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Lebanese American University

CHAPTER 30

Induction and Inductance

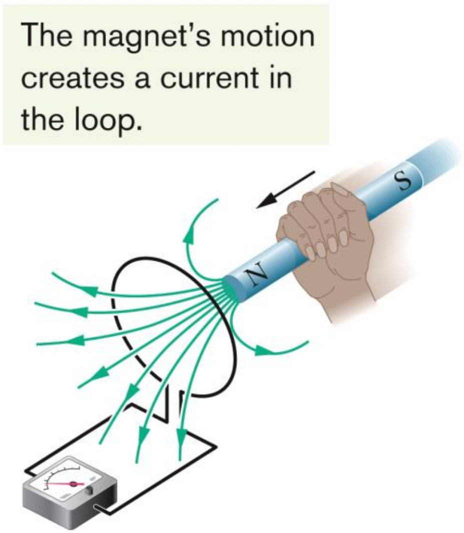
30-1

Faraday's Law and Lenz's Law

$$\text{Current} \;\underset{\text{Chapter 30}}{\overset{\text{Chapter 29}}{\rightleftarrows}}\; \text{Magnetic Field}$$

30-1 Faraday's Law and Lenz's Law

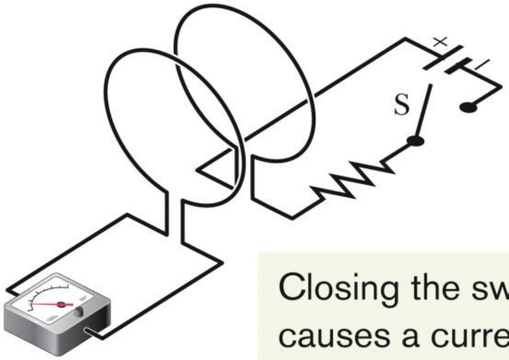
First Experiment. Figure shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit. The current disappears when the magnet stops moving. If we then move the magnet away, a current again suddenly appears, but now in the opposite direction. If we experimented for a while, we would discover the following:



1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion of the magnet produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions from the north pole effects.

30-1 Faraday's Law and Lenz's Law

Second Experiment. For this experiment we use the apparatus shown in the figure, with the two conducting loops close to each other but not touching. If we close switch S to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If the switch remains closed, no further current is observed. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction.



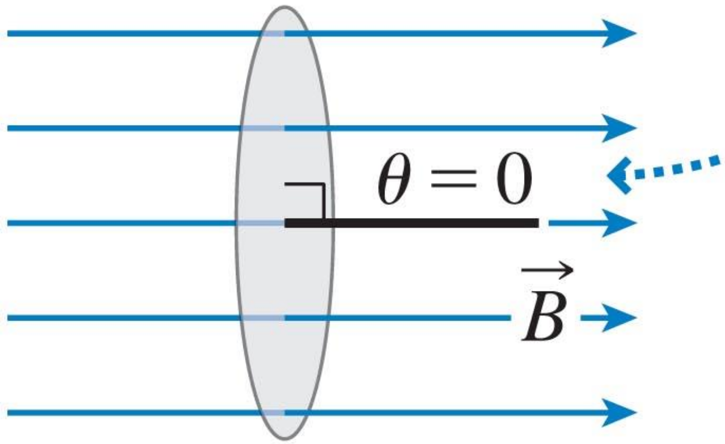
Closing the switch causes a current in the left-hand loop.

We get an induced current (from an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large). The induced emf and induced current in these experiments are apparently caused when something changes — but what is that "something"? Faraday knew.

Magnetic flux

- The current is actually induced by a change in the quantity called the *magnetic flux* rather than simply by a change in the magnetic field.
- Magnetic flux is proportional to both the strength of the magnetic field passing through the plane of a loop of wire and the area of the loop.

$$\Phi = BA\cos\theta \qquad \text{(Magnetic flux in a uniform field; SI unit: Wb)}$$

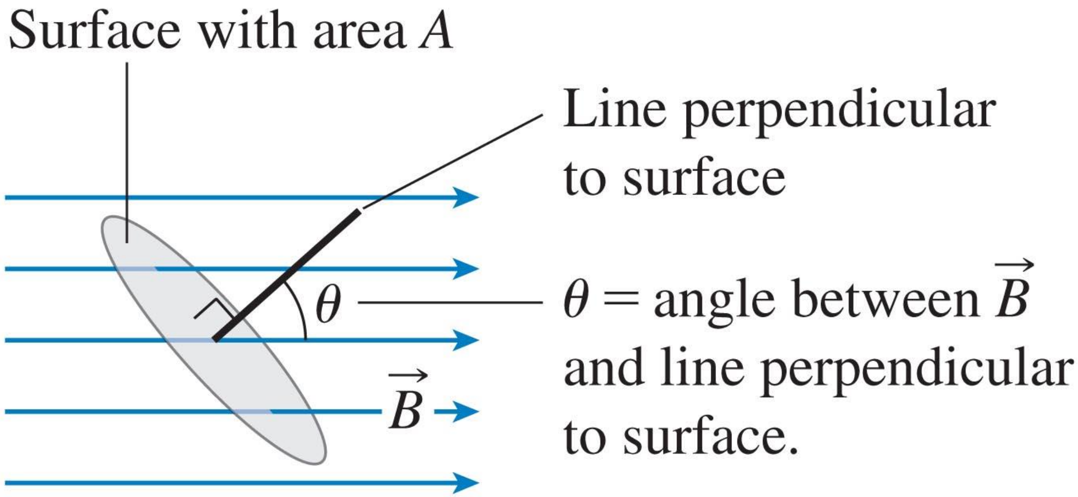


Flux Φ is greatest when surface is perpendicular to magnetic field. Then

$\theta = 0$ and $\cos\theta = 1$,
so $\Phi = BA$.

Induction and Faraday law

You are given a loop of wire. The wire is in a uniform magnetic field . The loop has an area A.



$$\Phi = BA\cos\theta \qquad \text{(Magnetic flux in a uniform field; SI unit: Wb)}$$

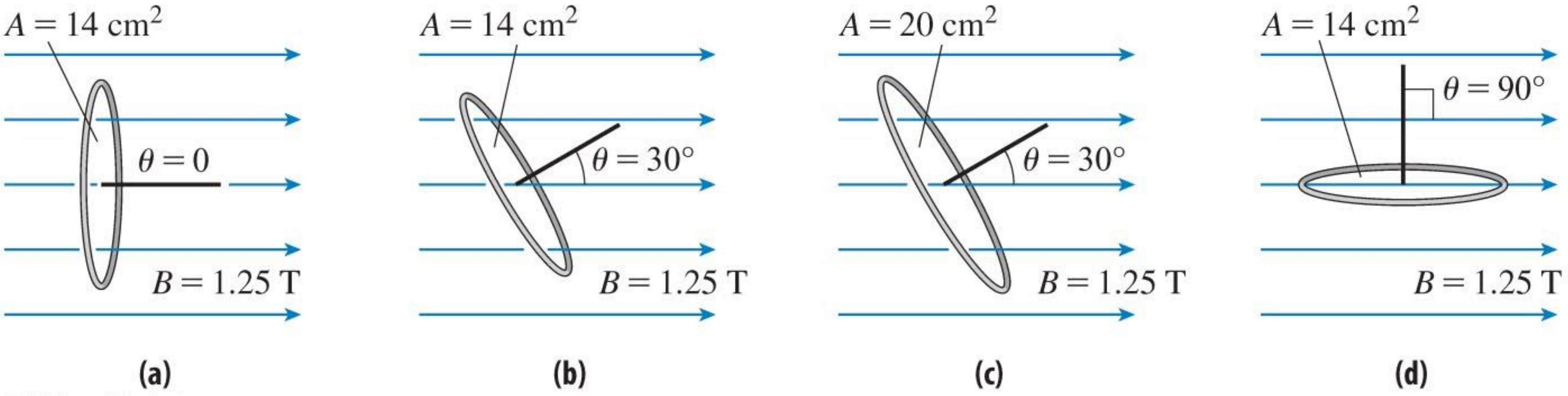
(θ is the angle between B and the normal to the plane)

For $B = 0.40$ T, $A = 0.10\ m^2$, and $\theta = 60°$; we find:

$\Phi = 0.20$ Wb; with 1 Wb = 1 $T.m^2$.

