Lebanese American University

CHAPTER 28

Magnetic Fields

History

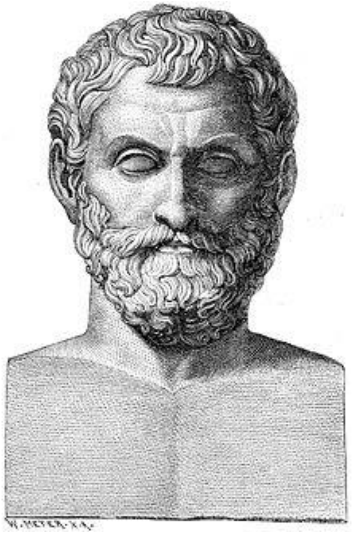
- One of the first references to lodestone's magnetic properties is by 6th century BCE Greek philosopher Thales of Miletus

- Thales attempted to explain natural phenomena without reference to mythology and was tremendously influential in this respect.

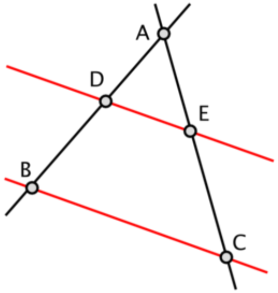
- In China, the earliest literary reference to magnetism lies in a 4th-century BC book called Book of the Devil Valley Master : "The lodestone makes iron come or it attracts it."

- By the 12th century, the lodestone compass was being used for navigation in medieval China.

Lodestone



Thales



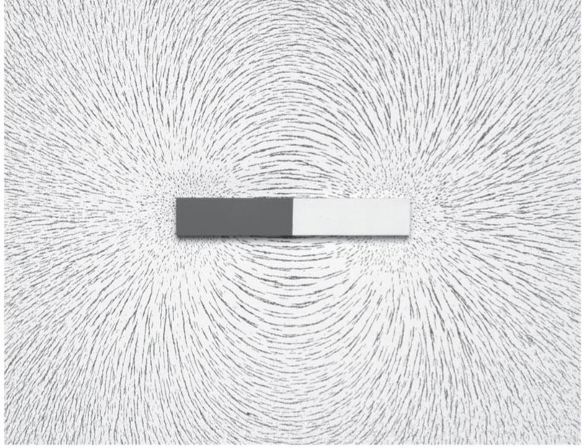
$$\frac{DE}{BC} = \frac{AE}{AC} = \frac{AD}{AB}$$

Magnets, Poles and Dipoles

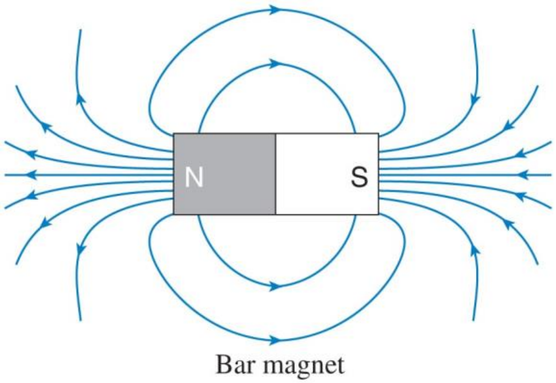
- Poles of a magnet are the ends where objects are most strongly attracted.
- Two poles, called **north** and **south**

Two Poles. The (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the north pole of the magnet; the other end, where field lines enter the magnet, is called the south pole. Because a magnet has two poles, it is said to be a **magnetic dipole**.

We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply: the direction of the tangent to a magnetic field line at any point gives the direction of B at that point



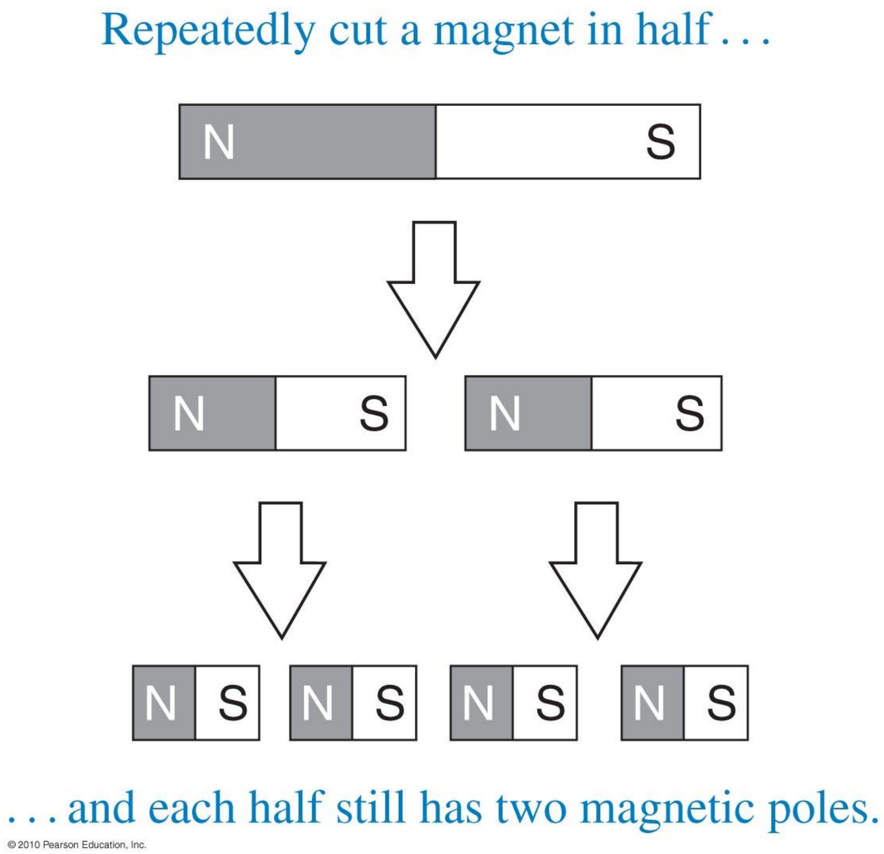
(a)



(b)

Opposite magnetic poles attract each other, and like magnetic poles repel each other.

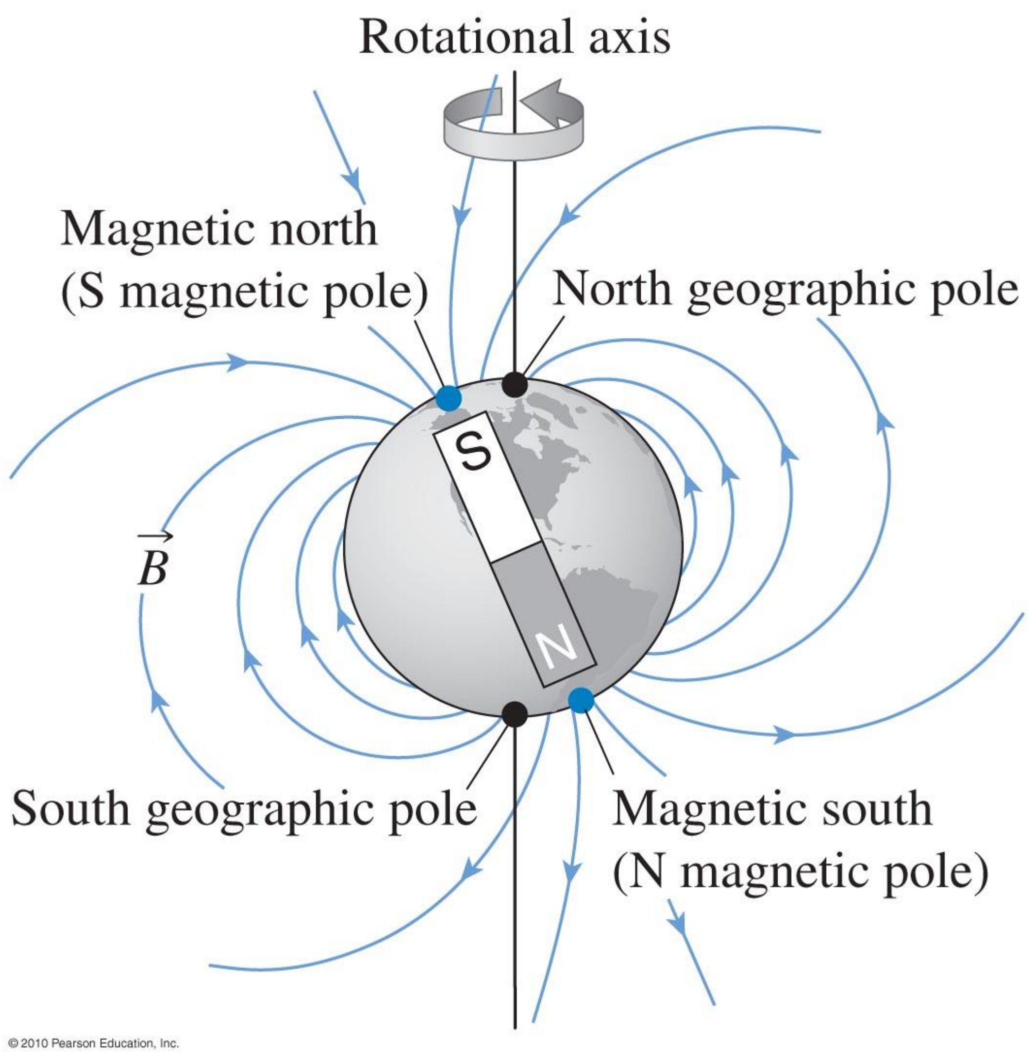
Magnets, Poles and Dipoles

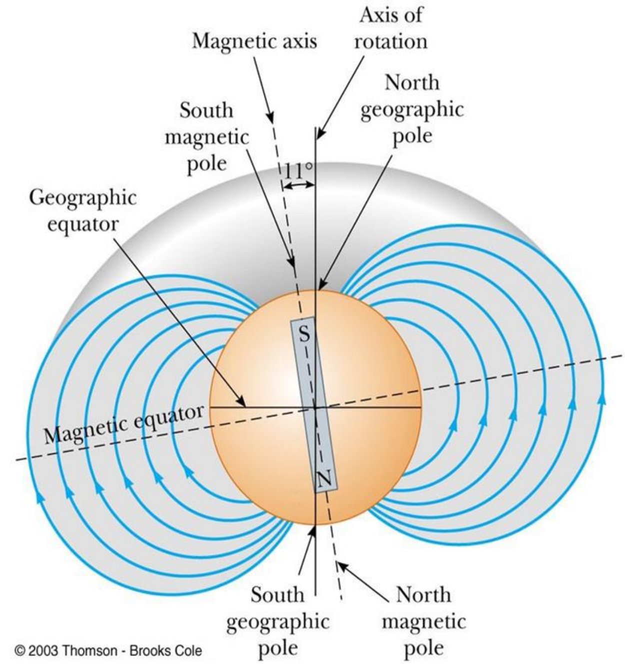


- Magnetic poles cannot be isolated
- If a permanent magnet is cut in half repeatedly, you will still have a north and a south pole
- This differs from electric charges
- There are no magnetic monopole (There is some theoretical basis for monopoles, but none have been detected)

