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Lebanese American University

CHAPTER 26

Current and Resistance

26-1

Electric Current

26-1 Electric Current

- In the last five chapters we discussed electrostatics—the physics of stationary charges.

- In this and the next chapter, we discuss the physics of electric currents—that is, charges in motion.

- Although an electric current is a stream of moving charges, not all moving charges constitute an electric current.

Free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of 10^6 m/s. If you pass a hypothetical plane through such a wire, conduction electrons pass through it in both directions at the rate of many billions per second—but there is no net transport of charge and thus no current through the wire.

26-1 Electric Current

As Fig. (a) reminds us, any isolated conducting loop—regardless of whether it has an excess charge — is all at the same potential. No electric field can exist within it or along its surface.

If we insert a battery in the loop, as in Fig. (b), the conducting loop is **no longer at a single potential**. **Electric fields act inside the material making up the loop, exerting forces on internal charges, causing them to move and thus establishing a current.** (The diagram assumes the motion of positive charges moving clockwise.)

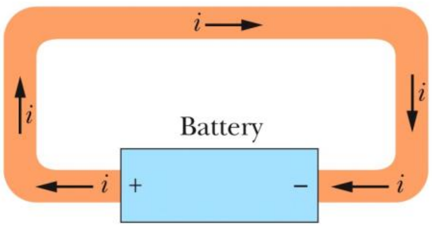
Figure c shows a section of a conductor, part of a conducting loop in which current has been established. If charge *dq* passes through a hypothetical plane (such as aa') in time *dt*, then the current *i* through that plane is defined as

$$i = \frac{dq}{dt} \quad \text{(definition of current).}$$

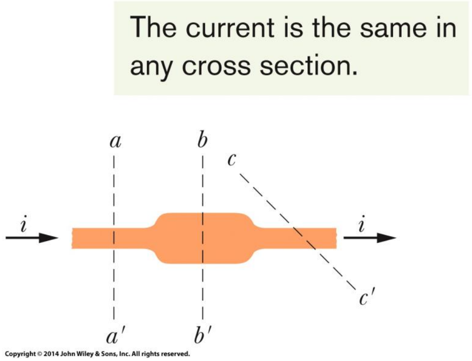
We can find the charge that passes through the plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i\, dt,$$

(a)



(b)



(c)

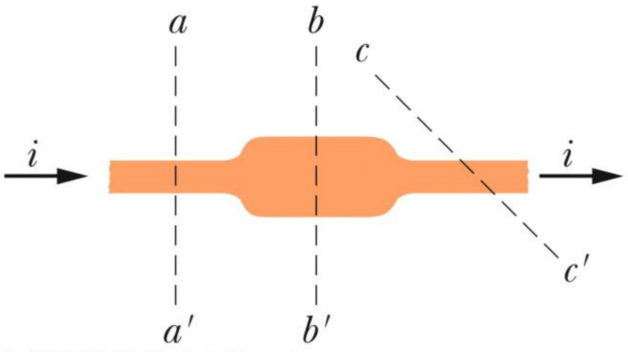
26-1 Electric Current

$$i = \frac{dq}{dt} \quad \text{(definition of current).}$$

We can find the charge that passes through the plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i\,dt,$$

The current is the same in any cross section.



Under steady-state conditions, the current is the same for planes aa’, bb’, and cc’ and indeed for all planes that pass completely through the conductor, no matter what their location or orientation.

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

26-1 Electric Current

Figure (a) shows a conductor with current i_0 splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

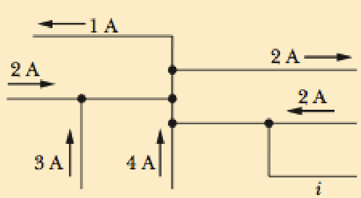
$$i_0 = i_1 + i_2.$$

Figure (b) suggests, bending or reorienting the wires in space does not change the validity of the above equation Current arrows show only a direction (or sense) of flow along a conductor, not a direction in space.

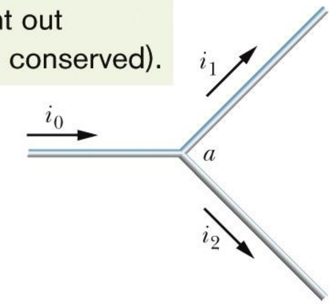
A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

Checkpoint 1

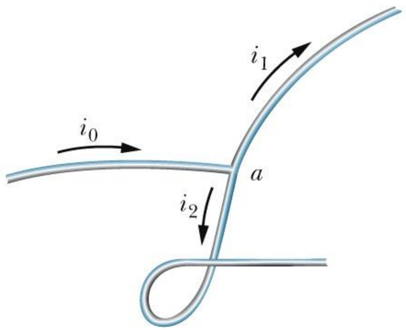
The figure here shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?



The current into the junction must equal the current out (charge is conserved).