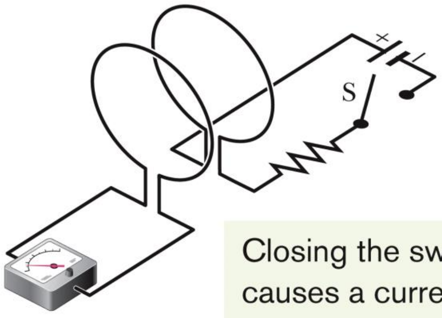
Induction and Faraday law

Rank the magnetic flux in decreasing order



30-1 Faraday’s Law and Lenz’s Law

Faraday’s Law of Induction



Closing the switch causes a current in the left-hand loop.

Faraday realized that an emf and a current can be induced in a loop, as in our two experiments, by changing the amount of magnetic field passing through the loop. He further realized that the “amount of magnetic field” can be visualized in terms of the magnetic field lines passing through the loop.

The magnetic flux Φ_B through an area *A* in a magnetic field ***B*** is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where *1 Wb = 1 T • m*^{2}.

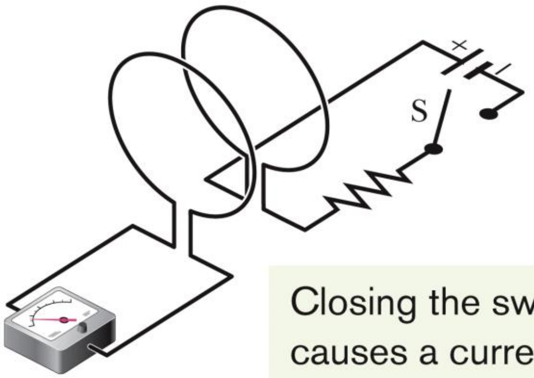
If ***B*** is perpendicular to the area and uniform over it, the flux is

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

30-1 Faraday’s Law and Lenz’s Law

Faraday’s Law of Induction

> The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.



Closing the switch causes a current in the left-hand loop.

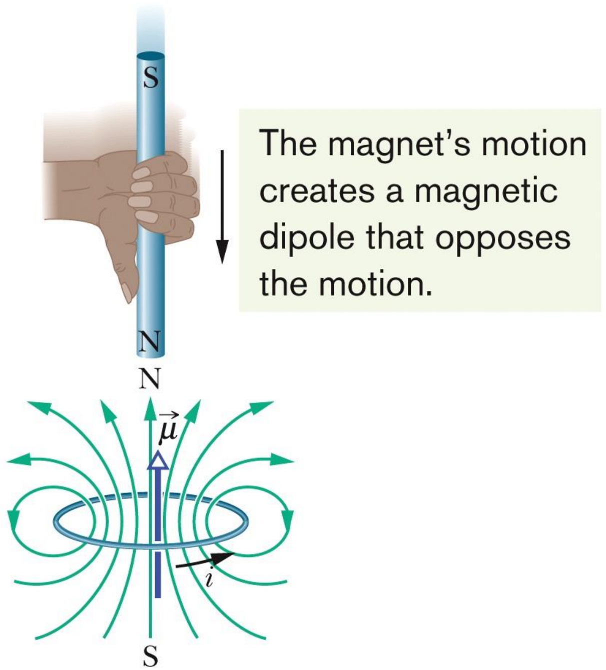
Faraday’s Law. With the notion of magnetic flux, we can state Faraday’s law in a more quantitative and useful way:

$$\mathscr{E} = -\frac{d\Phi_B}{dt}$$

the induced emf tends to oppose the flux change and the minus sign indicates this opposition. This minus sign is referred to as Lenz’s Law.

Lenz's Law

An induced current has a direction such that the magnetic field due to this induced current opposes the <u>change</u> in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.



Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment **μ** oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

Induction and Faraday law

$\Phi_B \nearrow \Leftrightarrow$ B and B_{in} have opposite directions

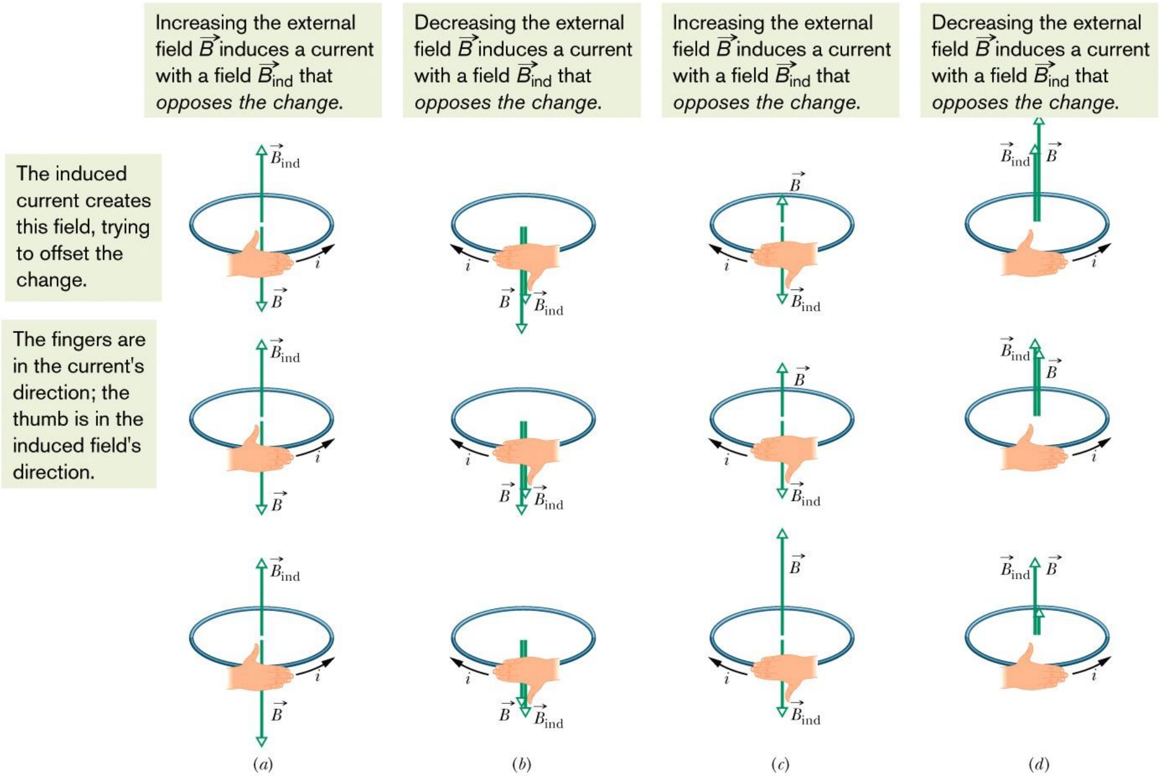
$\Phi_B \swarrow \Leftrightarrow$ B and B_{in} have same direction

When applying Lenz' Law, there are *two* magnetic fields to consider:

- B: The external changing magnetic field
- B induced: The magnetic field produced by the current in the loop.