Magnets, Poles and Dipoles

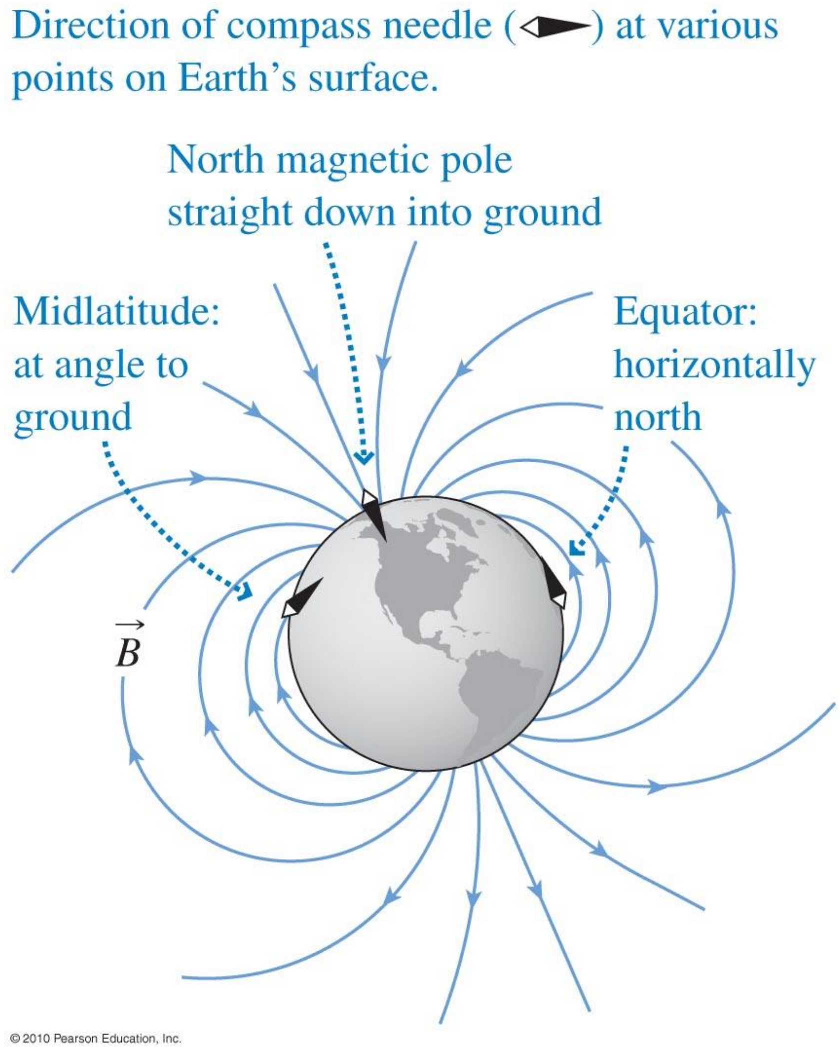
Earth Magnetic Field





The Earth's geographic north pole corresponds to a magnetic south pole
The Earth's geographic south pole corresponds to a magnetic north pole

Magnets, Poles and Dipoles



Your compass needle points toward magnetic north, not geographic north. Declination is the east/west deviation of the compass needle. Inclination is the vertical deviation.

Magnets, Poles and Dipoles

Birds, Bees and Bacteria use Earth's magnetic field to navigate. They have magnetite (Fe_3O_4), which responds to the field.

The dark specks on the photos are magnetite in a bacteria.

28-1

Defnition of $\vec{\mathbf{B}}$

28-1 Magnetic Fields and the Definition of *B*

The Definition of *B*

The Field. We can define a magnetic field ***B*** to be a vector quantity that exists when it exerts a force $\boldsymbol{F_B}$ on a charge moving with velocity ***v***. We can next measure the magnitude of $\boldsymbol{F_B}$ when ***v*** is directed perpendicular to that force and then define the magnitude of ***B*** in terms of that force magnitude:

$$B = \frac{F_B}{|q|v},$$

where *q* is the charge of the particle. We can summarize all these results with the following vector equation:

$$\vec{F}_B = q\vec{v} \times \vec{B};$$

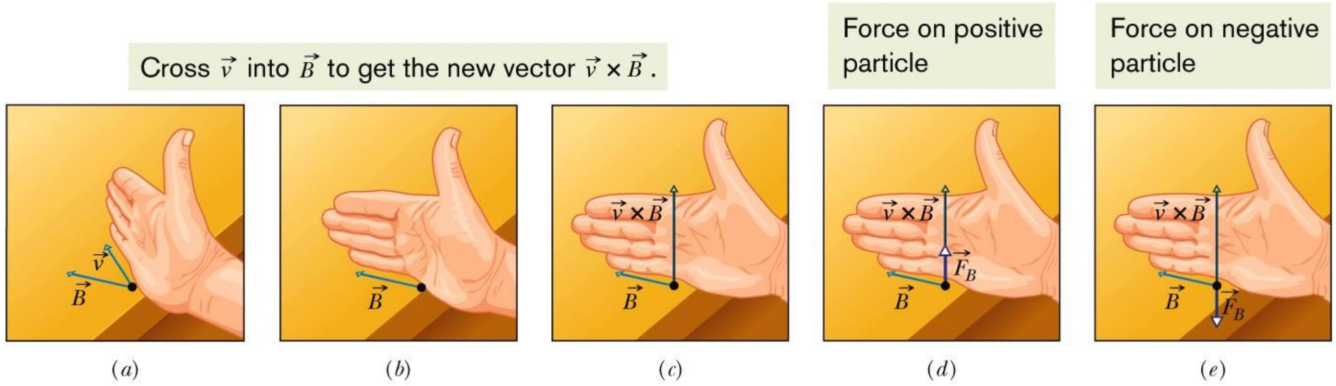
that is, the force $\boldsymbol{F_B}$ on the particle by the field ***B*** is equal to the charge *q* times the cross product of its velocity ***v*** and the field ***B*** (all measured in the same reference frame). We can write the magnitude of $\boldsymbol{F_B}$ as

$$F_B = |q|vB \sin \phi,$$

where ϕ is the angle between the directions of velocity ***v*** and magnetic field ***B***.

28-1 Magnetic Fields and the Definition of *B*

Finding the Magnetic Force on a Particle

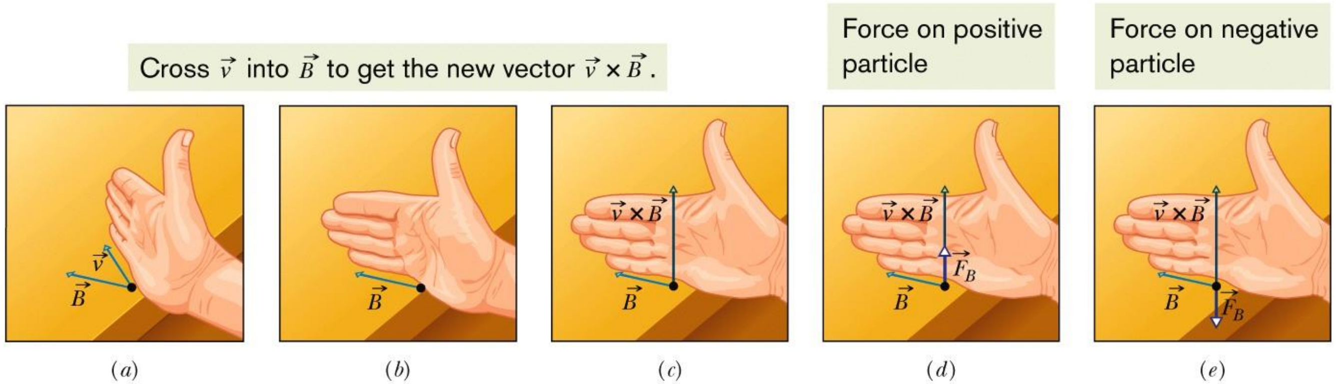


$$\vec{F}_B = q\vec{v} \times \vec{B};$$

This equation tells us the direction of ***F***. We know the cross product of ***v*** and ***B*** is a vector that is perpendicular to these two vectors. The right-hand rule (Figs. a-c) tells us that the thumb of the right hand points in the direction of ***v*** × ***B*** when the fingers sweep ***v*** into ***B***. If *q* is positive, then (by the above Eq.) the force $\boldsymbol{F_B}$ has the same sign as ***v*** × ***B*** and thus must be in the same direction; that is, for positive *q*, $\boldsymbol{F_B}$ is directed along the thumb (Fig. d). If *q* is negative, then the force $\boldsymbol{F_B}$ and cross product ***v*** × ***B*** have opposite signs and thus must be in opposite directions. For negative *q*, ***F*** is directed opposite the thumb (Fig. e).