(a)



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26-1 Electric Current

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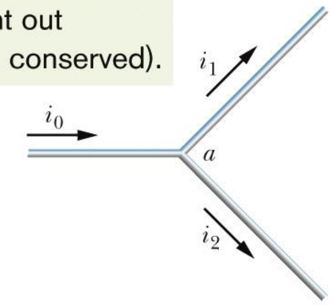
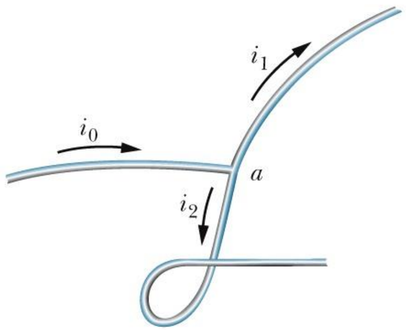


(a)



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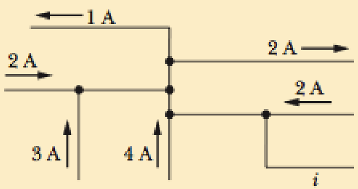


Answer: 8A with arrow pointing right

26-2

Electric Current

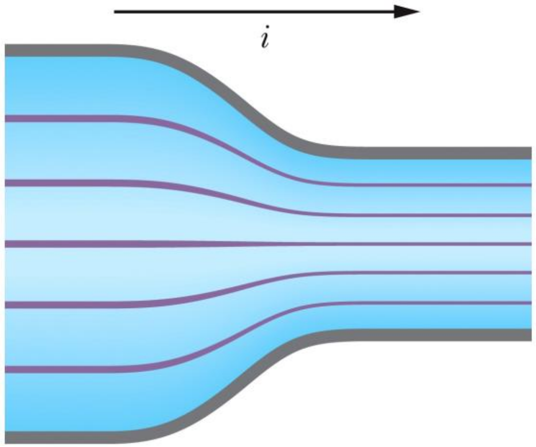
26-2 Current Density (Current per unit area)

Current *i* (a scalar quantity) is related to **current density *J*** (a vector quantity) by

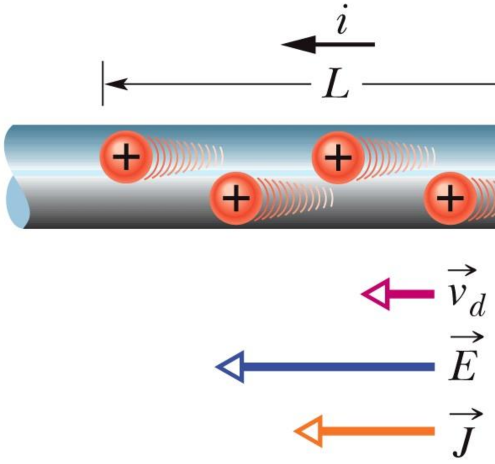
$$i = \int \vec{J} \cdot d\vec{A}.$$

where *d**A*** is a vector perpendicular to a surface element of area *d**A*** and the integral is taken over any surface cutting across the conductor. The current density ***J*** has the same direction as the velocity of the moving charges if they are positive charges and the opposite direction if the moving charges are negative.

Streamlines representing current density in the flow of charge through a constricted conductor.



26-2 Current Density

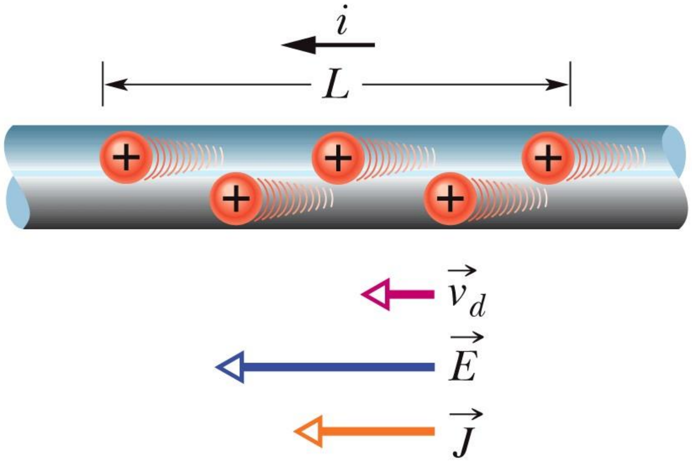


Conduction electrons are actually moving to the right but the conventional current *i* is said to move to the left.

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to drift with a drift speed v_d

For example, in the copper conductors of household wiring, electron drift speeds are perhaps 10^{-5} or 10^{-4} m/s, whereas the random-motion speeds are around 10^6 m/s

26-2 Current Density



Conduction electrons are actually moving to the right but the conventional current *i* is said to move to the left.

Current is said to be due to positive charges that are propelled by the electric field. In the figure, positive charge carriers drift at speed v_d in the direction of the applied electric field ***E*** which here is applied to the left. By convention, the direction of the current density ***J*** and the sense of the current arrow are drawn in that same direction, as is the drift speed v_d.

The drift velocity v_d is related to the current density by

$$\vec{J} = (ne)\vec{v}_d.$$

Here the product *ne*, whose SI unit is the coulomb per cubic meter (C/m^3), is the **carrier charge density**.