30-1 Faraday's Law and Lenz's Law

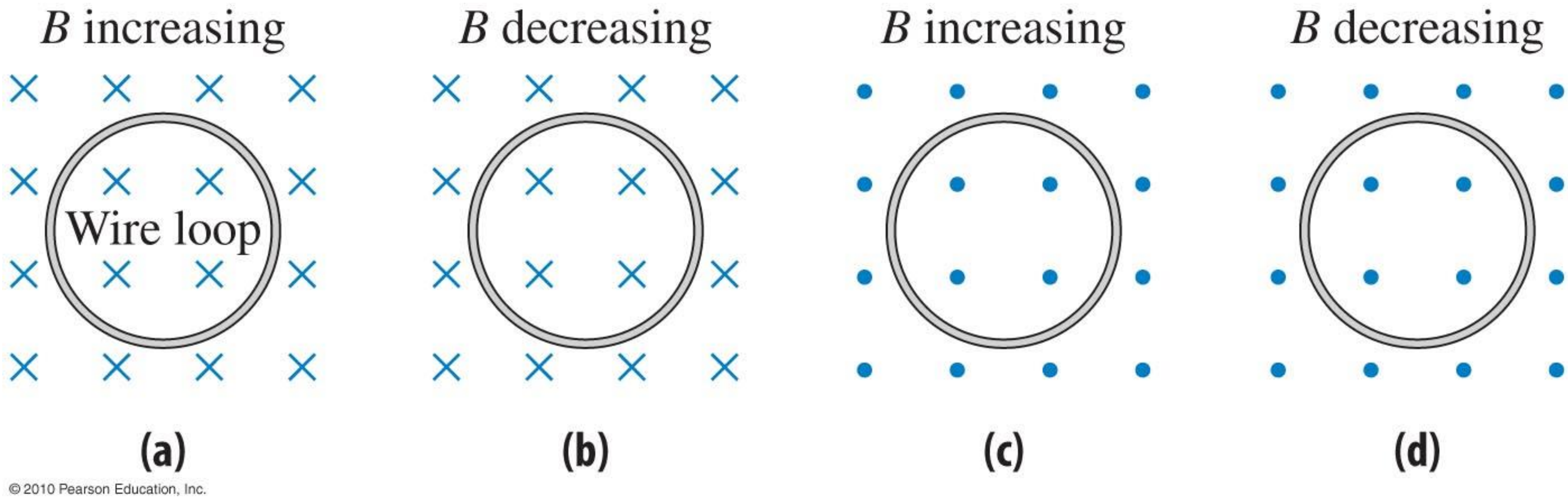
Lenz's Law



The direction of the current *i* induced in a loop is such that the current's magnetic field $\boldsymbol{B}_{ind}$ opposes the change in the magnetic field $\boldsymbol{B}$ inducing *i*. The field $\boldsymbol{B}_{ind}$ is always directed opposite an increasing field $\boldsymbol{B}$ (a, c) and in the same direction as a decreasing field $\boldsymbol{B}$ (b, d). The curled – straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

Induction and Faraday law

Find the direction of the induced current in each.

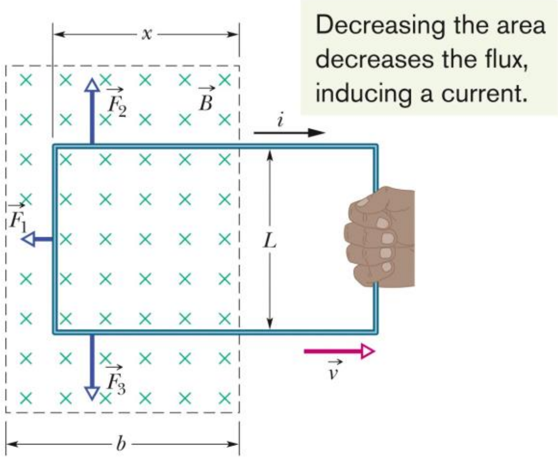


30-2

Induction and Energy Transfer

30-2 Induction and Energy Transfer

In the figure, a rectangular loop of wire of width *L* has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in the figure show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity ***v***.



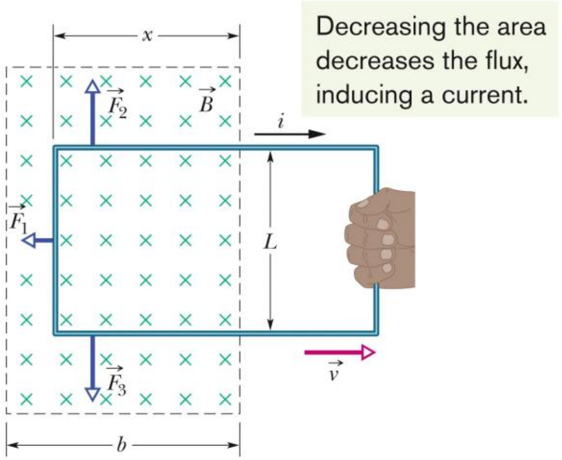
Flux change: Therefore, in the figure a magnetic field and a conducting loop are in relative motion at speed *v* and the flux of the field through the loop is changing with time (here the flux is changing as the area of the loop still in the magnetic field is changing).

Rate of Work: To pull the loop at a constant velocity ***v***, you must apply a constant force ***F*** to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you. The rate at which you do work — that is, the power — is then

$$P = Fv,$$

where F is the magnitude of the force you apply to the loop.

30-2 Induction and Energy Transfer



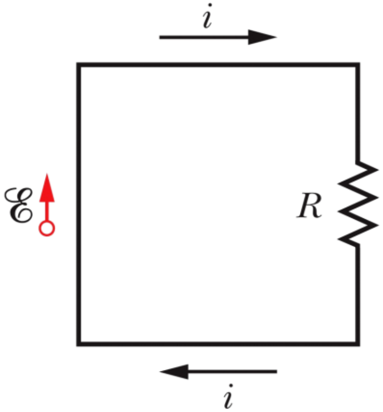
Induced emf: To find the current, we first apply Faraday's law. When *x* is the length of the loop still in the magnetic field, the area of the loop still in the field is *Lx*. Then, the magnitude of the flux through the loop is

$$\Phi_B = BA = BLx.$$

As *x* decreases, the flux decreases. Faraday's law tells us that with this flux *i* decrease, an *emf* is induced in the loop. We can write the magnitude of this emf as

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BL\frac{dx}{dt} = BLv,$$

in which we have replaced *dx/dt* with *v*, the speed at which the loop moves.



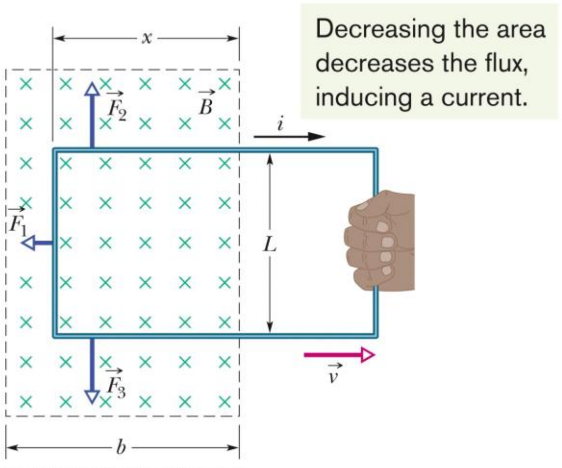
A circuit diagram for the loop of above figure while the loop is moving.

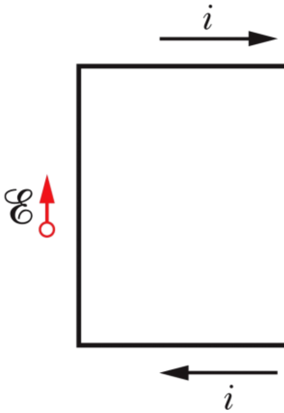
30-2 Induction and Energy Transfer

Induced Current: Figure (bottom) shows the loop as a circuit: induced *emf* is represented on the left, and the collective resistance R of the loop is represented on the right. To find the magnitude of the induced current, we can apply the equation $i = \mathscr{E}/R$, which gives

$$i = \frac{BLv}{R}.$$

In the Fig. (top), the deflecting forces acting on the three segments of the loop are marked $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$. Note, however, that from the symmetry, forces $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ are equal in magnitude and cancel. This leaves only force $\boldsymbol{F_1}$, which is directed opposite your force $\boldsymbol{F}$ on the loop and thus is the force opposing you.





A circuit diagram for the loop of above figure while the loop is moving.

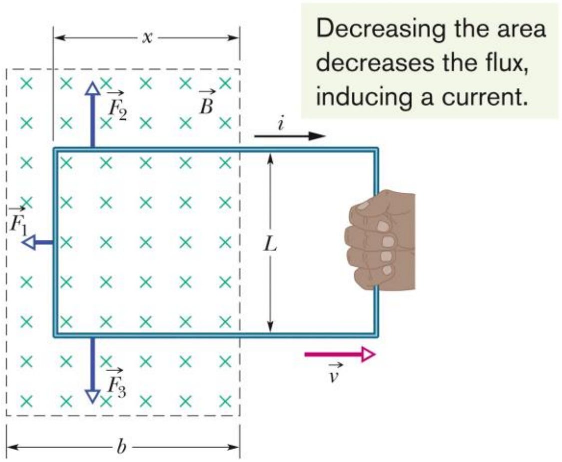
30-2 Induction and Energy Transfer

So, $\boldsymbol{F} = -\boldsymbol{F_1}$. the magnitude of $\boldsymbol{F_1}$ thus

$$\vec{F}_d = i\vec{L} \times \vec{B}.$$

$$F = F_1 = iLB \sin 90^\circ = iLB.$$

where the angle between $\boldsymbol{B}$ and the length vector $\boldsymbol{L}$ for the left segment is 90°. This gives us



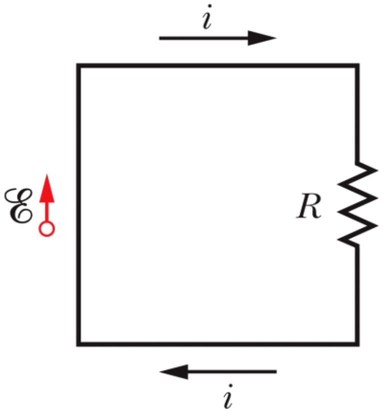
$$F = \frac{B^2L^2v}{R}.$$

Because B, L, and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

Rate of Work: We find the rate at which you do work on the loop as you pull it from the magnetic field:

$$P = Fv = \frac{B^2L^2v^2}{R}$$

NOTE: The work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop.