28-1 Magnetic Fields and the Definition of *B*

Finding the Magnetic Force on a Particle

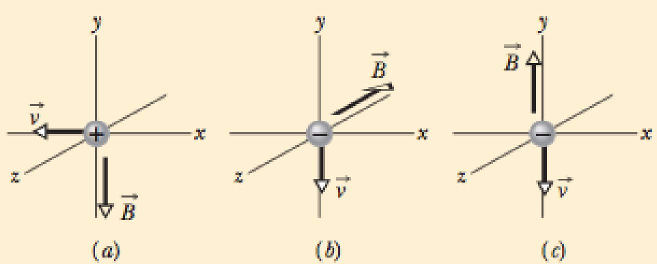


$$\vec{F}_B = q\vec{v} \times \vec{B};$$

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Checkpoint 1

The figure shows three situations in which a charged particle with velocity $\vec{v}$ travels through a uniform magnetic field $\vec{B}$. In each situation, what is the direction of the magnetic force $\vec{F}_B$ on the particle?



Answer:

(a) towards the positive z-axis

(b) towards the negative x-axis

(c) none (cross product is zero)

28-1 Magnetic Fields and the Definition of *B*

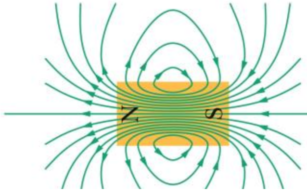
Magnetic Field Lines

We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply:

(1) the direction of the tangent to a magnetic field line at any point gives the direction of ***B*** at that point

(2) the spacing of the lines represents the magnitude of ***B*** —the magnetic field is stronger where the lines are closer together, and conversely.

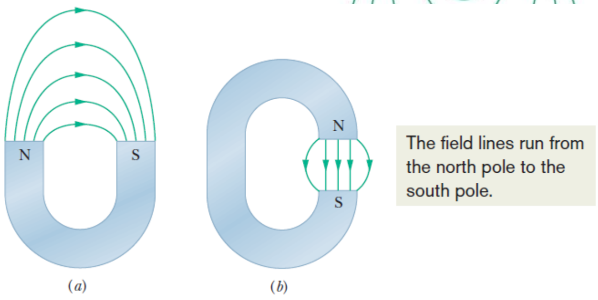
Two Poles. The (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the north pole of the magnet; the other end, where field lines enter the magnet, is called the south pole. Because a magnet has two poles, it is said to be a **magnetic dipole**.



Opposite magnetic poles attract each other, and like magnetic poles repel each other.

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

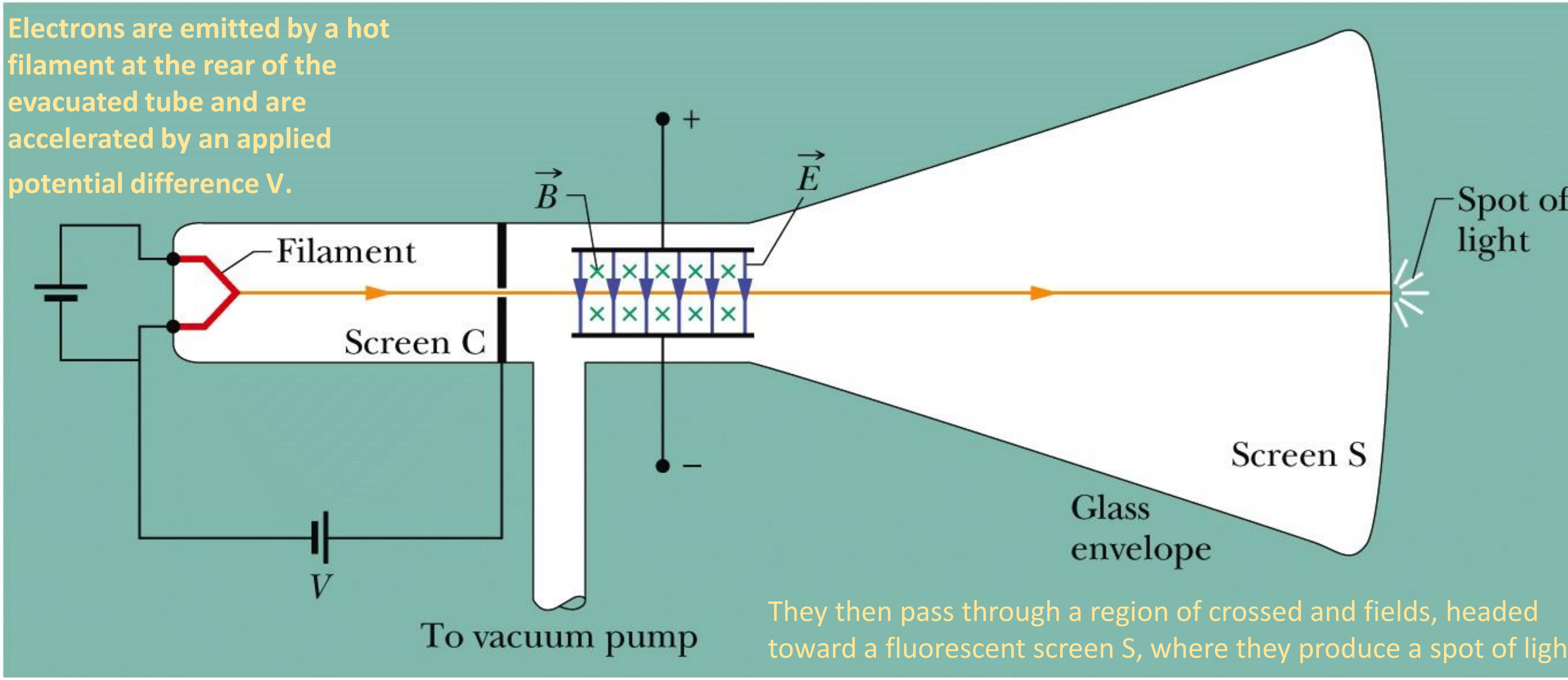


28-2

Crossed Fields: Discovery of The Electron

28-2 **Crossed Fields:** Discovery of The Electron

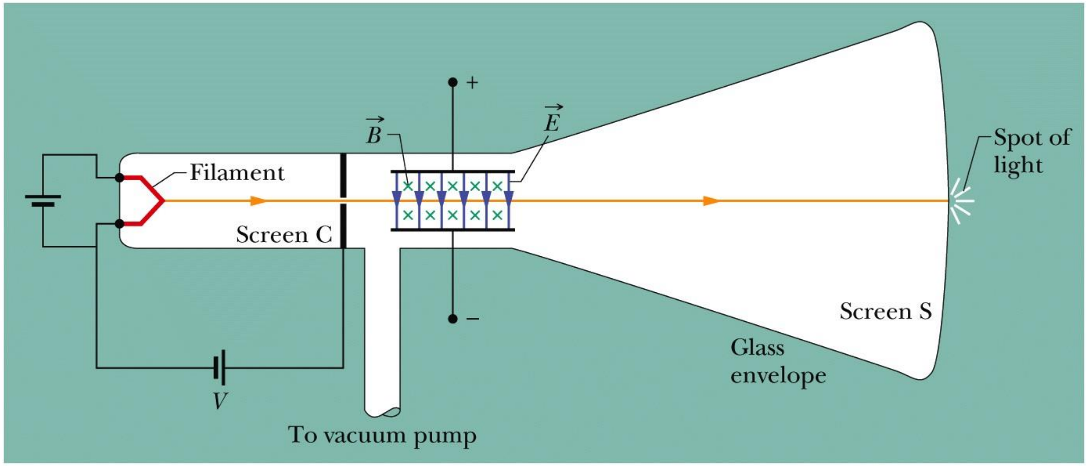
- When an electric field and a magnetic field are perpendicular to each other, they are said to be crossed fields.
- **What happens to charged particles— namely, electrons—as they move through crossed fields?**
- We use as our example the experiment that led to the discovery of the electron in 1897 by J. J. Thomson at Cambridge University.



After they pass through a slit in screen C, they form a narrow beam

28-2 Crossed Fields: Discovery of The Electron

A modern version of J.J. Thomson's apparatus (a *cathode ray tube)* for measuring the ratio of mass to charge for the electron. An electric field ***E*** is established by connecting a battery across the deflecting-plate terminals. The magnetic field ***B*** is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).



If a charged particle moves through a region containing both an electric field and a magnetic field, it can be affected by both an electric force and a magnetic force.

When the two fields are perpendicular to each other, they are said to be **crossed fields**.

If the forces are in opposite directions, one particular speed will result in no deflection of the particle.

$$y = \frac{|q|EL^2}{2mv^2}$$

where v is the particle's speed,m its mass, and q its charge, and L is the length of the plates

When the two fields are adjusted so that the two deflecting forces cancel , we have:

$$|q|E = |q|vB\sin(90^\circ) = |q|vB$$

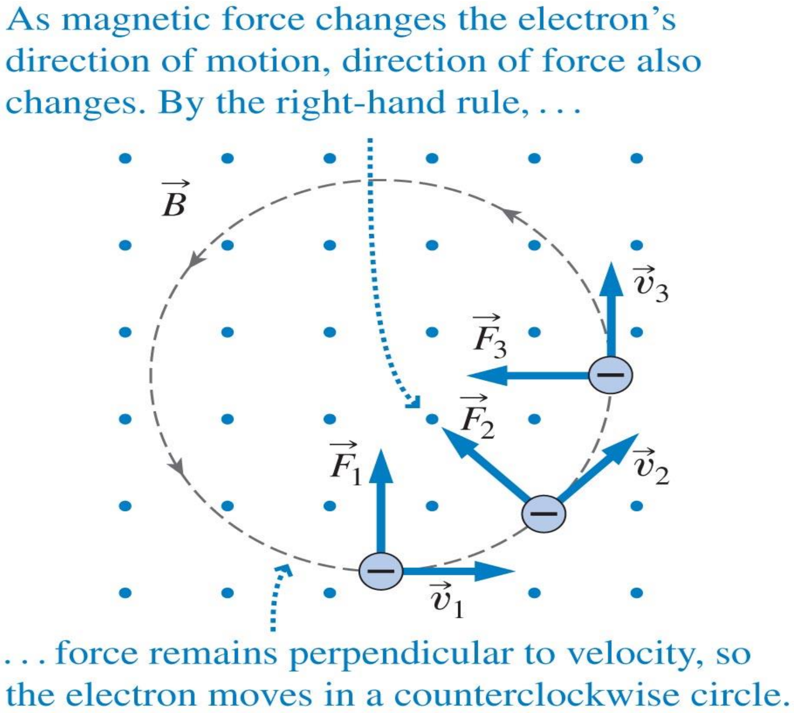
$$v = \frac{E}{B}.$$

$$\frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

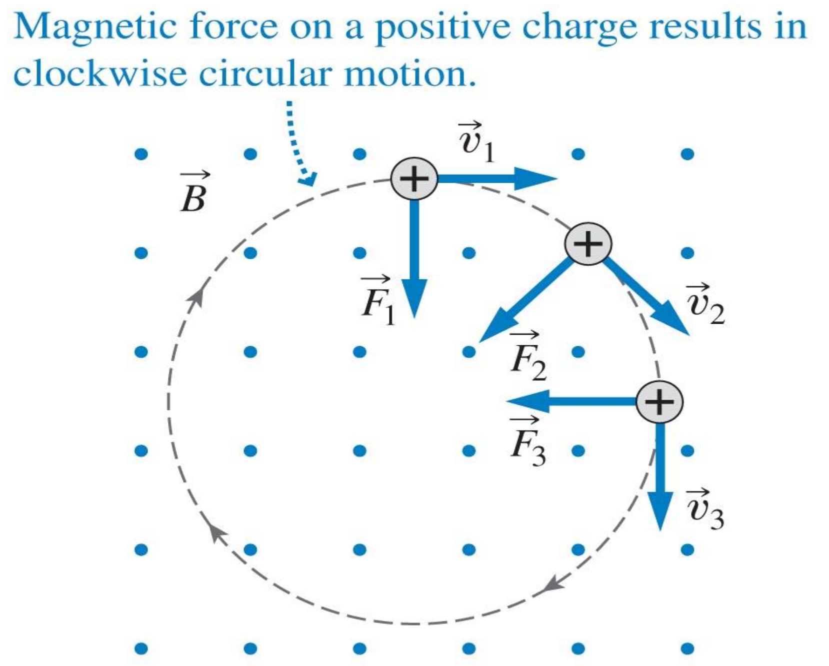
28-4

A Circulating Charged Particle

Magnetic Force on a Moving charge



(a) Motion of negative charge in magnetic field



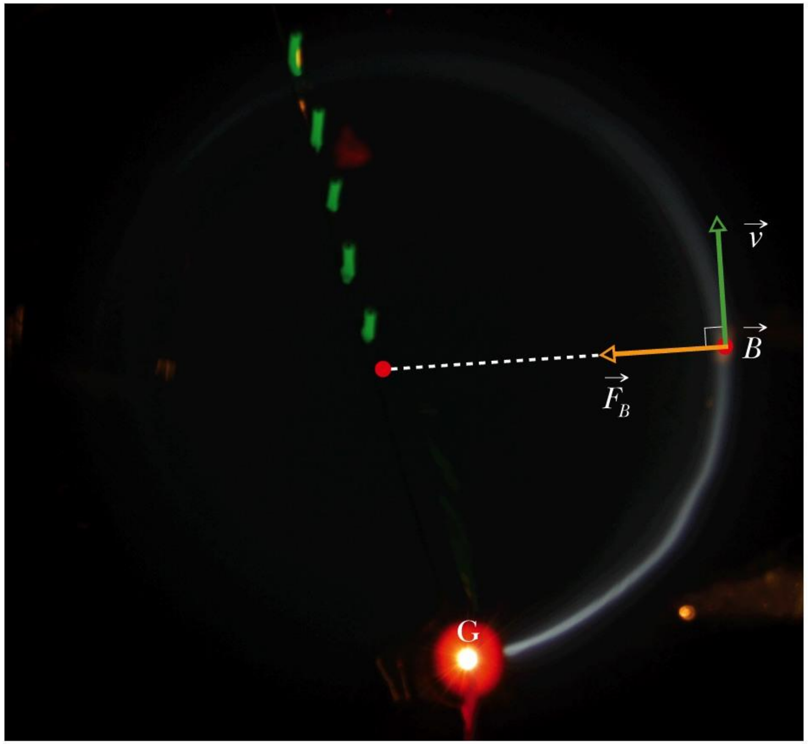
(b) Motion of positive charge in magnetic field

$$R = \frac{mv}{|q|B} \qquad \text{(Radius of charged particle's circular path; SI unit: m)} \qquad (18.2)$$

A charged particle moving through a magnetic field experiences a magnetic force perpendicular to the magnetic field and the particle's velocity.
A charged particle moving perpendicular to a uniform magnetic field has a **circular trajectory**.

28-4 A Circulating Charged Particle

A beam of electrons is projected into a chamber by an electron gun G. The electrons enter in the plane of the page with speed *v* and then move in a region of uniform magnetic field ***B*** directed out of that plane. As a result, a magnetic force $F_{B=}\ q\ (\boldsymbol{v}\times\boldsymbol{B})$ continuously deflects the electrons, and because ***v*** and B are always perpendicular to each other, this deflection causes the electrons to follow a circular path. The path is visible in the photo because atoms of gas in the chamber emit light when some of the circulating electrons collide with them.



Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}.$$

Therefore the radius *r* of the circle is $r = \dfrac{mv}{|q|B}.$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period)}$$

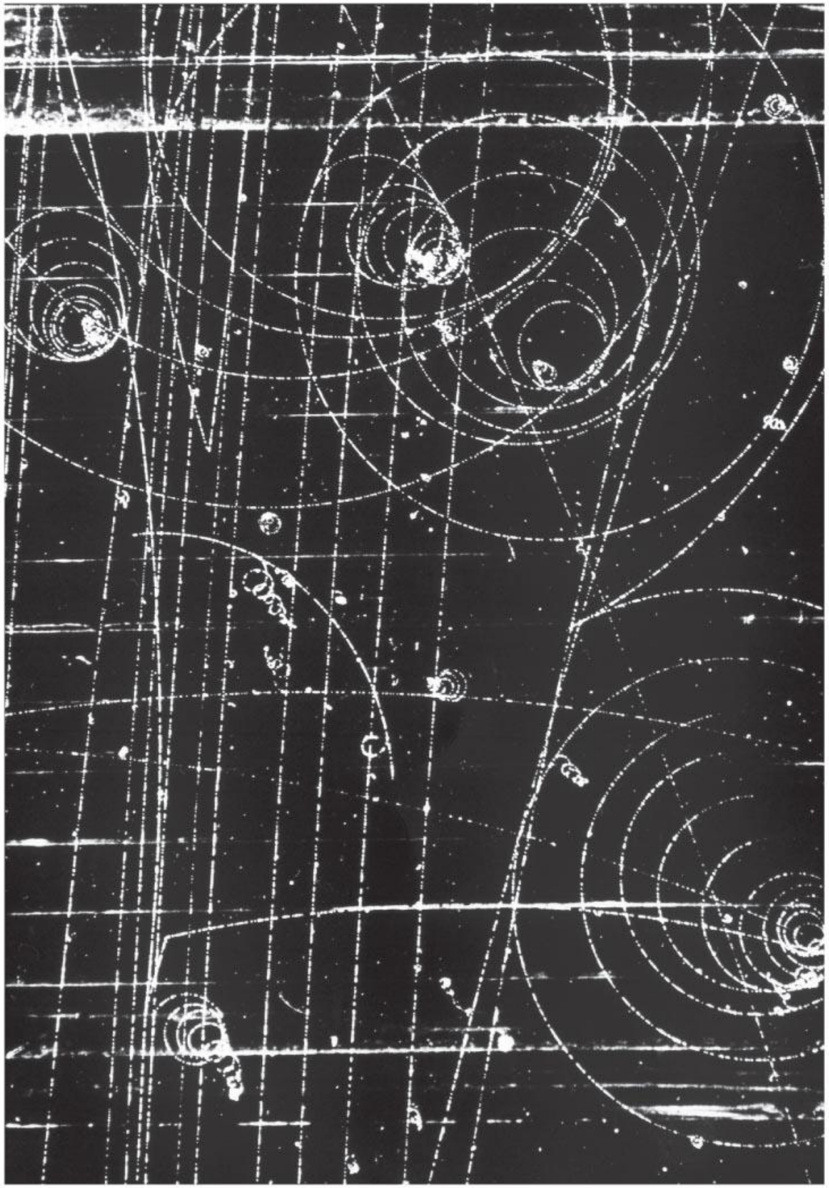
$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency)}$$

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency)}$$

Magnetic Force on a Moving charge

General Knowledge: Bubble Chamber

When a fast-moving charged particle passes through, it vaporizes liquid along its path, leaving a visible trail.



28-5

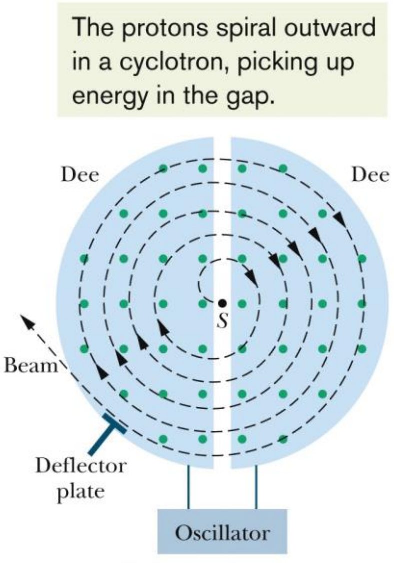
Cyclotrons and Synchrotrons

A Synchrotron is an advanced cyclotron

Proton Synchrotron: The magnetic field ***B*** and the oscillator frequency f_{osc}, instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle (relativistic effects).