26-3

Resistance and Resistivity

26-3 Resistance and Resistivity

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its **electrical resistance**. The resistance *R* of a conductor is defined as

$$R = \frac{V}{i} \quad \text{(definition of } R\text{)}.$$

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere}$$

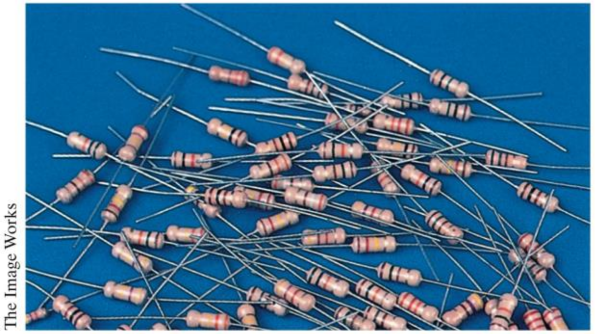
where *V* is the potential difference across the conductor and *i* is the current through the conductor. Instead of the resistance R of an object, we may deal with the **resistivity** *ρ* of the material:

Objects: Resistance
Materials: Resistivity

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{)}.$$

The reciprocal of resistivity is **conductivity** *σ* of the material:

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}.$$



Assortment of Resistors

A conductor whose function in a circuit is to provide a specified resistance is called a resistor

26-3 Resistance and Resistivity

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

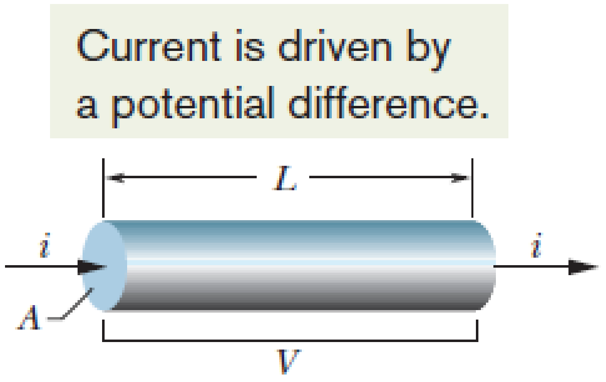
[a]An alloy specifically designed to have a small value of α.
[b]Pure silicon doped with phosphorus impurities to a charge carrier density of $10^{23}\ m^{-3}$.
[c]Pure silicon doped with aluminum impurities to a charge carrier density of $10^{23}\ m^{-3}$.

Resistance is a property of an object. Resistivity is a property of a material.

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{)}.$$

$$\frac{\text{unit } (E)}{\text{unit } (J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\,\text{m} = \Omega \cdot \text{m}.$$

26-3 Resistance and Resistivity



A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

$$\vec{E} = \rho \vec{J}.$$

This equation hold only for isotropic materials—materials whose electrical properties are the same in all directions

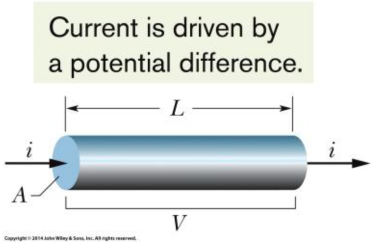
26-3 Resistance and Resistivity

The resistance R of a conducting wire of length L and uniform cross section is

$$E = V/L \quad \text{and} \quad J = i/A$$

$$R = \rho \frac{L}{A}.$$

Here A is the cross-sectional area.

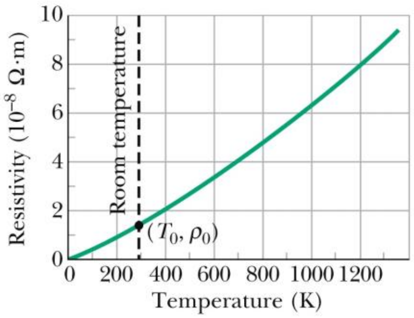


A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

The resistivity ρ for most materials changes with temperature. For many materials, including metals, the relation between ρ and temperature T is approximated by the equation

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

Here T_0 is a reference temperature, ρ_0 is the resistivity at T_0, and α is the temperature coefficient of resistivity for the material.



The resistivity of copper as a function of temperature.

26-3

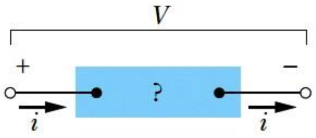
Ohm's Law

26-4 Ohm's Law

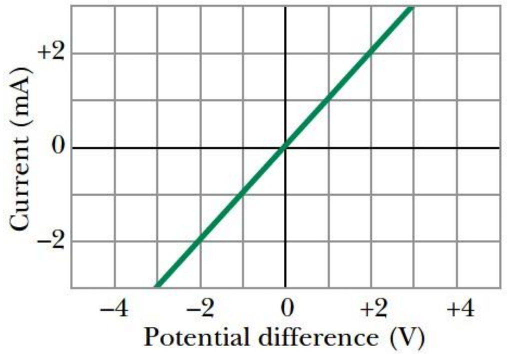
Figure (a) shows how to distinguish among devices. A potential difference *V* is applied across the device being tested, and the resulting current *i* through the device is measured as *V* is varied in both magnitude and polarity.

Figure (b) is a plot of *i* versus *V* for one device. This plot is a straight line passing through the origin, so the ratio *i/V* (which is the slope of the straight line) is the same for all values of V. This means that the resistance *R = V/i* of the device is independent of the magnitude and polarity of the applied potential difference *V*.

