqrt{n\left(\sum x^2\right)-\left(\sum x\right)^2}\right]\cdot\left[\sqrt{n\left(\sum y^2\right)-\left(\sum y\right)^2}\right]}$$

Linear regression equation: $\hat{Y} = a + bX$

Slope of the regression line: $b = r\,\frac{S_y}{S_x}$

or use equivalently: $b = \frac{n\left(\sum xy\right)-\left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right)-\left(\sum x\right)^2}$

Intercept of the regression line: $a = \bar{Y} - b\bar{X}$

1. X = grade in QBA 201

normally distributed, $\mu = 80$ $\sigma = 16$

$$Z = \frac{X - \mu}{\sigma}$$

a) $P(40 \leq X \leq 100)$

$$= P\left(\frac{40-80}{16} \leq Z \leq \frac{100-80}{16}\right)$$

$$= P(-2.50 \leq Z \leq 1.25)$$

$$= P(Z \leq 1.25) - P(Z \leq -2.50)$$

$$= 0.8944 - 0.0620$$

$$= \underline{0.8324}$$

d.f	0.00	0.05
1.2		0.8944
-2.5	0.0620	

b) G = grade above which 93.7% of the students score.

$$P(X > G) = 93.7\%$$

$$P\left(Z > \frac{G-80}{16}\right) = 0.9370$$

$$P\left(Z \leq \frac{G-80}{16}\right) = 1 - 0.9370$$

$$P\left(Z \leq \frac{G-80}{16}\right) = 0.0630$$

$$\frac{G-80}{16} = -1.53$$

d.f.	.03
-1.5	$\boxed{0.0630}$

$$G - 80 = -1.53(16)$$

$$G = 80 - 1.53(16)$$

$$\underline{G = 55.52}$$

2. X = annual salary in Lebanon.
Normally distributed

$\mu = 12200 \qquad \sigma = 2500$

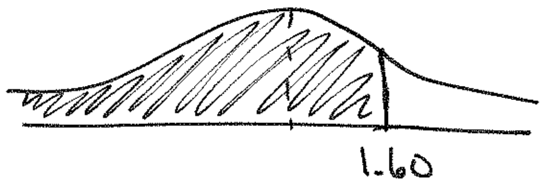
a) $n = 25$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

d.f.	0.00
1.6	→ 0.945

$P(\bar{x} < 13000)$

$= P\left(z < \frac{13000 - 12{,}200}{2500/\sqrt{25}}\right) \Rightarrow P(z < 1.60) = 0.945$



b) $n = 16 \quad \text{Total} = \Sigma x \qquad \bar{x} = \frac{\Sigma x}{n}$

$P(\Sigma x > 180{,}000)$

$= P\left(\frac{\Sigma x}{n} > \frac{180{,}000}{16}\right)$

$= P(\bar{x} > 11{,}250)$

$= P\left(z > \frac{11{,}250 - 12{,}200}{2500/\sqrt{16}}\right)$

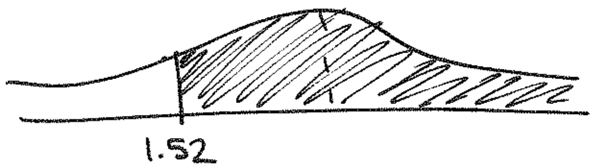
$= P(z > -1.52)$

$= 1 - P(z \leq -1.52)$

$= 1 - 0.0643$

$= \underline{0.9357}$

d.f.	.02
-1.5	→ 0.0643



3. C.I. for population proportion

$n = 200 \quad X = 170 \quad 80\% \Rightarrow 1-\alpha = 0.80$

$\Rightarrow \alpha = 0.2$

$\Rightarrow \alpha/2 = 0.1$

$\alpha/2 = 0.1$, d.f. $= \infty$

d.f.	$t_{0.1}$
∞	1.282

$Z_{\alpha/2} = Z_{0.1} = 1.282$

C.I. for P is (proportion):

$$\hat{P} - Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} \le P \le \hat{P} + Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{\sqrt{n}}}$$

$$0.85 - 1.282\sqrt{\frac{(0.85)(0.15)}{200}} \le P \le 0.85 + 1.282\sqrt{\frac{(0.85)(0.15)}{200}}$$

$$0.8176 \le P \le 0.8823$$

According to the sample data, we are 80% confident that the proportion of workers that were interrupted by phone messages is between 81.76% and 88.23%.

4. $n=10 \quad \bar{x}=65 \quad S=19.22 \quad \sigma=\text{unknown} \Rightarrow \text{case 2}$

Confidence Interval for μ is:

$$\bar{x}-t_{\alpha/2}\left(S/\sqrt{n}\right) \leq \mu \leq \bar{x}+t_{\alpha/2}\left(S/\sqrt{n}\right)$$

$99\% \Rightarrow 1-\alpha=0.99 \quad \alpha=0.01 \quad \alpha/2=0.005$

$t_{\alpha/2}=t_{0.005}$, degree of freedom $=n-1=9$

d.f	$t_{0.05}$
9	3.250

99% C.I. for μ is:

$$65-3.25\left(\frac{19.22}{\sqrt{10}}\right) \leq \mu \leq 65+3.25\left(\frac{19.22}{\sqrt{10}}\right)$$

$$45.25 \leq \mu \leq 84.75$$

According to our sample data, we are 99% confident that the avg. number of cars sold is between 45.25 and 84.75.