LHC - Synchrotron

28-5 Cyclotrons and Synchrotrons



The Cyclotron: The figure is a top view of the region of a cyclotron in which the particles (protons, say) circulate. The two hollow D-shaped objects (each open on its straight edge) are made of sheet copper. These dees, as they are called, are part of an electrical oscillator that alternates the electric potential difference across the gap between the dees. The electrical signs of the dees are alternated so that the electric field in the gap alternates in direction, first toward one dee and then toward the other dee, back and forth. The dees are immersed in a large magnetic field directed out of the plane of the page. The magnitude B of this field is set via a control on the electromagnet producing the field.

The key to the operation of the cyclotron is that the frequency f at which the proton circulates in the magnetic field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$$f = f_{\text{osc}} \quad \text{(resonance condition).}$$

This resonance condition says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field.

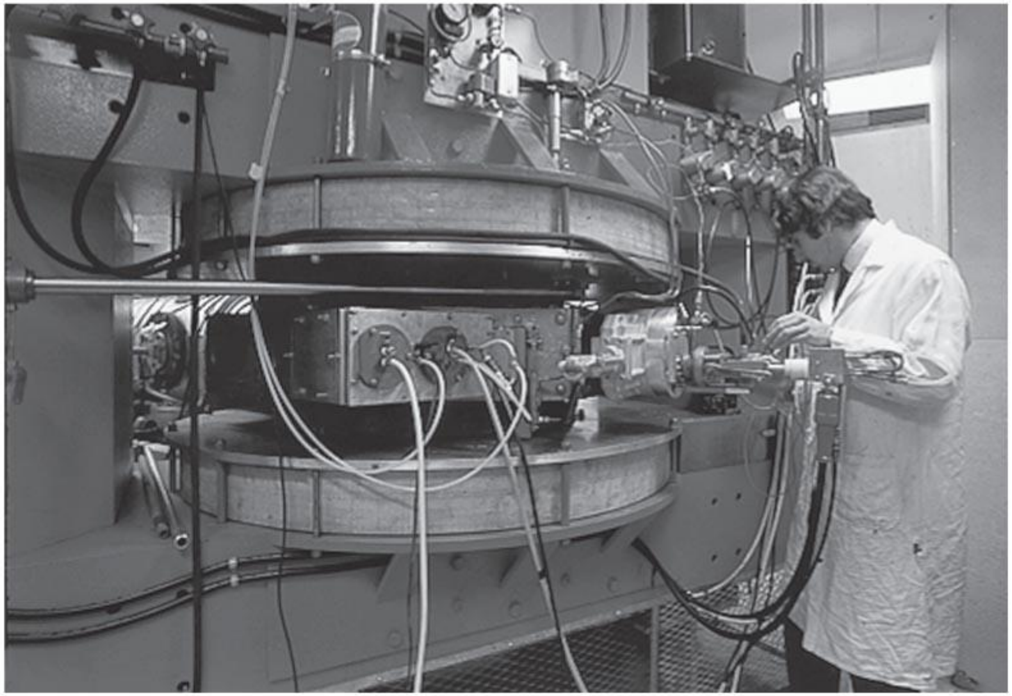
28-5 Cyclotrons and Synchrotrons

Proton Synchrotron: The magnetic field $\boldsymbol{B}$ and the oscillator frequency f_{osc}, instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle. When this is done properly, (1) the frequency of the circulating protons remains in step with the oscillator at all times, and (2) the protons follow a circular — not a spiral — path. Thus, the magnet need extend only along that circular path, not over some 4×10^6 m^2. The circular path, however, still must be large if high energies are to be achieved.

Applications of Magnetic Forces

C. Cyclotrons and Synchrotrons

Medical Cyclotron:
Radioisotopes production

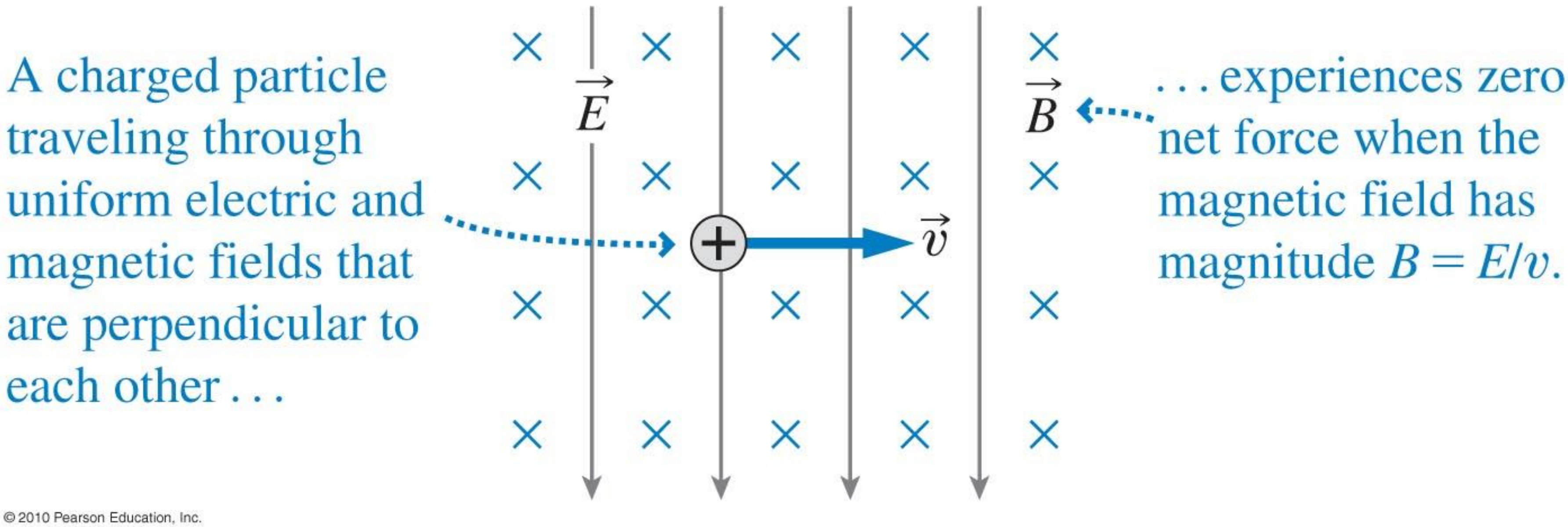


The medical imaging technique known as PET scanning (for *p*ositron *e*mission *t*omography) often uses radioactive isotopes with such short lifetimes that they have to be produced just prior to a PET scan. Bombarding the appropriate materials with high-energy particles from a cyclotron can induce nuclear reactions that produce the desired isotopes. Larger hospitals often have in-house cyclotrons for just that purpose.

Mass Spectrometer

Magnetic Force on a Moving charge

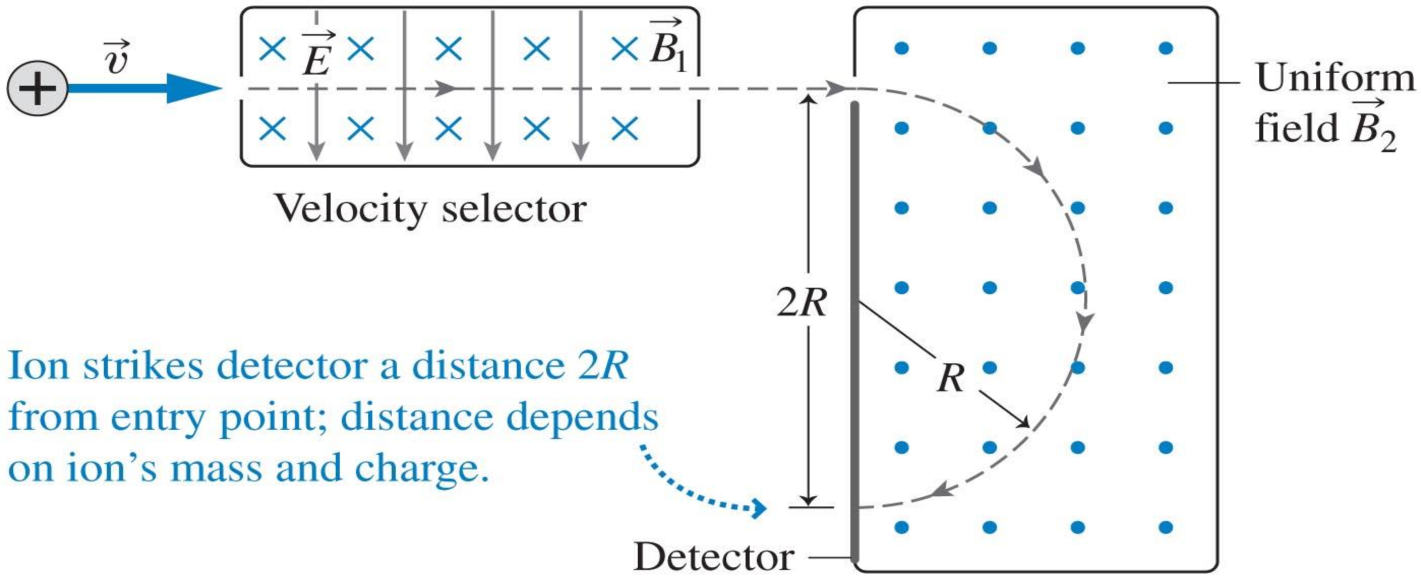
Velocity selector



Applications of Magnetic Forces

A. Mass Spectrometer

- The mass spectrometer is an instrument for separating and identifying atoms and molecules by mass. It relies on magnetic forces on charged particles