A circuit diagram for the loop of above figure while the loop is moving.

30-4

Inductors and Inductance

30-4 Inductors and Inductance

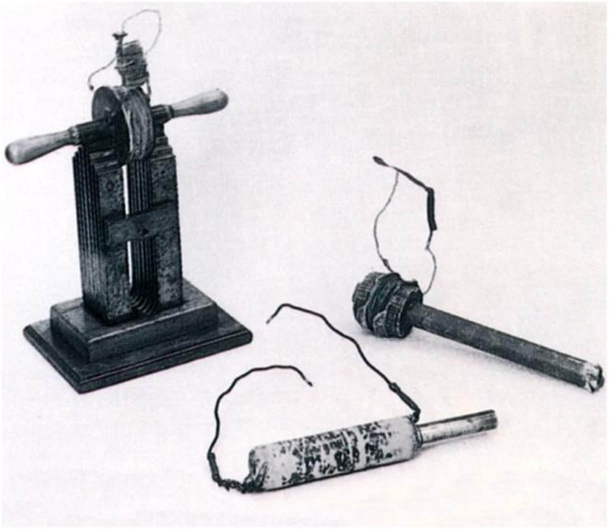
An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current *i* is established through each of the *N* windings of an inductor, a magnetic flux Φ_B links those windings. The inductance *L* of the inductor is

$$L = \frac{N\Phi_B}{i}$$

The SI unit of inductance is the *henry (H)*, where *1 henry = 1H=1T•m²/A*.

The inductance per unit length near the middle of a long solenoid of cross-sectional area *A* and *n* turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$



The Royal Institution/Bridgeman Art Library/NY

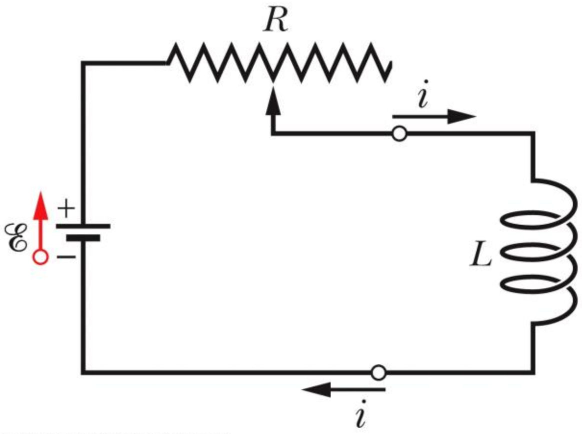
The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats.

30-5

Self-Induction

30-5 Self-Induction

If two coils — which we can now call inductors — are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced *emf* appears in the second coil according to Faraday's law. An induced *emf* appears in the first coil as well. This process (see Figure) is called self-induction, and the *emf* that appears is called a



self-induced *emf*. It obeys Faraday's law of induction just as other induced emfs do.
For any inductor,

$$N\Phi_B = Li.$$

Faraday's law tells us that

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

By combining these equations, we can write

$$\mathscr{E}_L = -L\frac{di}{dt} \quad \text{(self-induced emf).}$$

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

Note: Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced *emf* appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced *emf*; only the rate of change of the current counts.

30-6

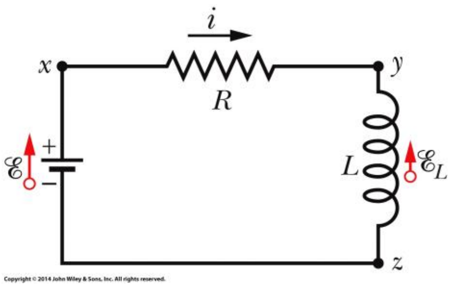
RL Circuits

A first order non-homogeneous differential equation: $a\frac{di(t)}{dt} + bi(t) + c = 0$ where a,b,c are constants	
General solution: $\mathrm{i}(t) = Ke^{-\frac{b}{a}t} + \frac{c}{b}[e^{-\frac{b}{a}t} - 1]$	
$i(t) = \frac{\mathcal{E}}{R}\,[1 - \ e^{-\frac{R}{L}t}]$	$\mathrm{i}(t) = \frac{\mathcal{E}}{R}\ e^{-\frac{R}{L}t}$

30-6 RL Circuits

If a constant emf $\mathscr{E}$ is introduced into a single-loop circuit containing a resistance *R* and an inductance *L*, the current rises to an equilibrium value of $\mathscr{E}$/R according to

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L})$$

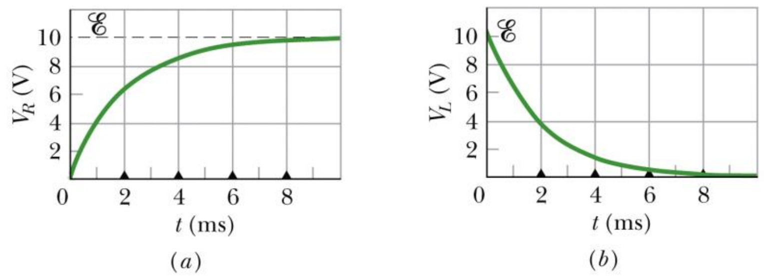


An RL circuit.

Here τ_L, the **inductive time constant**, is given by

$$\tau_L = \frac{L}{R}$$

The resistor's potential difference turns on.
The inductor's potential difference turns off.



The variation with time of (a) V_R, the potential difference across the resistor in the circuit (top), and (b) V_L, the potential difference across the inductor in that circuit.

Plot (a) and (b) shows how the potential differences V_R (= *iR*) across the resistor and V_L (= *L di/dt*) across the inductor vary with time for particular values of $\mathscr{E}$, *L*, and *R*.

When the source of constant *emf* is removed and replaced by a conductor, the **current decays** from a value i_0 according to

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

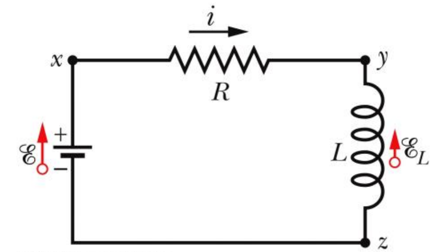
30-7

Energy Stored in a Magnetic Field

30-7 Energy Stored in a Magnetic Field

If an inductor L carries a current i, the inductor's magnetic field stores an energy given by

$$U_B = \tfrac{1}{2}Li^2$$

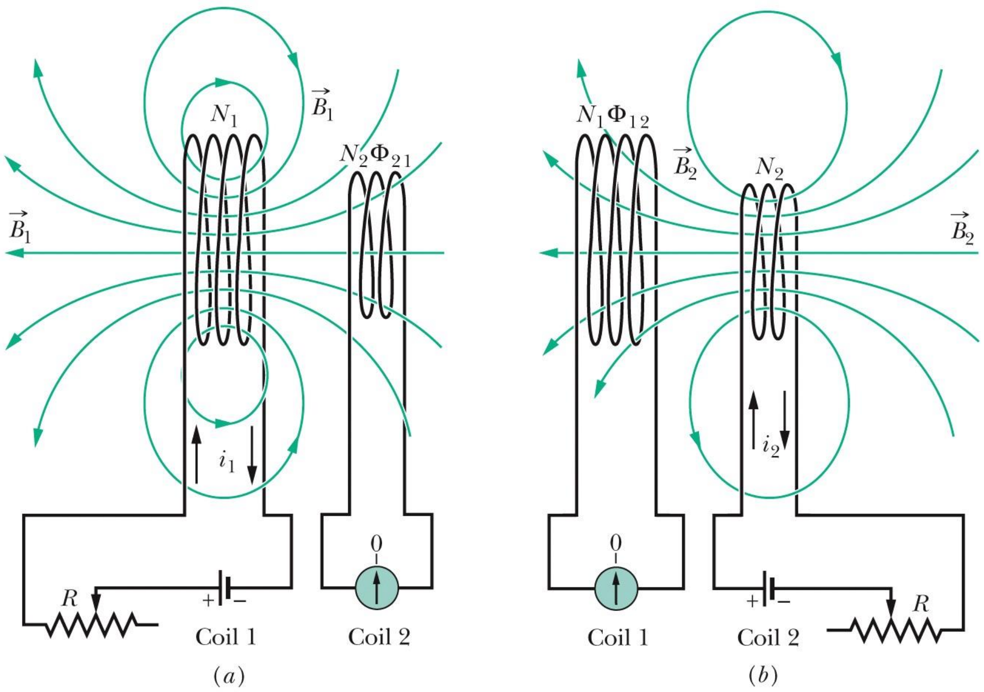


An RL circuit.

