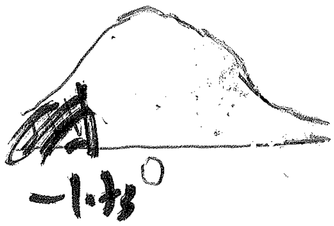
Final Exam

QBA 201 - Fall 2013-2014

1) X = size of computer chip (normal)

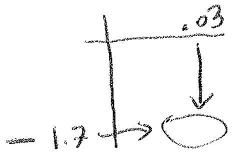
$\mu = 1$, $\sigma = 0.1$, $n = 12$

a) $P(\bar{X} < 0.95) = P\left(Z < \dfrac{0.95 - 1}{0.1/\sqrt{12}}\right)$



$= P(Z < 1.73)$

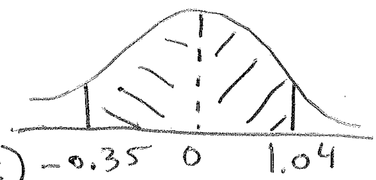
$= 0.0418$



b) $P(0.99 \leq X \leq 1.03)$

$= P\left(\dfrac{0.99 - 1}{0.1/\sqrt{12}} \leq Z \leq \dfrac{1.03 - 1}{0.1/\sqrt{12}}\right)$

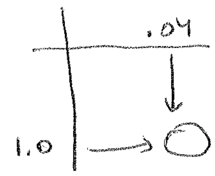
$= P(-0.35 \leq Z \leq 1.04)$

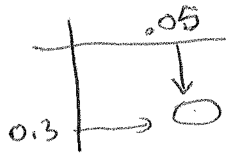


$= P(Z \leq 1.04) - P(Z \leq -0.35)$

$= 0.8508 - 0.3632$

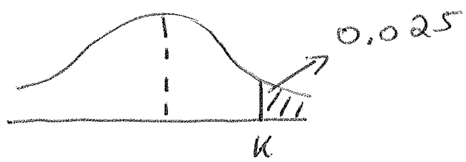
$= 0.4876$





c) $P(\bar{X} > W) = 2.5\%$

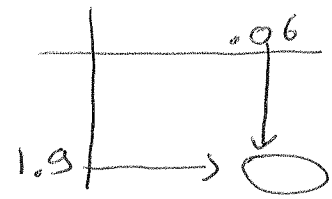
$P\left(Z > \dfrac{W-1}{\frac{0.1}{\sqrt{12}}}\right) = 0.025$



$P(Z > K) = 0.025$

$P(Z \leq K) = 1 - 0.025$

$P(Z \leq k) = 0.9750$



$K = 1.96$

$\dfrac{W-1}{0.1/\sqrt{12}} = 1.96$

$W = 1 + 1.96\left(\dfrac{0.1}{\sqrt{12}}\right)$

$W = 1.057$

2) μ = Avg. amount credit card customers spend.

$n = 15$, $\bar{X} = \$50.50$, $s^2 = 400$, $s = 20$

σ unknown $\Rightarrow$ case 2.

a) $(1-\alpha) \times 100\%$ C.I for μ is

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

95% C.I $\Rightarrow 1 - \alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

...

Continuation of 2)

a)... for $t_{\frac{\alpha}{2}}$, d.f $= n-1 = 15-1 = 14$

d.f	$t_{0.025}$
14	2.145

$$t_{\frac{\alpha}{2}} = t_{0.025} = 2.145$$

The 95% C.I is

$$50.50 - 2.145\left(\frac{20}{\sqrt{15}}\right) \le \mu \le 50.50 + 2.145\left(\frac{20}{\sqrt{15}}\right)$$

$$39.42 \le \mu \le 61.58$$

According to the sample data, we are 95% confident that the avg. amount of credit card customers spent is between \$39.42 and \$61.58.

b) Sample size for population mean is

$$n = \left(\frac{s \cdot z_{\frac{\alpha}{2}}}{E}\right)^2$$

$s = 20$, $E = \$3$

$$1-\alpha = 0.95 \Rightarrow \frac{\alpha}{2} = 0.025$$

for $z_{\frac{\alpha}{2}}$, d.f $= \infty$

	$t_{0.025}$
∞	1.96

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

$$n = \left(\frac{(20)(1.96)}{3}\right)^2 = 170.74 \simeq 171 \text{ (Round up)}$$

3) p = proportion of all students who use a PC

$n = 300$, $X = 225$

X = # of students in the sample out of 300 found to use PC.

sample population is $\hat{p} = \frac{225}{300} = 0.75 = 75\%$

The $(1-\alpha) \times 100\%$ C.I for p is

$$\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$90\% \Rightarrow 1 - \alpha = 0.9$

$\Rightarrow \alpha = 0.1$

$\Rightarrow \frac{\alpha}{2} = 0.05$

	$t_{0.05}$
∞	1.645

For $z_{\frac{\alpha}{2}}$, d.f $= \infty$

$$z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$$

The 90% C.I for p is

$$0.75 - 1.645 \sqrt{\frac{(.75)(.25)}{300}} \leq p \leq 0.75 + 1.645 \sqrt{\frac{(.75)(.25)}{300}}$$

$$0.7089 \leq p \leq 0.7911$$

According to the sample data, we are 90% confiden that the ~~true~~ true proportion of students who use PC is between 70.89% and 79.11%.

b) p = proportion of all students who do not use PC

$n = 300$, 225 use PC, $300 - 225 = 75$ do not.