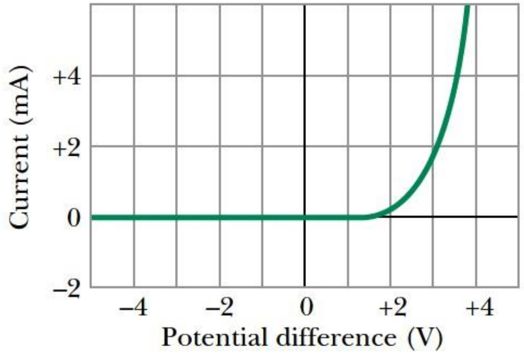
Figure (c) is a plot for another conducting device. Current can exist in this device only when the polarity of *V* is positive and the applied potential difference is more than about 1.5 *V*. When current does exist, the relation between *i* and *V* is not linear; it depends on the value of the applied potential difference *V*.



(a)



(b)



(c)

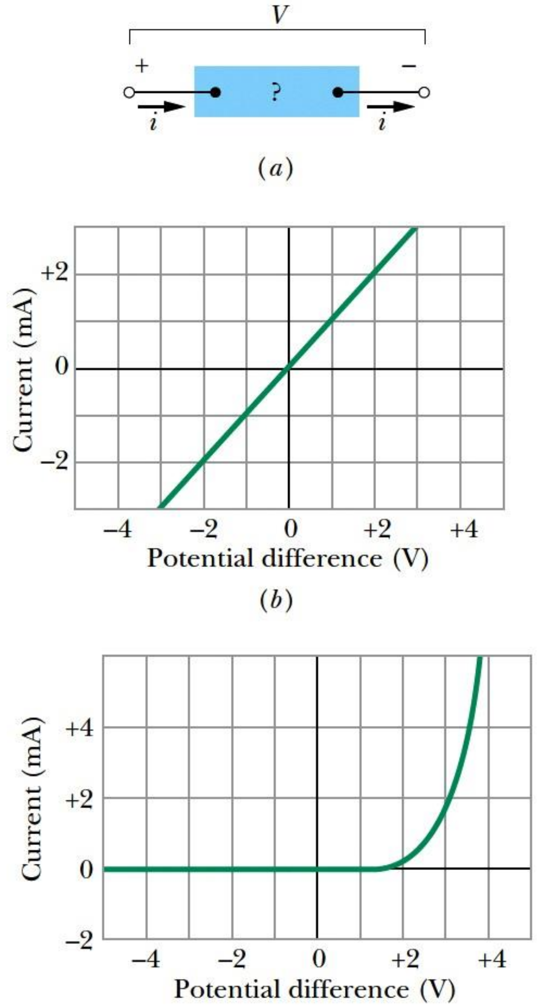
26-4 Ohm's Law

Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

$$I \sim V \quad \text{or} \quad I = V/R$$

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

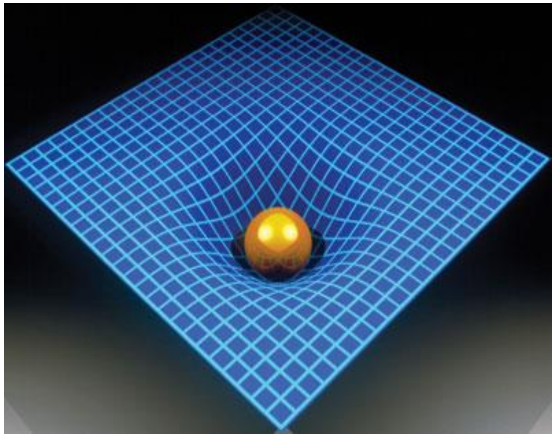
A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

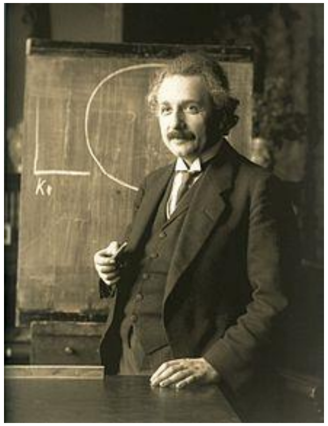


(c)

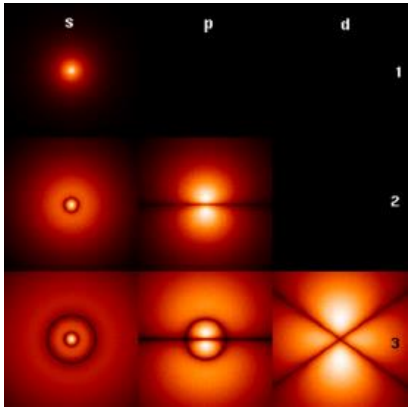
Bonus slide (not the exam)!