30-8

Mutual inductance

30-8 Mutual Induction



Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

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CHAPTER 31

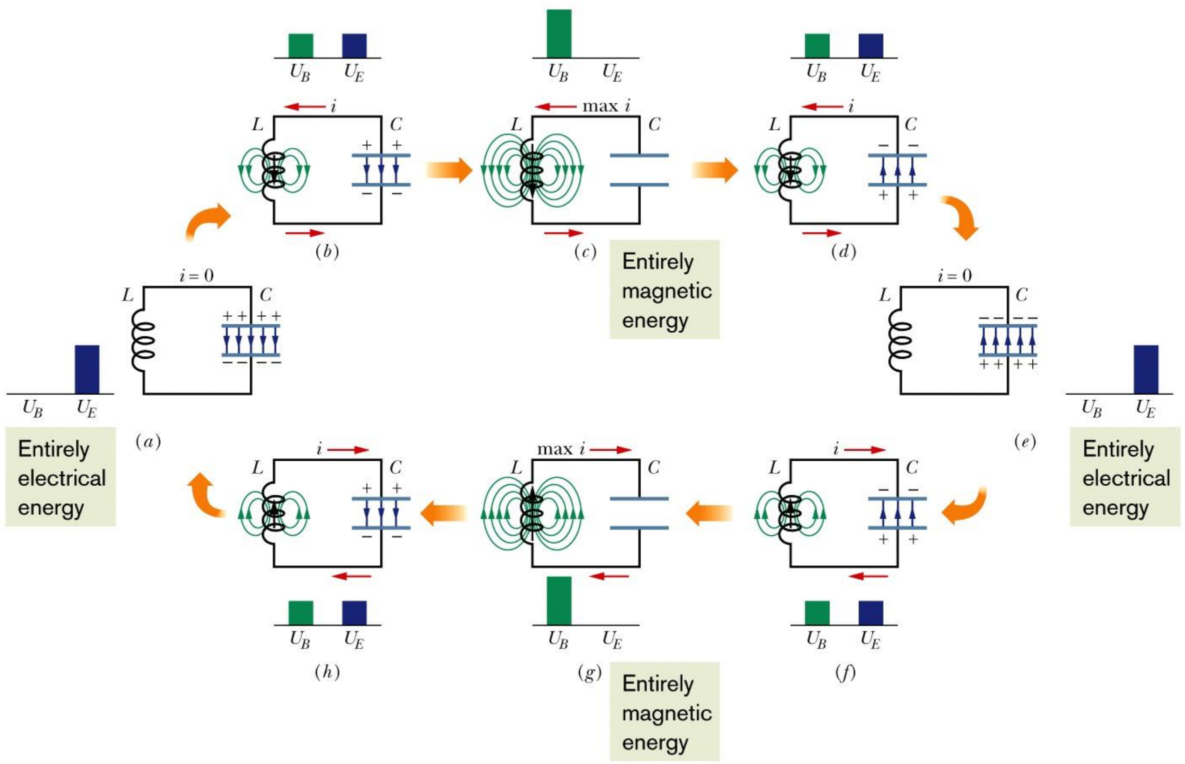
Electromagnetic Oscillations and Alternating Current

31-1

Electromagnetic Oscillations

31-1 Electromagnetic Oscillations

Eight stages in a single cycle of oscillation of a resistance less LC circuit. The bar graphs by each figure show the stored magnetic and electrical energies. The magnetic field lines of the inductor and the electric field lines of the capacitor are shown. (a) Capacitor with maximum charge, no current. (b) Capacitor discharging, current increasing. (c) Capacitor fully discharged, current maximum. (d) Capacitor charging but with polarity opposite that in (a), current decreasing.



(e) Capacitor with maximum charge having polarity opposite that in (a), no current. (f) Capacitor discharging, current increasing with direction opposite that in (b). (g) Capacitor fully discharged, current maximum. (h) Capacitor charging, current decreasing.

31-1 Electromagnetic Oscillations

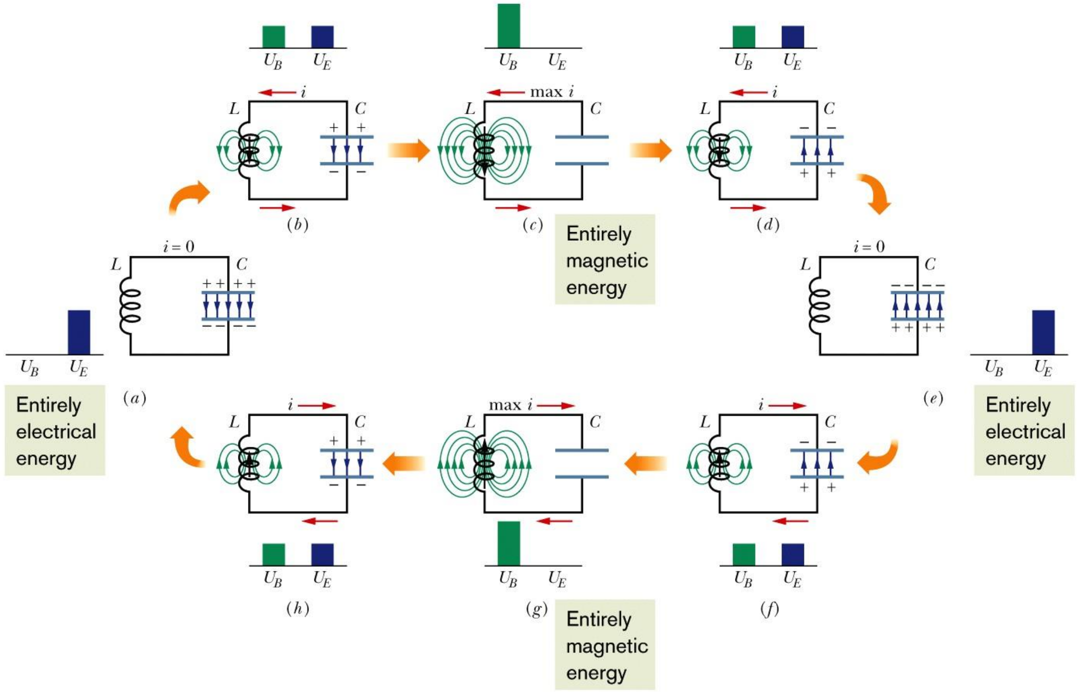
Parts (a) through (h) of the Figure show succeeding stages of the oscillations in a simple LC circuit. The energy stored in the electric field of the **capacitor** at any time is

$$U_E = \frac{q^2}{2C}$$

where q is the charge on the capacitor at that time. The energy stored in the magnetic field of the **inductor** at any time is

$$U_B = \frac{Li^2}{2}$$

where i is the current through the inductor at that time.



The resulting oscillations of the capacitor's electric field and the inductor's magnetic field are said to be **electromagnetic oscillations**.

31-1 Electromagnetic Oscillations

Table 31-1 Comparison of the Energy in Two Oscillating Systems

Block–Spring System		LC Oscillator	
Element	Energy	Element	Energy
Spring	Potential, $\frac{1}{2}kx^2$	Capacitor	Electrical, $\frac{1}{2}(1/C)q^2$
Block	Kinetic, $\frac{1}{2}mv^2$	Inductor	Magnetic, $\frac{1}{2}Li^2$
$v = dx/dt$		$i = dq/dt$	

From the table we can deduce the correspondence between these systems. Thus,

q corresponds to *x*, $1/C$ corresponds to *k*,
i corresponds to *v*, and *L* corresponds to *m*.