Sample proportion $\hat{p} = \frac{75}{300} = 0.25$

90% C.I for P is

$$0.25 - 1.645\sqrt{\frac{(.25)(.75)}{300}} \le P \le 0.25 + 1.645\sqrt{\frac{(.25)(.}{300}}$$

$$0.2089 \le P \le 0.2911$$

4) Claim: $\mu > 11$

$n = 15$, $\bar{x} = 11.13$ ~~ounces (0.2)~~, $s = 0.15$

σ unknown.

$\alpha = 0.01$

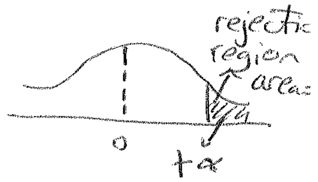
Step 1: $H_0: \mu \le 11$

$H_1: \mu > 11$ (claim)] Upper tailed t-test

Step 2: $\alpha = 0.01$

Step 3: Test statistics is

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{11.13 - 11}{0.15/\sqrt{15}} = 3.357$$



Step 4: Reject H_0 if $t_0 > t_\alpha$

For t_α, $d.f = n-1 = 15-1 = 14$

$t_\alpha = t_{0.01} = 2.624$

df	$t_{0.01}$
14	→ 2.624

Step 5: Since the test statistics falls in the rejection region, we reject H_0.

According to the sample data, we can conclude the claim that the mean weight of printer cartridges shipped is more than 11 ounces.

5) $n = 10$, $\bar{x} = 17$, $(s = 2.055)$ (6)

$\sigma = \$3$ (given) $\Rightarrow$ σ known.

a) $(1-\alpha)\times 100\%$ C.I for μ, (when σ is known), is

$$= \bar{x} - z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$$

95% C.I $\Rightarrow$ $1-\alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

For $z_{\alpha/2}$, d.f $= \infty$

	$t_{0.025}$
∞	1.96

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

The 95% C.I for μ is

$$17 - 1.96\left(\frac{3}{\sqrt{10}}\right) \le \mu \le 17 + 1.96\left(3/\sqrt{10}\right)$$

$$15.14 \le \mu \le 18.86$$

b) Claim $\mu \ne 15$

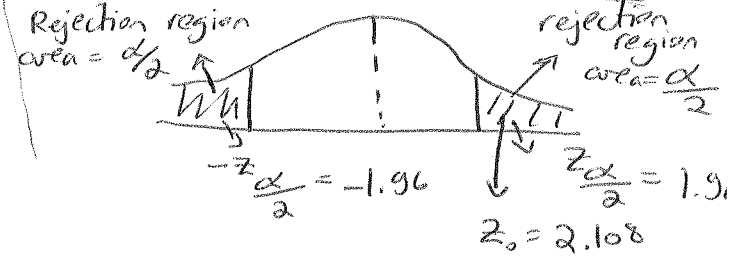
$n = 10$, $\bar{x} = 17$, $\sigma = 3$, $\alpha = 0.05$. σ is known $\Rightarrow$ z test.

Step 1: $H_0: \mu = 15$
$H_1: \mu \ne 15$ (claim)] two tailed z test

Step 2: $\alpha = 0.05$

Step 3: Test statistics is $z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{17-15}{3/\sqrt{10}} = 2.108$

Step 4: Reject H_0 if $z_0 < -z_{\frac{\alpha}{2}}$ or $z_0 > z_{\alpha/2}$.



For $z_{\alpha/2}$, d.f $= \infty$. $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$,

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

	$t_{0.025}$
∞	1.96

Step 5: Since the test statistics falls in the rejection region, we reje[ct] H_0.

According to the sample data, we can conclude the claim that the population avg. hourly earnings is different than \$15.

6) $n = 46$, $\bar{x} = \$25{,}000$, $s = \$2000$

Find 80% C.I for the avg. income.