- **General relativity:** The current description of gravitation in modern physics





- **Quantum Physics:** The physics that governs the behavior of matter and light at the atomic (and subatomic) scale



Schrödinger's cat

Good documentaries to watch: The Elegant Universe (Nova, PBS)

26-4 Ohm's Law

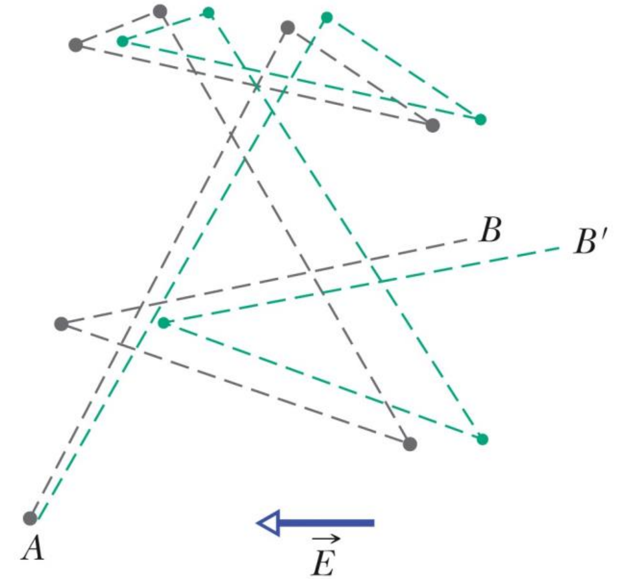
A Microscopic View

The assumption that the conduction electrons in a metal are free to move like the molecules in a gas leads to an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Here n is the number of free electrons per unit volume and τ is the mean time between the collisions of an electron with the atoms of the metal.

Metals obey Ohm's law because the mean free time τ is approximately independent of the magnitude E of any electric field applied to a metal.



The gray lines show an electron moving from A to B, making six collisions en route. The green lines show what the electron's path might be in the presence of an applied electric field ***E***. Note the steady drift in the direction of -***E***.

26-3

Power, Semiconductors, Superconductors

26-5 Power, Semiconductors, Superconductors

The battery at the left supplies energy to the conduction electrons that form the current.

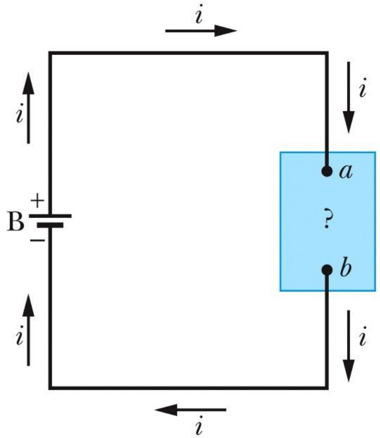


Figure shows a circuit consisting of a battery B that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude V across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal b.

The power *P*, or rate of energy transfer, in an electrical device across which a potential difference *V* is maintained is

$$P = iV \quad \text{(rate of electrical energy transfer).}$$

If the device is a resistor, the power can also be written as

$$P = i^2R \quad \text{(resistive dissipation)}$$

or,

$$P = \frac{V^2}{R} \quad \text{(resistive dissipation).}$$

26-5 Power, Semiconductors, Superconductors

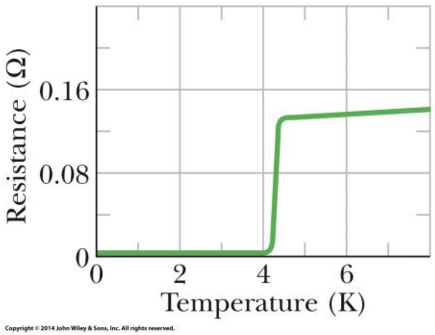
Semiconductors are materials that have few conduction electrons but can become conductors when they are doped with other atoms that contribute charge carriers.

In a semiconductor, *n* (number of free electrons) is small (unlike conductor) but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a decrease of resistivity with increasing temperature, as indicated by the negative temperature coefficient of resistivity for silicon in Table 26-2.

Table 26-2 Some Electrical Properties of Copper and Silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	8.49×10^{28}	1×10^{16}
Resistivity, $\Omega \cdot m$	1.69×10^{-8}	2.5×10^{3}
Temperature coefficient of resistivity, K^{-1}	$+4.3 \times 10^{-3}$	-70×10^{-3}

Superconductors are materials that lose all electrical resistance below some critical temperature. Most such materials require very low temperatures, but some become superconducting at temperatures as high as room temperature.



The resistance of mercury drops to zero at a temperature of about 4 K.

26 Summary

Current

- The electric current i in a conductor is defined by

$$i = \frac{dq}{dt}. \qquad \text{Eq. 26-1}$$

Current Density

- Current is related to current density by

$$i = \int \vec{J} \cdot d\vec{A}, \qquad \text{Eq. 26-4}$$

Drift Speed of the Charge Carriers

- Drift speed of the charge carriers in an applied electric field is related to current density by

$$\vec{J} = (ne)\vec{v}_d, \qquad \text{Eq. 26-7}$$

Resistance of a Conductor

- Resistance R of a conductor is defined by

$$R = \frac{V}{i} \qquad \text{Eq. 26-8}$$

- Similarly the resistivity and conductivity of a material is defined by

$$\rho = \frac{1}{\sigma} = \frac{E}{J} \qquad \text{Eq. 26-10\&12}$$

- Resistai ucting wire of length L and uniform cross section is

$$R = \rho\frac{L}{A} \qquad \text{Eq. 26-16}$$

Change of ρ with Temperature

- The resistivity of most material changes with temperature and is given as

$$\rho - \rho_0 = \rho_0\alpha(T - T_0). \qquad \text{Eq. 26-17}$$

26 Summary

Ohm's Law

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance R (defined by **Eq. 26-8** as *V/i*) is independent of the applied potential difference *V.*