The correspondences listed above suggest that to find the angular frequency of oscillation for an ideal (resistanceless) LC circuit, *k* should be replaced by $1/C$ and *m* by *L*, yielding

$$\omega = \frac{1}{\sqrt{LC}} \quad (LC \text{ circuit}).$$

31-1 Electromagnetic Oscillations

LC Oscillator

The total energy *U* present at any instant in an oscillating LC circuit is given by

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

in which U_B is the energy stored in the magnetic field of the inductor and U_E is the energy stored in the electric field of the capacitor. Since we have assumed the circuit resistance to be zero, no energy is transferred to thermal energy and *U* remains constant with time. In more formal language, *dU/dt* must be zero. This leads to

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

However, i = dq/dt and di/dt = d^2q/dt^2. With these substitutions, we get

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

This is the **differential equation** that describes the oscillations of a resistanceless LC circuit.

Charge and Current Oscillation

The solution for the differential equation equation that describes the oscillations of a resistanceless LC circuit is

$$q = Q\cos(\omega t + \phi)$$

where Q is the amplitude of the charge variations, ω is the angular frequency of the electromagnetic oscillations, and ϕ is the phase constant. Taking the first derivative of the above Eq. with respect to time gives us the current:

$$i = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi)$$

Checkpoint 2

A capacitor in an LC oscillator has a maximum potential difference of 17 V and a maximum energy of 160 μJ. When the capacitor has a potential difference of 5 V and an energy of 10 μJ, what are (a) the emf across the inductor and (b) the energy stored in the magnetic field?

Answer: (a) ε_L= 12 V
(b) U_B=150 μJ

31-1 Electromagnetic Oscillations

Electrical and Magnetic Energy Oscillations

The electrical energy stored in the LC circuit at time *t* is,

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi).$$

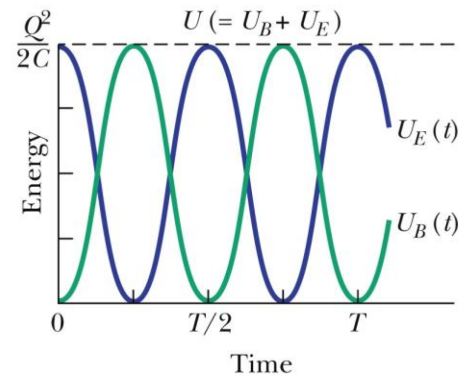
The magnetic energy is,

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

Figure shows plots of U_E (t) and U_B (t) for the case of $\phi=0$. Note that

1. The maximum values of U_E and U_B are both $Q^2/2C$.
2. At any instant the sum of U_E and U_B is equal to $Q^2/2C$, a constant.
3. When U_E is maximum, U_B is zero, and conversely.

The electrical and magnetic energies vary but the total is constant.



The stored magnetic energy and electrical energy in the RL circuit as a function of time.

31-2

Damped Oscillation in an RLC circuit

31-2 Damped Oscillation in an RLC circuit

To analyze the oscillations of this circuit, we write an equation for the total electromagnetic energy U in the circuit at any instant. Because the resistance does not store electromagnetic energy, we can write

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}.$$

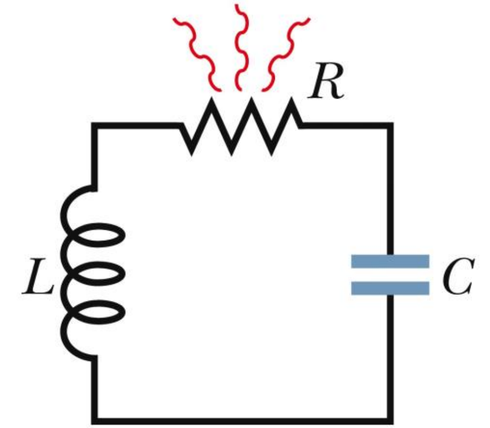
Now, however, this total energy decreases as energy is transferred to thermal energy. The rate of that transfer is,

$$\frac{dU}{dt} = -i^2R,$$

where the minus sign indicates that *U* decreases. By differentiating *U* with respect to time and then substituting the result we eventually get,

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

which is the differential equation for **damped oscillations** in an RLC circuit.



A series RLC circuit. As the charge contained in the circuit oscillates back and forth through the resistance, electromagnetic energy is dissipated as thermal energy, damping (decreasing the amplitude of) the oscillations.

Charge Decay. The solution to above Eq. is $q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$

in which $\omega' = \sqrt{\omega^2 - (R/2L)^2}$ and $\omega = 1/\sqrt{LC}$,

31-3

Forced Oscillations of Three Simple Circuits

Forced Oscillations

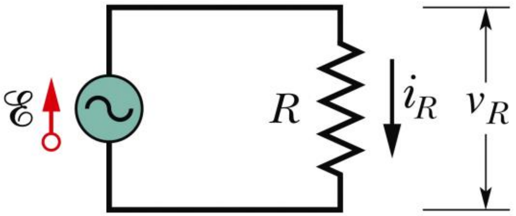
Whatever the natural angular frequency ω of a circuit may be, forced oscillations of charge, current, and potential difference in the circuit always occur at the driving angular frequency ω_d.

31-3 Forced Oscillations of Three Simple Circuits

Resistive Load

The alternating potential difference across a resistor has amplitude

$$V_R = I_R R \quad \text{(resistor)}.$$



A resistor is connected across an alternating-current generator.

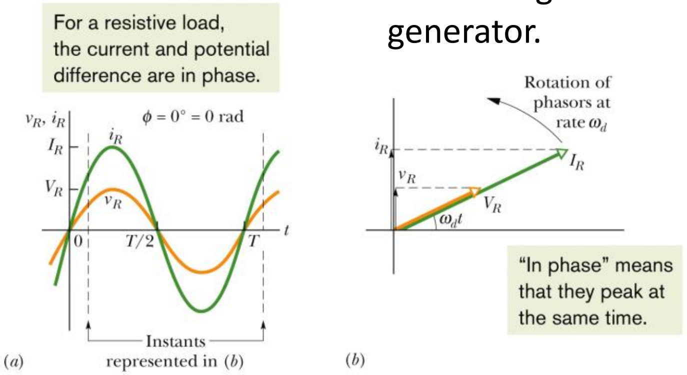
where V_R and I_R are the amplitudes of alternating current i_R and alternating potential difference v_r across the resistance in the circuit.

Angular speed: Both current and potential difference phasors rotate counterclockwise about the origin with an angular speed equal to the angular frequency ω_d of v_R and i_R.

Length: The length of each phasor represents the amplitude of the alternating quantity: V_R for the voltage and I_R for the current.

Projection: The projection of each phasor on the vertical axis represents the value of the alternating quantity at time t: v_R for the voltage and i_R for the current.

Rotation angle: The rotation angle of each phasor is equal to the phase of the alternating quantity at time t.



(a) The current i_R and the potential difference v_R across the resistor are plotted on the same graph, both versus time t. They are in phase and complete one cycle in one period T. (b) A phasor diagram shows the same thing as (a).

31-3 Forced Oscillations of Three Simple Circuits

Inductive Load

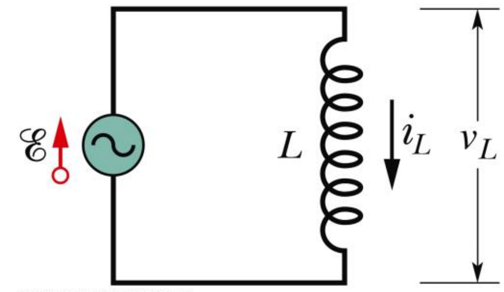
The **inductive reactance** of an inductor is defined as

$$X_L = \omega_d L$$

Its value depends not only on the inductance but also on the driving angular frequency ω_d.

The voltage amplitude and current amplitude are related by

$$V_L = I_L X_L$$



An inductor is connected across an alternating-current generator.

For an inductive load, the current lags the potential difference by 90°.