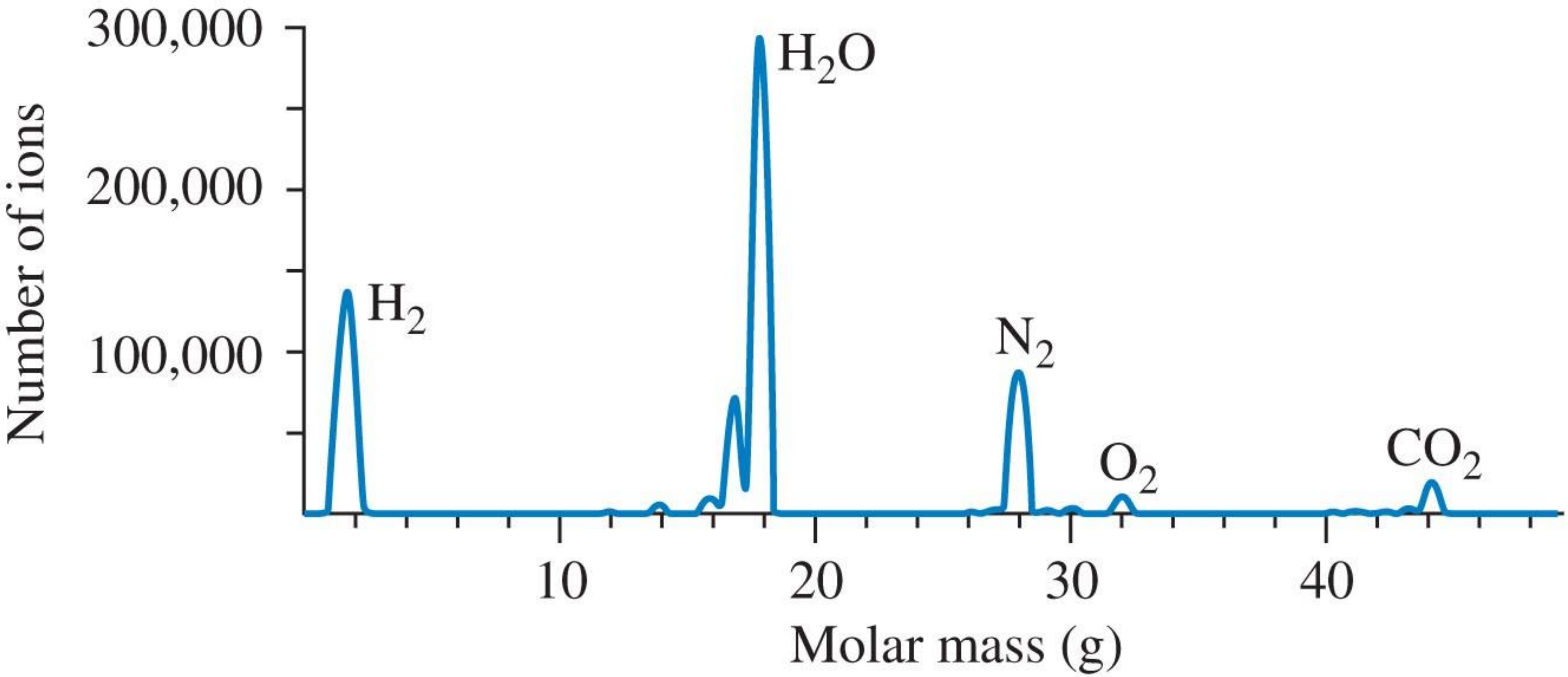


(a) How a mass spectrometer works

$$\frac{|q|}{m} = \frac{E}{B_1 B_2 R}$$ (Charge-to-mass ratio in a mass spectrometer; SI unit: C/kg)

Applications of Magnetic Forces

A. Mass Spectrometer

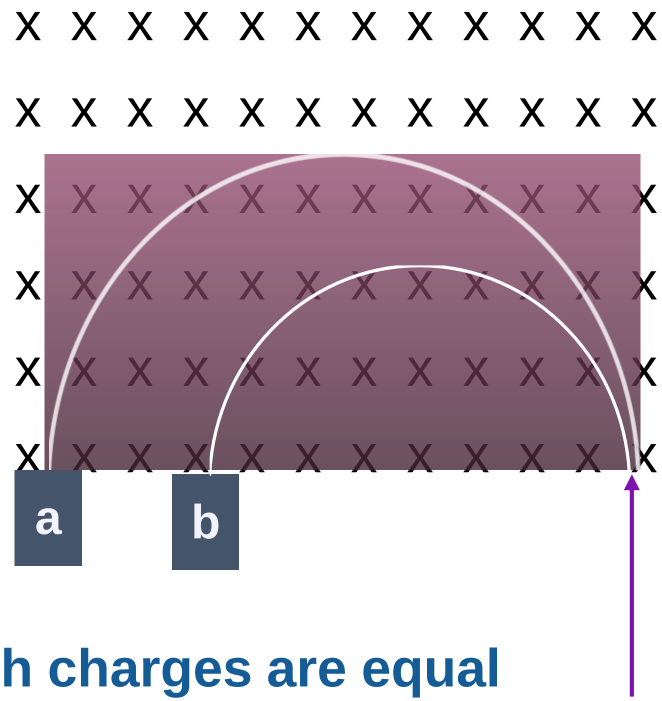


(b) Typical output of mass spectrometer. Curve shows number of ions as a function of mass.

Applications of Magnetic Forces

A. Mass Spectrometer

Two particles of the *same mass* enter a magnetic field with the *same speed* and follow the paths shown. Which particle has the *bigger charge*?



c) both charges are equal

d) impossible to tell from the picture

Answer b: the bigger the charge, the smaller the radius

28-6

Magnetic Force on Current-Carrying Wire

28-6 Magnetic Force on a Current-Carrying Wire

A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

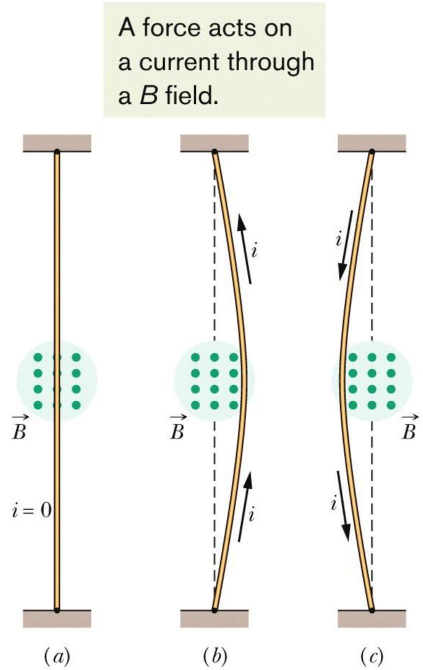
$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).}$$

Here ***L*** is a length vector that has magnitude *L* and is directed along the wire segment in the direction of the (conventional) current.

Crooked Wire. If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}.$$

and the direction of length vector ***L*** or *d**L*** is in the direction of *i.*



A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (a) Without current in the wire, the wire is straight. (b) With upward current, the wire is deflected rightward. (c) With downward current, the deflection is leftward.

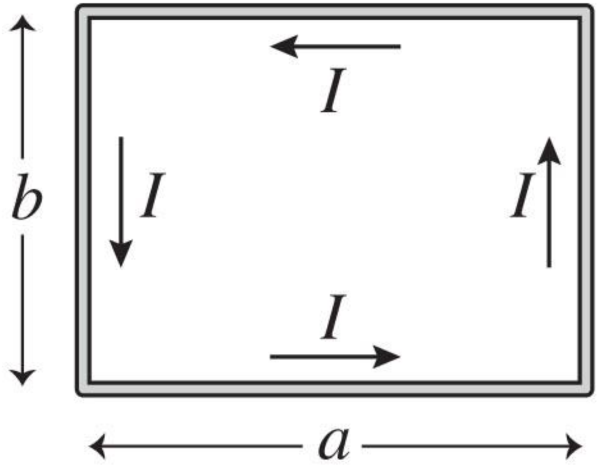
28-7

Torque on a Current Loop

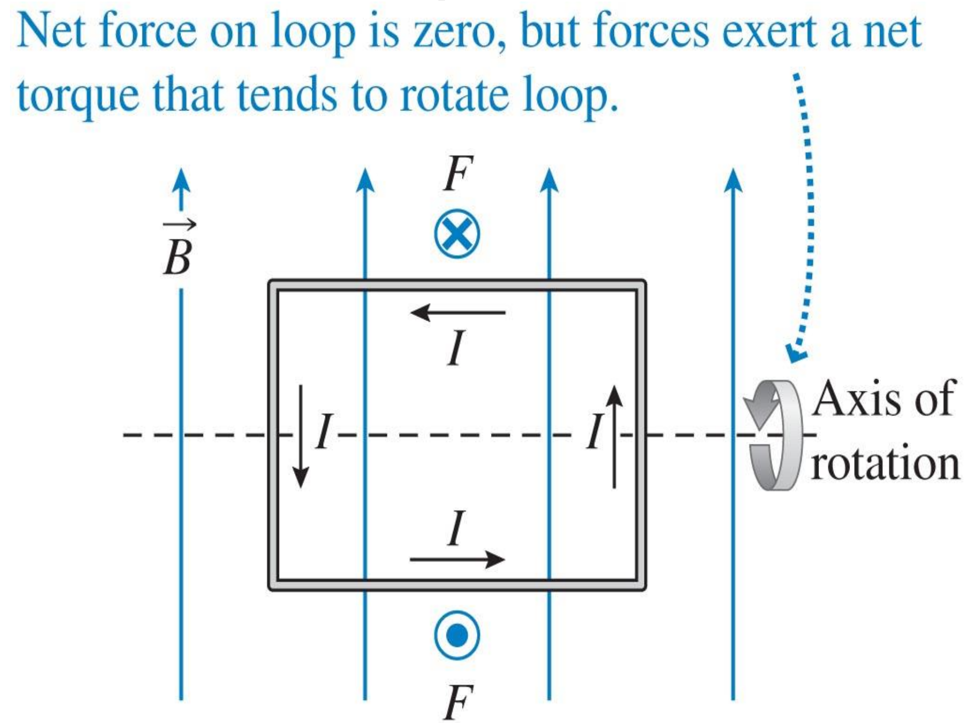
Magnetic Forces on Conducting Wires

Torque (Fr: Moment d'une force)