5. $n = 25 \quad \bar{x} = 9.5$

a) Point Estimate for $\mu \Rightarrow \bar{x} = 9.5$

b) 98% confidence interval $\qquad \sigma = 2.6 \Rightarrow \sigma$ is known (case 1)

$\Rightarrow 1 - \alpha = 0.98$

$\Rightarrow \alpha = 0.02$

$\alpha/2 = 0.01$

$z_{\alpha/2} = z_{0.01} = 2.326$
(d.f. $= \infty$)

d.f.	$t_{0.01}$
∞	2.326

The 98% C.I. (confidence interval) for μ (case 1) is:

$$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$9.5 - 2.326\left(\frac{2.6}{\sqrt{25}}\right) \le \mu \le 9.5 + 2.326\left(\frac{2.6}{\sqrt{25}}\right)$$

$$8.29 \le \mu \le 10.71$$

According to the sample data, we are 98% confident that the average time a statistics professor spends correcting an exam is between 8.29 and 10.71 minutes.

6. $n = ?$ 95% confident $\Rightarrow z_{\alpha/2}$

$\sigma = \$1000$ $E = \$200$ (keyword: <u>within</u>)

$95\% \Rightarrow 1-\alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$z_{\alpha/2} = z_{0.025} = 1.96$

d.f. $= \infty$

d.f.	$t_{0.025}$
∞	1.96

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{(1.96)(1000)}{(200)} \right)^2 = 96.04$$

$\simeq 97$ (always round up)

7. Claim: $\mu = 3.1$

$n = 9 \quad \bar{x} = 3.5 \quad \sigma = 0.54 \Rightarrow \sigma$ is known $\Rightarrow$ z-test

Step1: $H_0: \mu = 3.1$ (claim)

$H_1: \mu \neq 3.1$

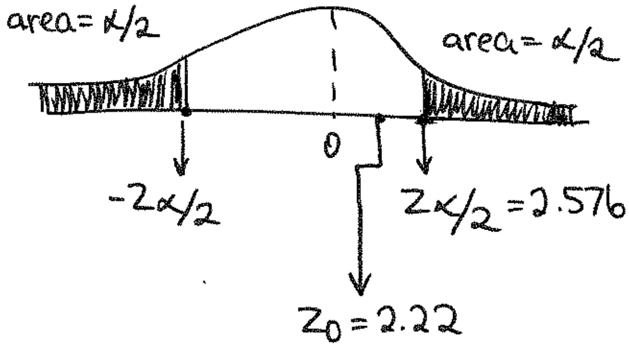
two-tailed test

Step2: $\alpha = 0.01$

Step3: Test statistics for the z-test is:

$$Z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.5 - 3.1}{0.54/\sqrt{9}} = 2.22$$

Step4: Reject H_0 if $Z_0 < -Z_{\alpha/2}$ or $Z_0 > Z_{\alpha/2}$



$\alpha = 0.01$, z-test, degree of freedom $= \infty$

$\alpha/2 = 0.005 \quad Z_{\alpha/2} = Z_{0.005} = 2.576$

On table:

d.f.	$t_{0.005}$
∞	2.576

Step5: Since Z_0 does not fall in the rejection region, we do not reject H_0. According to the sample, we can conclude that the average coffee drinker drinks on avg. 3.1 cups/day

8. Claim: $\mu < 3$

$n = 41 \quad \bar{x} = 2.7 \quad S = 1.2 \quad \sigma = \text{unknown} \Rightarrow$ t-test

Step1: $H_0: \mu \geq 3$

$H_1: \mu < 3$ (claim)

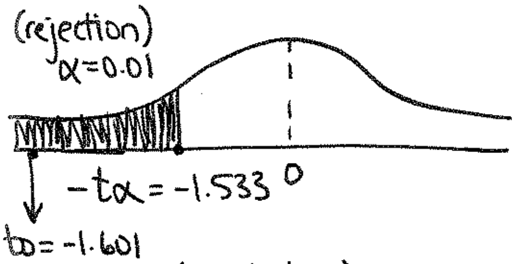
lower-tailed test

Step2: $\alpha = 0.01$

Step3: Test Statistics (t-test) is:

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{2.7 - 3}{1.2/\sqrt{41}} = -1.601$$

Step4: Reject H_0 if $t_0 < -t_\alpha$



$\alpha = 0.01$, t-test, degree of freedom $= n-1$
$= 41 - 1$
$= 40$

$t_\alpha = t_{0.1} = \underline{1.533}$

(on table)

d.f.	$t_{0.1}$
40	1.533

Step5: Since t_0 falls in the rejection region, we reject H_0. According to the sample data, we can conclude that the avg. waiting time is less than 3min.