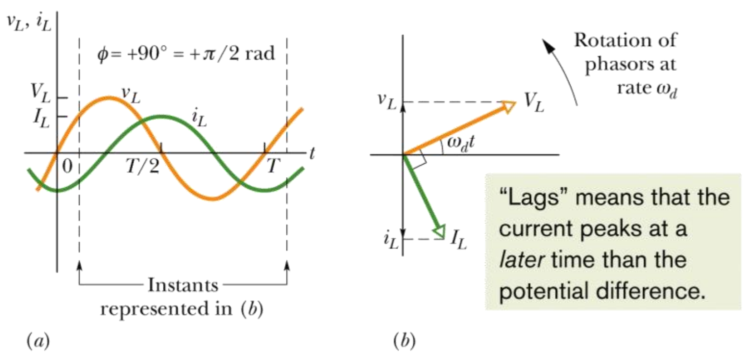


Fig. (left), shows that the quantities i_L and v_L are 90° out of phase. In this case, however, i_L lags v_L; that is, monitoring the current i_L and the potential difference v_L in the circuit of Fig. (top) shows that i_L reaches its maximum value after v_L does, by one-quarter cycle.

31-3 Forced Oscillations of Three Simple Circuits

Capacitive Load

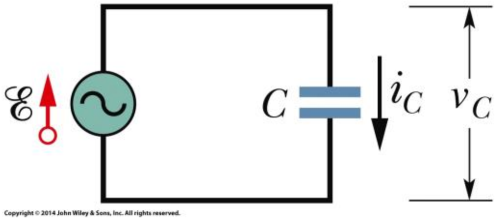
The **capacitive reactance** of a capacitor, defined as

$$X_C = \frac{1}{\omega_d C}$$

Its value depends not only on the capacitance but also on the driving angular frequency ω_d.

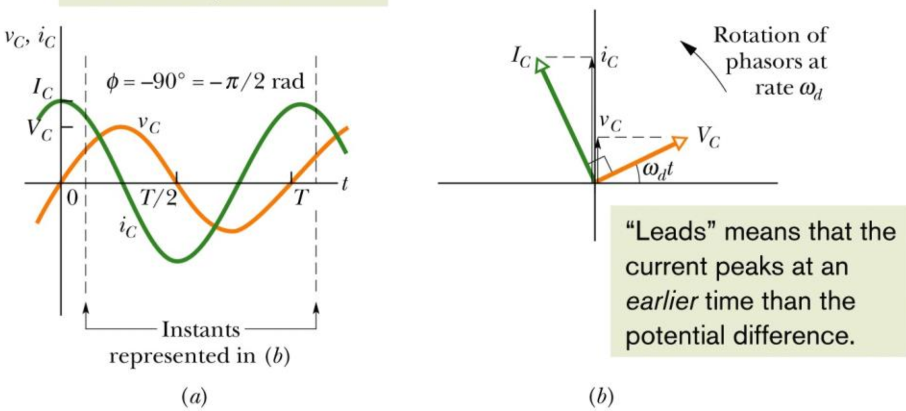
The voltage amplitude and current amplitude are related by

$$V_C = I_C X_C$$



A capacitor is connected across an alternating-current generator.

For a capacitive load, the current leads the potential difference by 90°.



In the phasor diagram we see that i_C leads v_C, which means that, if you monitored the current i_C and the potential difference v_C in the circuit above, you would find that i_C reaches its maximum before v_C does, by one-quarter cycle.

31-4

The Series RLC Circuits

31-4 The Series RLC Circuit

For a series RLC circuit with an external emf given by

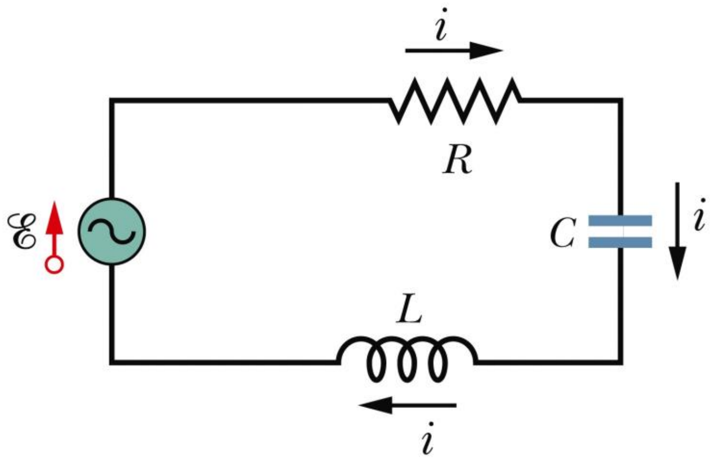
$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t$$

The current is given by

$$i = I \sin(\omega_d t - \phi)$$

the current amplitude is given by

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}.$$



Series RLC circuit with an external emf

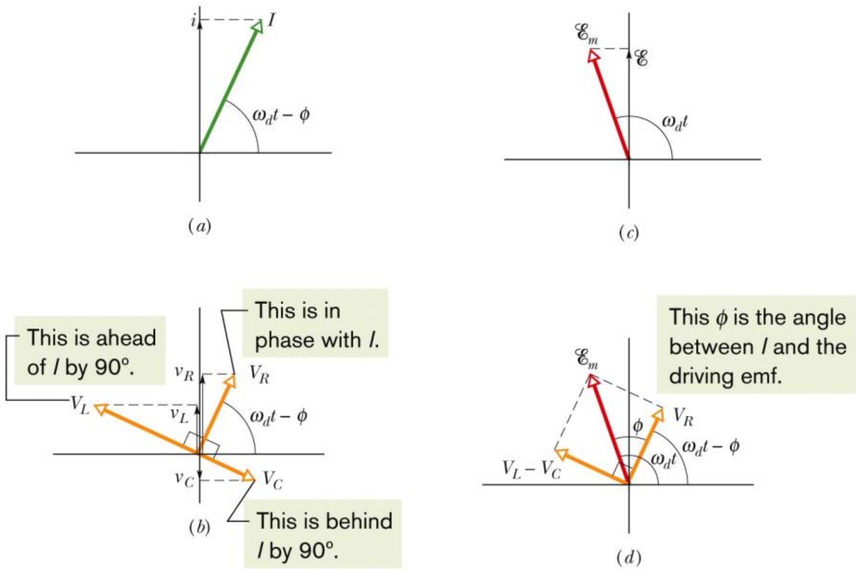
The denominator in the above equation is called the impedance *Z* of the circuit for the driving angular frequency ω_d.

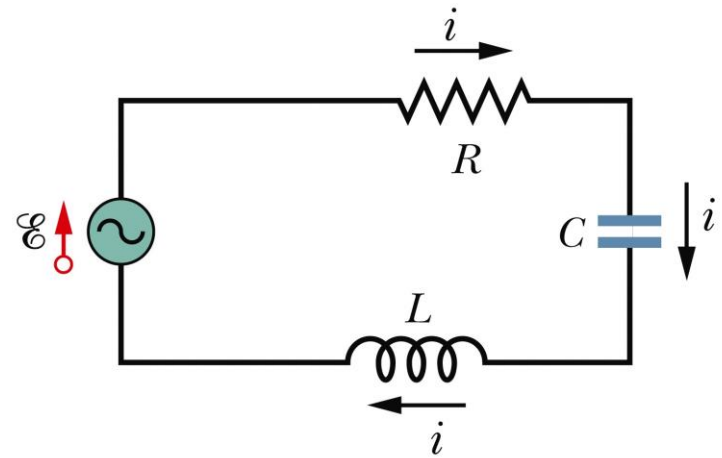
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If we substitute the value of X_L and X_C in the equation for current (I), the equation becomes:

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

31-4 The Series RLC Circuits





Series RLC circuit with an external emf

From the right-hand phasor triangle in Fig.(d) we can write

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR},$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Phase Constant

The current amplitude I is maximum when the driving angular frequency ω_d equals the natural angular frequency ω of the circuit, a condition known as **resonance**. Then $X_C = X_L$, $\phi = 0$, and the current is in phase with the emf.