Torque on a current loop



(a) Current-carrying wire loop



(c) Net torque on loop

$$F = IaB.$$

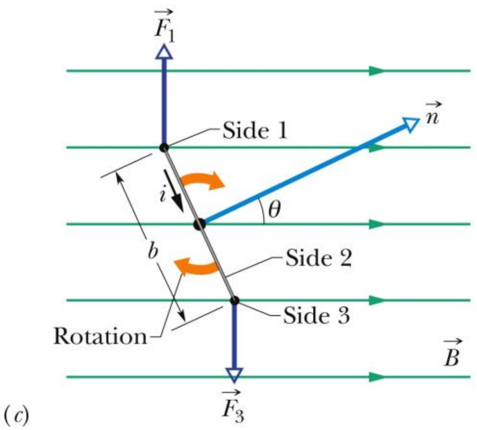
$$\tau = \frac{b}{2}IaB + \frac{b}{2}IaB = IabB = IAB$$

where $A = ab$ is the loop area

(b) Forces on loop when placed in a magnetic field oriented parallel to two of its sides

$$\tau = NIAB\sin\theta \qquad \text{(Torque on a current loop; SI unit: N} \cdot \text{m)}$$

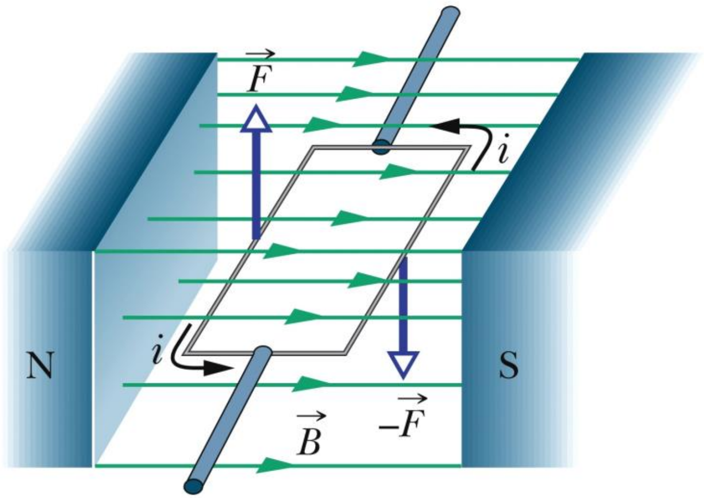
28-7 Torque on a Current Loop



As shown in the figure (right) the net force on the loop is the vector sum of the forces acting on its four sides and comes out to be zero. The net torque acting on the coil has a magnitude given by

$$\tau = NiAB \sin \theta,$$

where *N* is the number of turns in the coil, *A* is the area of each turn, *i* is the current, *B* is the field magnitude, and ϑ is the angle between the magnetic field ***B*** and the normal vector to the coil ***n***.



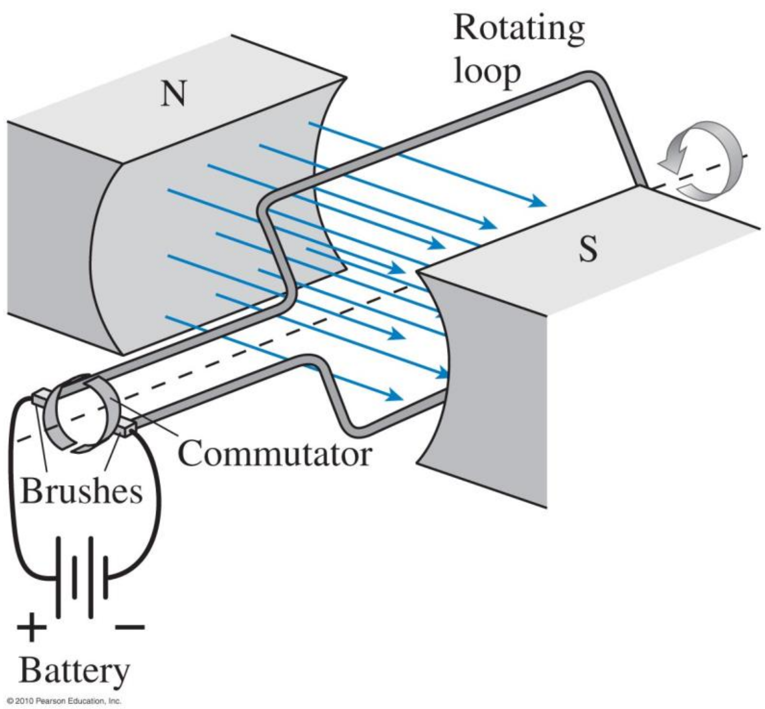
The elements of an electric motor: A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.

Magnetic Forces on Conducting Wires

Electric Motor

The torque on a current loop is what makes electric motors run

Another key element in the motor, the commutator. This rotating electrical contact reverses the direction of the current after every 180^{o} turn of the armature, keeping it rotating continually in the same direction.



Schematic diagram of an electric motor

28-8

The Magnetic Dipole Moment

28-8 The Magnetic Dipole Moment

A coil (of area *A* and *N* turns, carrying current *i*) in a uniform magnetic field ***B*** will experience a torque ***τ*** given by

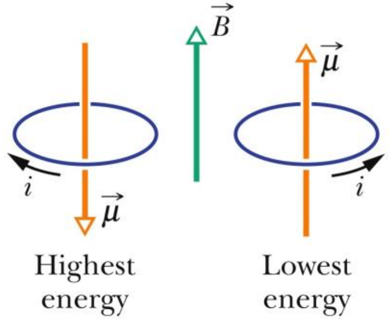
$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

where ***μ*** is the **magnetic dipole moment** of the coil, with magnitude *μ = NiA* and direction given by the right- hand rule.

The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

The magnetic moment vector attempts to align with the magnetic field.



If an external agent rotates a magnetic dipole from an initial orientation ϑ_i to some other orientation ϑ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i.$$

28

Summary

28 Summary

The Magnetic Field *B*

- Defined in terms of the force F_B acting on a test particle with charge *q* moving through the field with velocity ***v***

$$\vec{F}_B = q\vec{v} \times \vec{B}. \qquad \textbf{Eq. 28-2}$$

A Charge Particle Circulating in a Magnetic Field

- Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r} \qquad \textbf{Eq. 28-15}$$

- from which we find the radius r of the orbit circle to be

$$r = \frac{mv}{|q|B}. \qquad \textbf{Eq. 28-16}$$

Magnetic Force on a Current Carrying wire

- A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}. \qquad \textbf{Eq. 28-26}$$

- The force acting on a current element i dL in a magnetic field is

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}. \qquad \textbf{Eq. 28-28}$$

Torque on a Current Carrying Coil

- A coil (of area *A* and *N* turns, carrying current *i*) in a uniform magnetic field ***B*** will experience a torque ***τ*** given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \qquad \textbf{Eq. 28-37}$$

28 Summary

The Hall Effect

- When a conducting strip carrying a current i is placed in a uniform magnetic field $\boldsymbol{B}$, some charge carriers (with charge e) build up on one side of the conductor, creating a potential difference V across the strip. The polarities of the sides indicate the sign of the charge carriers.

Orientation Energy of a Magnetic Dipole

- The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \qquad \textbf{Eq. 28-38}$$

- If an external agent rotates a magnetic dipole from an initial orientation ϑ_i to some other orientation ϑ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i. \qquad \textbf{Eq. 28-39}$$

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Lebanese American University

CHAPTER 29

Magnetic Fields due to Currents

29-1

Magnetic Field due to a Current

29-1 Magnetic Field due to a Current

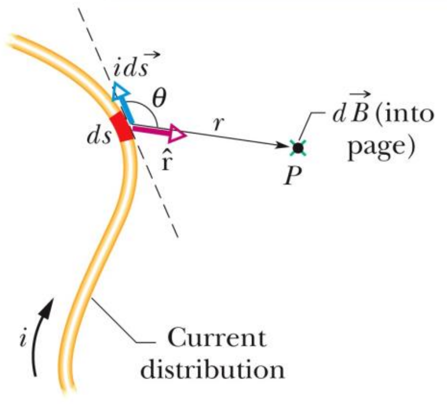
The magnitude of the field $d\boldsymbol{B}$ produced at point P at distance r by a current-length element $d\boldsymbol{s}$ turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

where ϑ is the angle between the directions of $d\boldsymbol{s}$ and $\hat{\boldsymbol{r}}$, a unit vector that points from $d\boldsymbol{s}$ toward P. Symbol μ_0 is a constant, called the permeability constant, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\,\mathrm{T\cdot m/A}.$$

This element of current creates a magnetic field at *P*, into the page.



The direction of $d\boldsymbol{B}$, shown as being into the page in the figure, is that of the cross product $d\boldsymbol{s}\times\hat{\boldsymbol{r}}$. We can therefore write the above equation containing $d\boldsymbol{B}$ in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{\mathrm{r}}}{r^2}$$

This vector equation and its scalar form are known as the **law of Biot and Savart.**

29-1 Magnetic Field due to a Current

For a **long straight wire** carrying a current i, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire,

$$B = \frac{\mu_0 i}{2\pi R}$$

Figure: The magnetic field lines produced by a current in along straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the X.

The magnetic field vector at any point is tangent to a circle.

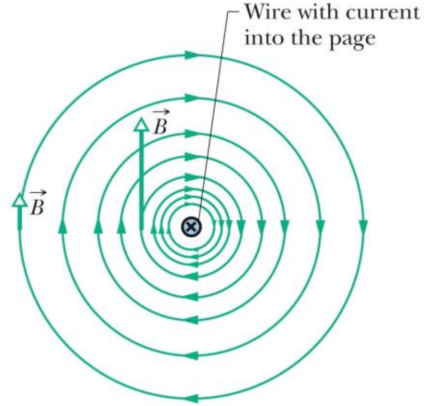
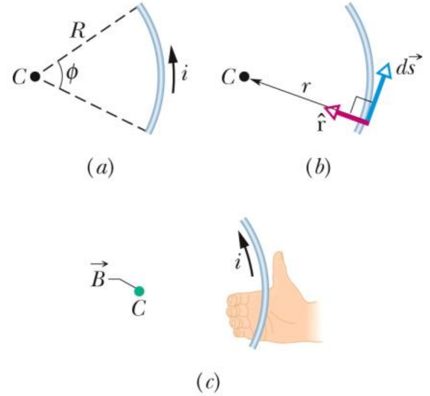


Fig. 29-3 Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current. *(Courtesy Education Development Center)*

Curled–straight right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

The magnitude of the **magnetic field at the center of a circular arc**, of radius *R* and central angle ϕ (in radians), carrying current *i,* is

$$B = \frac{\mu_0 i \phi}{4\pi R}$$



The right-hand rule reveals the field's direction at the center.