Resistivity of a Metal

- By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Eq. 26-22

Power

- The power P, or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV$$

Eq. 26-26

- If the device is a resistor, we can write

$$P = i^2 R = \frac{V^2}{R}$$

Eq. 26-27&28

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CHAPTER 27

Circuits

27-1

Electric Current

27-1 Single-Loop Circuits

emf: electromotive force

To produce a steady flow of charge, you need a "**charge pump**," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals. We call such a device an ***emf* device**, and the device is said to provide an *emf* $\mathscr{E}$, <u>which means that it does work on charge carriers</u>.

An emf device supplies the energy for the motion (of charge) via the work it does and thus maintain a potential difference between its terminal

Emf devices:
- Battery
- Electric generator
- Solar cells
- Fuel cells
- Human beings
- Electric eels (600 V)

Hydrogen Fuel cells (Honda CFX clarity)

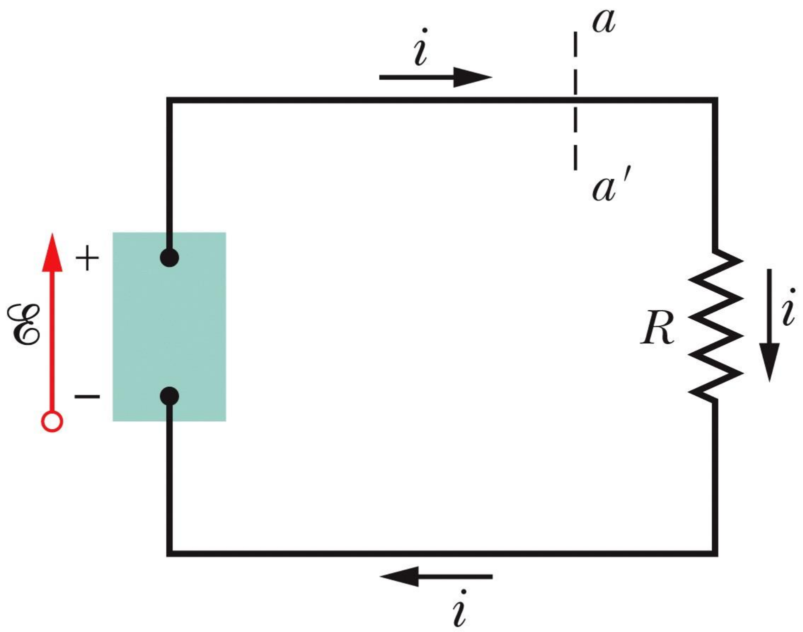

Electric eels

Battery (Chevy Volt)


Solar cells

27-1 Single-Loop Circuits

Figure shows an *emf* device (consider it to be a battery) that is part of a simple circuit containing a single resistance *R*. The *emf* device keeps one of its terminals (called the positive terminal and often labeled +) at a higher electric potential than the other terminal (called the negative terminal and labeled -). We can represent the *emf* of the device with an arrow that points from the negative terminal toward the positive terminal as in Figure. A small circle on the tail of the *emf* arrow distinguishes it from the arrows that indicate current direction.



27-1 Single-Loop Circuits

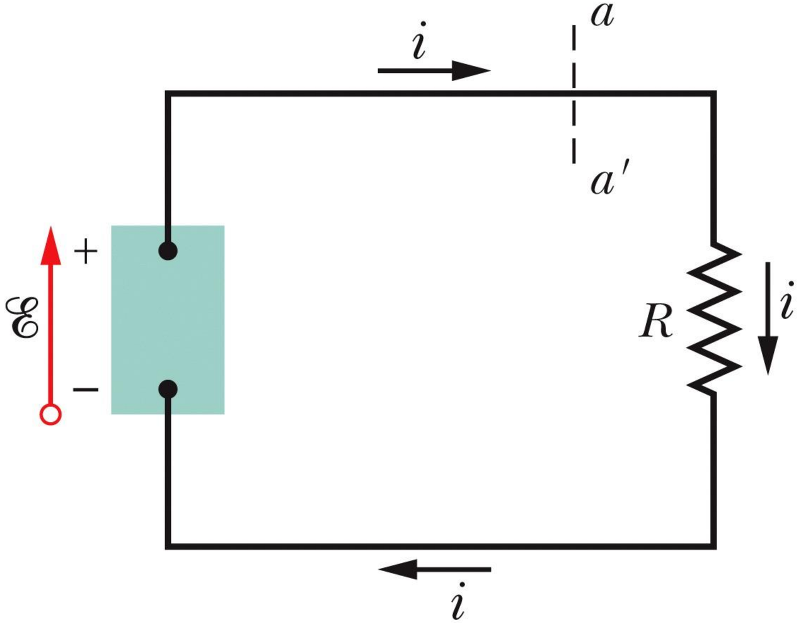
An emf device does work on charges to maintain a potential difference between its output terminals. If *dW* is the work the device does to force positive charge *dq* from the negative to the positive terminal, then the *emf* (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{)}.$$

emf is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal.