σ unknown $\Rightarrow$ case 2

$$\bar{x} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$25000 - 1.301\left(\frac{2000}{\sqrt{46}}\right) \le \mu \le 2000 + 1.301\left(\frac{2000}{\sqrt{46}}\right)$$

$$\$24{,}616.36 \le \mu \le \$25{,}383.64$$

$t_{0.1}$

$45 \longrightarrow 1.301$

7)

	Model A	Model B	Available
steel	12 lbs/tub	10 lb/tub	240 lbs
zinc	2 lb/tub	3 lb/tub	48 lbs
Profit	\$90/tub	\$70/tub	

a) x_1 = # of tubs in Model A produced

x_2 = # of " " B "

Max $Z = 90x_1 + 70x_2$

subject to $12x_1 + 10x_2 \le 240$

$2x_1 + 3x_2 \le 48$

$x_1, x_2 \ge 0$

b) $12x_1 + 10x_2 = 240$

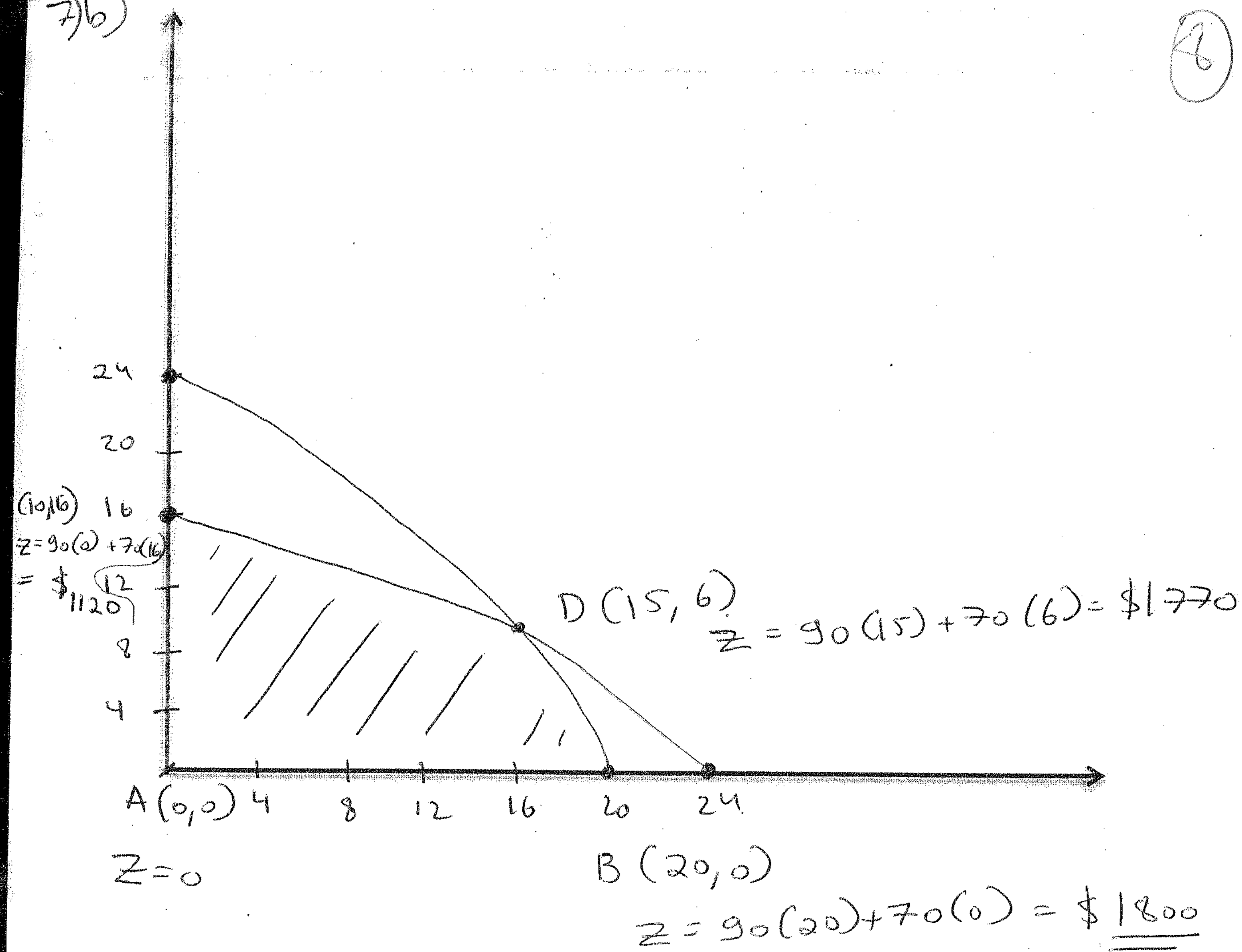
x_1	x_2
20	0
0	24

$2x_1 + 3x_2 = 48$

x_1	x_2
24	0
0	16

$$\left.\begin{aligned} 12x_1 + 10x_2 &= 240 \\ 2x_1 + 3x_2 &= 48 \end{aligned}\right\} \Rightarrow x_1 = 15,\ x_2 = 6$$

7)b)



The optimal solution is
$x_1 = 20$, $x_2 = 0$
maximum value is $Z = \$1800$

7)(c) Produce 20 tubs of model A
none of model B,
to realize the maximum profit is $1770