29-2 Force Between Two Parallel Currents

Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length L of either wire is

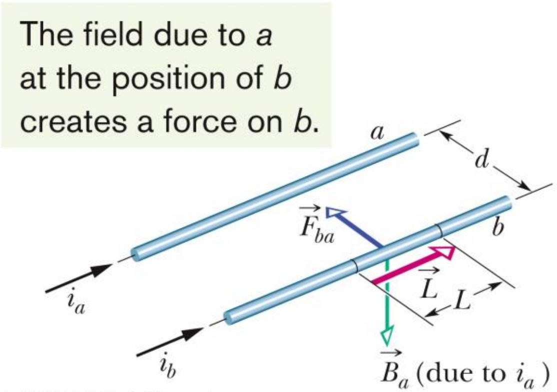
$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d},$$

where d is the wire separation, and i_a and i_b are the currents in the wires.

The general procedure for finding the force on a current-carrying wire is this:

> To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Similarly, if the two currents were anti-parallel, we could show that the two wires repel each other.



Two parallel wires carrying currents in the same direction attract each other.

29-3

Ampere's Law

29-3 Ampere's Law

Ampere's law states that

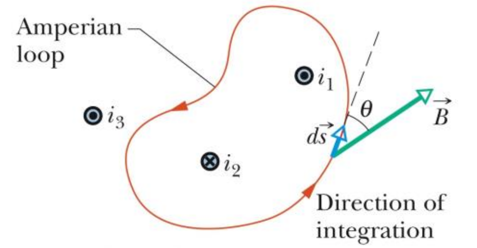
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current *i* on the right side is the net current encircled by the loop.

A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop.

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

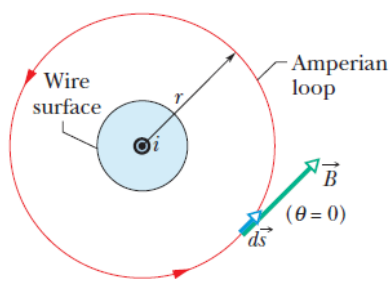
Only the currents encircled by the loop are used in Ampere's law.



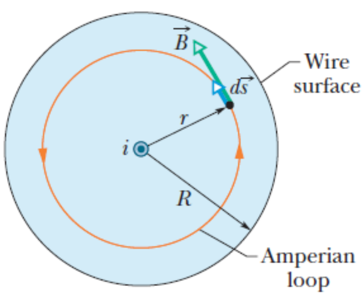
Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

Magnetic Fields of a long straight wire with current:

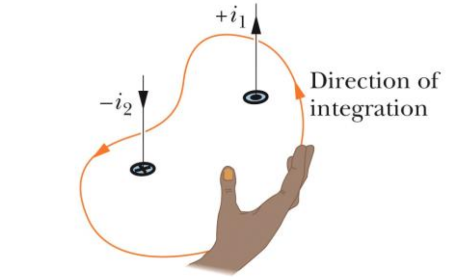
$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire).}$$



$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \quad \text{(inside straight wire).}$$



This is how to assign a sign to a current used in Ampere's law.

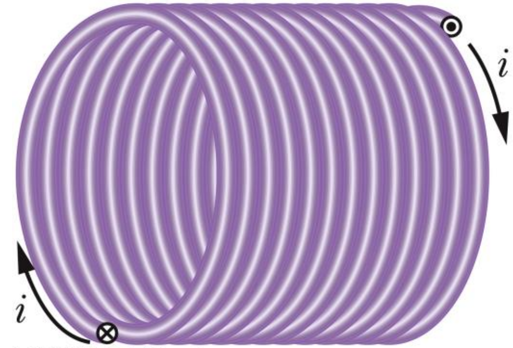


29-4

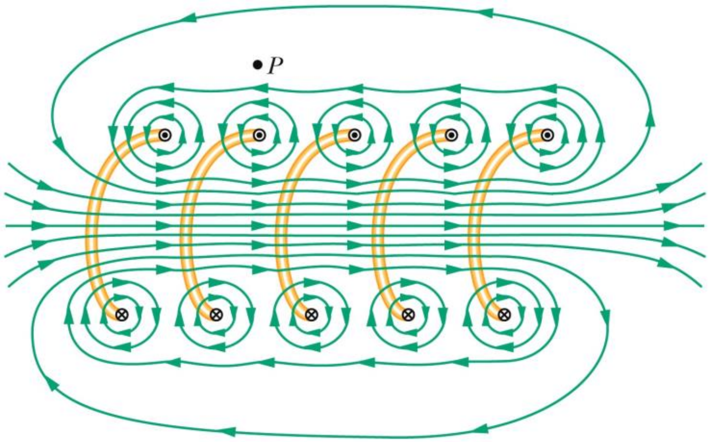
Solenoids and Toroids

Magnetic Field of a Solenoid

Figure (a) is a solenoid carrying current *i*. Figure (b) shows a section through a portion of a "stretched-out" solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (windings) that make up the solenoid. For points very close to a turn, the wire behaves magnetically almost like a long straight wire, and the lines of ***B*** there are almost concentric circles. Figure (b) suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire, ***B*** is approximately parallel to the (central) solenoid axis.



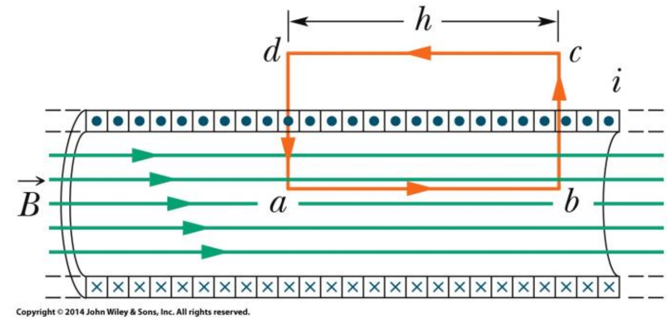
(a)



(b)

Magnetic Field of a Solenoid

Let us now apply Ampere's law, $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc},$



(a)

to the ideal solenoid of Fig. (a), where ***B*** is uniform within the solenoid and zero outside it, using the rectangular Amperian loop *abcda*. We write $\oint \vec{B} \cdot d\vec{s}$ as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

The first integral on the right of equation is *Bh*, where *B* is the magnitude of the uniform field ***B*** inside the solenoid and *h* is the (arbitrary) length of the segment from a to b. The second and fourth integrals are zero because for every element *ds* of these segments, ***B*** either is perpendicular to *ds* or is zero, and thus the product ***B•ds*** is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because *B=0* at all external points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value *Bh*.
Inside a long solenoid carrying current i, at points not near its ends, the magnitude B of the magnetic field is

$$B = \mu_0 i n \quad \text{(ideal solenoid).}$$

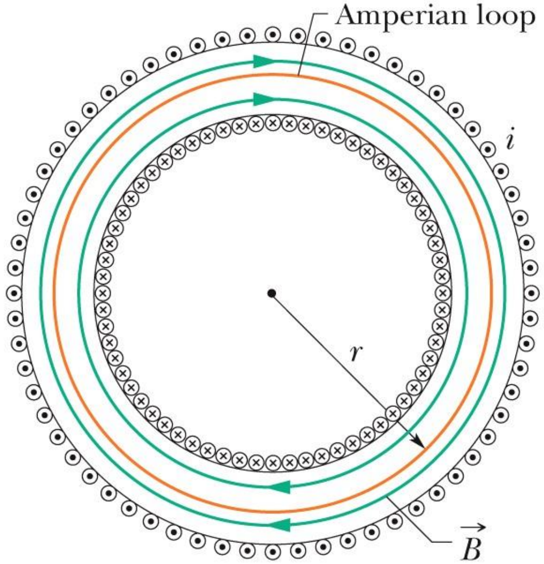
Magnetic Field of a Toroid

Figure (a) shows a toroid, which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet. What magnetic field ***B*** is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet. From the symmetry, we see that the lines of ***B*** form concentric circles inside the toroid, directed as shown in Fig. (b). Let us choose a concentric circle of radius *r* as an Amperian loop and traverse it in the clockwise direction. Ampere's law yields

where *i* is the current in t $(B)(2\pi r) = \mu_0 i N,$ s (and is positive for those windings enclosed by the Amperian loop) and *N* is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \quad \text{(toroid).}$$

(a)



(b)

In contrast to the situation for a solenoid, *B* is **not constant** over the cross section of a toroid.

Summary

29 Summary

The Biot-Savart Law

- The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathrm{r}}}{r^2} \qquad \textbf{Eq. 29-3}$$

- The quantity μ_0, called the permeability constant, has the value

$$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}.$$

Magnetic Field of a Long Straight Wire

- For a long straight wire carrying a current *i*, the Biot–Savart law gives,

$$B = \frac{\mu_0 i}{2\pi R} \qquad \textbf{Eq. 29-4}$$

Magnetic Field of a Circular Arc

- The magnitude of the magnetic field at the center of a circular arc,

$$B = \frac{\mu_0 i \phi}{4\pi R} \qquad \textbf{Eq. 29-9}$$

Force Between Parallel Currents

- The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d} \qquad \textbf{Eq. 29-13}$$

Ampere's Law

- Ampere's law states that,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \qquad \textbf{Eq. 29-14}$$

29 Summary

Fields of a Solenoid and a Toroid

- Inside a long solenoid carrying current *i*, at points not near its ends, the magnitude *B* of the magnetic field is

$$B = \mu_0 i n$$

Eq. 29-23

- At a point inside a toroid, the magnitude *B* of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

Eq. 29-24

Field of a Magnetic Dipole

- The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point P located a distance *z* along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$

Eq. 29-9