An **ideal *emf* device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the *emf*.

A **real *emf* device** has internal resistance. The potential difference between its terminals is equal to the *emf* only if there is no current through the device.



27-1 Single-Loop Circuits

Calculating Current in a Single-Loop Circuits

Potential Method

In the figure, let us start at point a, whose potential is V_a, and mentally walk clockwise around the circuit until we are back at point a, keeping track of potential changes as we move. Our starting point is at the low-potential terminal of the battery. Because the battery is ideal, the potential difference between its terminals is equal to $\mathscr{E}$. When we pass through the battery to the high-potential terminal, the change in potential is $+\mathscr{E}$.

After making a complete loop, our initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \mathscr{E} - iR = V_a.$$

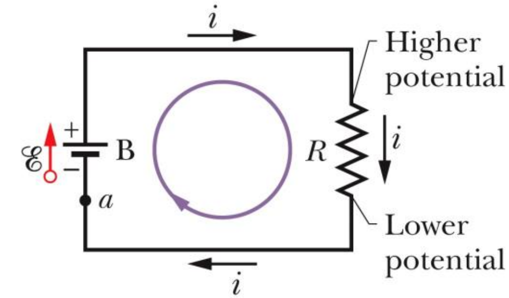
The value of V_a cancels from this equation, which becomes

$$\mathscr{E} - iR = 0.$$

Which gives us

$$i = \frac{\mathscr{E}}{R}.$$

The battery drives current through the resistor, from high potential to low potential.



Calculating Current in a Single-Loop Circuits

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Also known as Kirchhoff' loop rule

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

27-1 Single-Loop Circuits

Internal Resistance

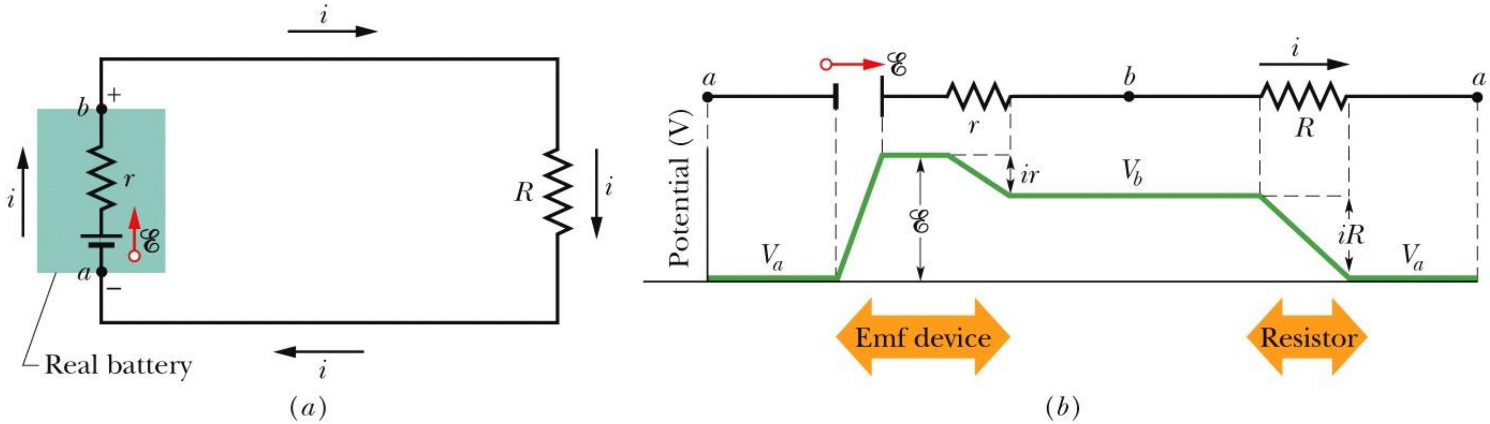


Figure (a) shows a real battery, with internal resistance *r*, wired to an external resistor of resistance *R*. The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an unremovable feature of the battery. Figure (b) shows graphically the changes in electric potential around the circuit. Now if we apply the loop rule clockwise beginning at point a, the changes in potential give us

$$\mathscr{E} - ir - iR = 0.$$

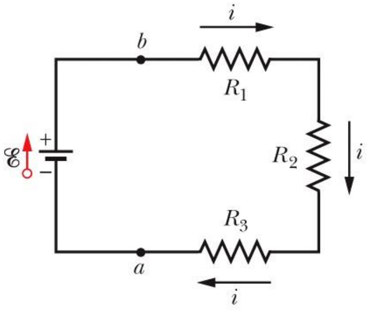
Solving for the current we find,

$$i = \frac{\mathscr{E}}{R + r}.$$

27-1 Single-Loop Circuits

Resistance in Series

Figure (a) shows three resistances connected in series to an ideal battery with *emf* $\mathscr{E}$. The resistances are connected one after another between a and b, and a potential difference is maintained across a and b by the battery. The potential differences that then exist across the resistances in the series produce identical currents *i* in them. To find total resistance R_{eq} in Fig. (b), we apply the loop rule to both circuits. For Fig. (a), starting at a and going clockwise around the circuit, we find

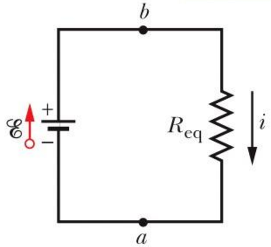


(a)

Series resistors and their equivalent have the same current ("ser-i").