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \quad \text{(resonance)}.$$

31-5

Power in Alternating-Current Circuits

31-5 Power in Alternating-Current Circuits

The instantaneous rate at which energy is dissipated in the resistor can be written as

$$P = i^2R = [I\sin(\omega_d t - \phi)]^2 R = I^2 R \sin^2(\omega_d t - \phi).$$

Over one complete cycle, the average value of sinϑ, where ϑ is any variable, is zero (Fig.a) but the average value of sin$^2\vartheta$ is 1/2(Fig.b). Thus the power is,

$$P_{avg} = \frac{I^2R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R.$$

The quantity I/ √2 is called the **root-mean-square**, or rms, value of the current *i*:

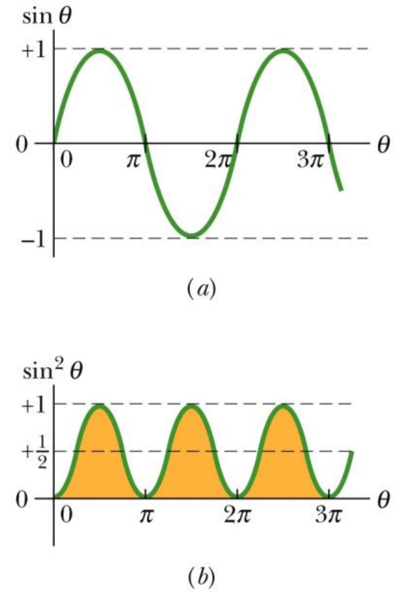
$$I_{rms} = \frac{I}{\sqrt{2}} \qquad P_{avg} = I_{rms}^2 R$$

We can also define rms values of voltages and emfs for alternating-current circuits:

$$V_{rms} = \frac{V}{\sqrt{2}} \quad \text{and} \quad \mathscr{E}_{rms} = \frac{\mathscr{E}_m}{\sqrt{2}}$$

In a series RLC circuit, the average power P_{avg} of the generator is equal to the production rate of thermal energy in the resistor:

$$P_{avg} = \mathscr{E}_{rms} I_{rms} \cos\phi$$



(a) A plot of sinϑ versus ϑ. The average value over one cycle is zero.

(b) A plot of sin$^2\vartheta$ versus ϑ. The average value over one cycle is 1/2.

31-6

Transformers

31-6 Transformers

A transformer (assumed to be ideal) is an iron core on which are wound a primary coil of N_p turns and a secondary coil of N_s turns. If the primary coil is connected across an alternating-current generator, the primary and secondary voltages are related by

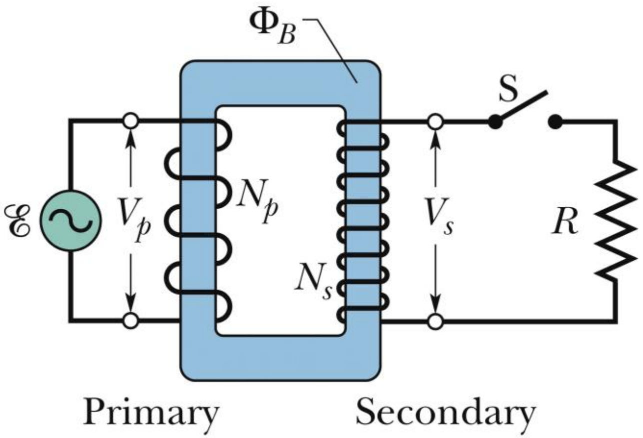
$$V_s = V_p \frac{N_s}{N_p}$$

Energy Transfers. The rate at which the generator transfers energy to the primary is equal to $I_p V_p$. The rate at which the primary then transfers energy to the secondary (via the alternating magnetic field linking the two coils) is $I_s V_s$. Because we assume that no energy is lost along the way, conservation of energy requires that

$$I_p V_p = I_s V_s. \Rightarrow I_s = I_p \frac{N_p}{N_s}$$

The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R.$$



An ideal transformer (two coils wound on an iron core) in a basic trans- former circuit. An ac generator produces current in the coil at the left (the primary). The coil at the right (the secondary) is connected to the resistive load R when switch S is closed.

31-6

Summary

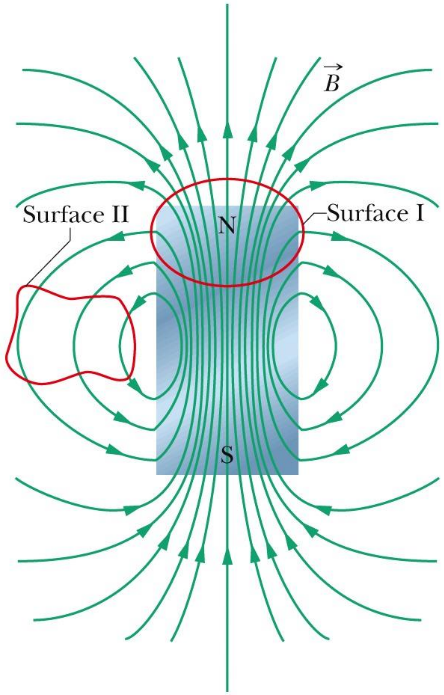
CHAPTER 32

Maxwell Equations;
Magnetism of Matter

32-1

Gauss' Law for Magnetic Fields

32-1 Induced Magnetic Fields



The field lines for the magnetic field ***B*** of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

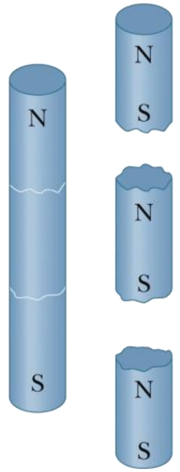
Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux Φ_B through <u>any</u> closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Contrast this with Gauss' law for electric fields,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

Gauss' law for magnetic fields says that there can be no net magnetic flux through the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface.



If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

32-2

Induced Magnetic Field

32-2 Induced Magnetic Fields

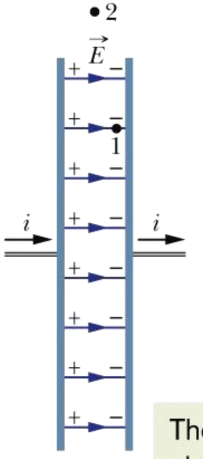
A changing electric flux induces a magnetic field ***B***. Maxwell's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Relates the magnetic field induced along a closed loop to the changing electric flux ϕ_E through the loop.

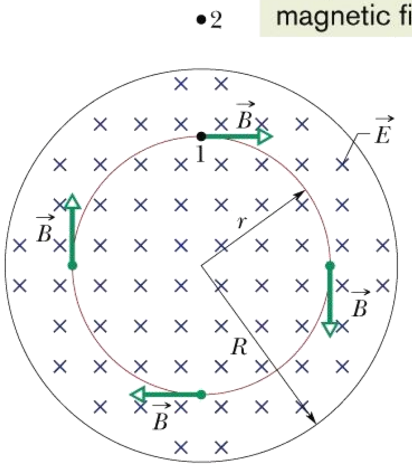
Charging a Capacitor.

As an example of this sort of induction, we consider the charging of a parallel-plate capacitor with circular plates. The charge on our capacitor is being increased at a steady rate by a constant current *i* in the connecting wires. Then the electric field magnitude between the plates must also be increasing at a steady rate.



(a)

The changing of the electric field between the plates creates a magnetic field.



(b)

32-2 Induced Magnetic Fields

A changing electric flux induces a magnetic field ***B***. Maxwell's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Relates the magnetic field induced along a closed loop to the changing electric flux ϕ_E through the loop.

Ampere-Maxwell Law

Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

gives the magnetic field generated by a current i_{enc} encircled by a closed loop.

Thus, the two equations (the other being Maxwell's Law) that specify the magnetic field ***B*** produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

32-3

Displacement Current

32-3 Displacement Current

If you compare the two terms on the right side of Eq. (Ampere-Maxwell Law), you will see that the product $\varepsilon_0(d\phi_E/dt)$ must have the dimension of a current. In fact, that product has been treated as being a fictitious current called the **displacement current** i_d:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Ampere-Maxwell Law then becomes,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$

where $i_{d,enc}$ is the displacement current encircled by the integration loop.

32-3 Displacement Current

The four fundamental equations of electromagnetism, called Maxwell's equations and are displayed in Table 32-1.

Table 32-1 Maxwell's Equations[a]

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \dfrac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$	Relates induced magnetic field to changing electric flux and to current

[a]Written on the assumption that no dielectric or magnetic materials are present.

These four equations explain a diverse range of phenomena, from why a compass needle points north to why a car starts when you turn the ignition key. They are the basis for the functioning of such electromagnetic devices as electric motors, television transmitters and receivers, telephones, scanners, radar, and microwave ovens.