$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0, \qquad i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}.$$ or

For Fig. (b), with the three resistances replaced with a single equivalent resistance R_{eq}, we find



(b)

$$\mathscr{E} - iR_{eq} = 0, \qquad i = \frac{\mathscr{E}}{R_{eq}}.$$ or

Equating them, we get,

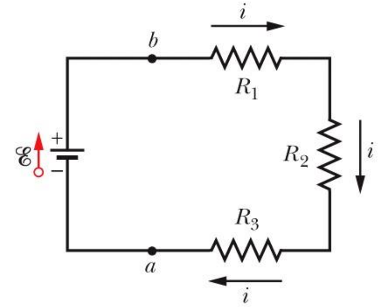
$$R_{eq} = R_1 + R_2 + R_3.$$

$$R_{eq} = \sum_{j=1}^{n} R_j \quad (n \text{ resistances in series}).$$

Resistance in Series

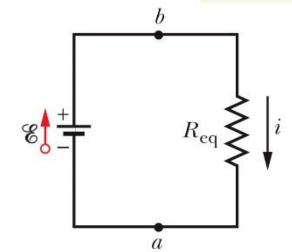
When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.



(a)

Series resistors and their equivalent have the same current ("ser-i").



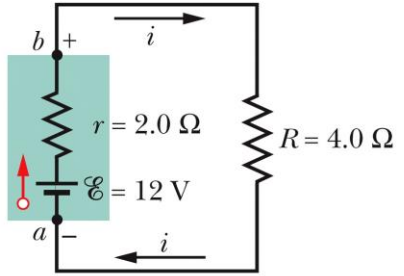
(b)

27-1 Single-Loop Circuits

Potential Difference

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

The internal resistance reduces the potential difference between the terminals.



Potential Difference across a real battery: In the Figure, points a and b are located at the terminals of the battery. Thus, the potential difference V_b - V_a is the terminal-to-terminal potential difference V across the batter and is given by:

$$V = \mathscr{E} - ir.$$

Grounding a Circuit: Grounding a circuit usually means connecting one point in the circuit to a conducting path to Earth's surface (actually to the electrically conducting moist dirt and rock below ground)

Power of *emf* Device: The rate P_{emf} at which the *emf* device transfers energy both to the charge carriers and to internal thermal energy is

$$P_{\text{emf}} = i\mathscr{E} \quad \text{(power of emf device).}$$

27-2

Multiloop Circuits

27-2 Multiloop Circuits

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

The current into the junction must equal the current out (charge is conserved).

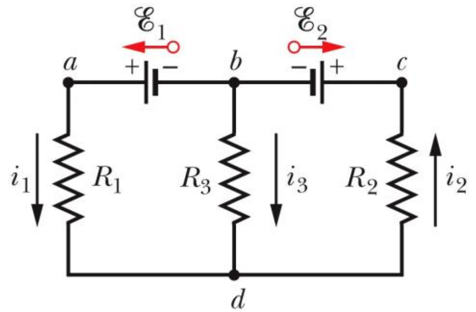


Figure shows a circuit containing more than one loop. If we traverse the left-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$$

If we traverse the right-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from b) the equation

$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

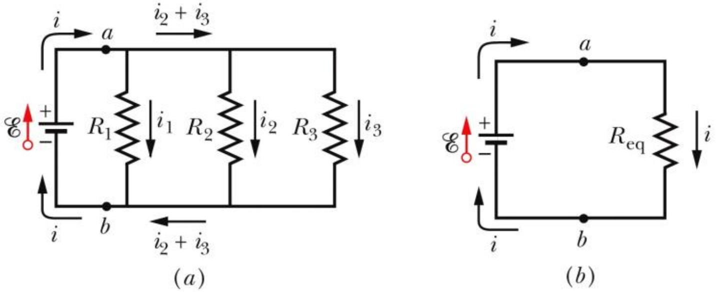
which is the sum of two small loops equations.

27-2 Multi-Loop Circuits

Resistances in Parallel

Figure (a) shows three resistances connected in parallel to an ideal battery of *emf* $\mathcal{E}$. The applied potential difference V is maintained by the battery. Fig. b, the three parallel resistances have been replaced with an equivalent resistance R_{eq}.

Parallel resistors and their equivalent have the same potential difference ("par-V").



To derive an expression for R_{eq} in Fig. (b), we first write the current in each actual resistance in Fig. (a) as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b. If we apply the junction rule at point a in Fig. (a) and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

If we replaced the parallel combination with the equivalent resistance R_{eq} (Fig. b), we would have $i = \frac{V}{R_{eq}}$. and thus substituting the value of *i* from above equation we get,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

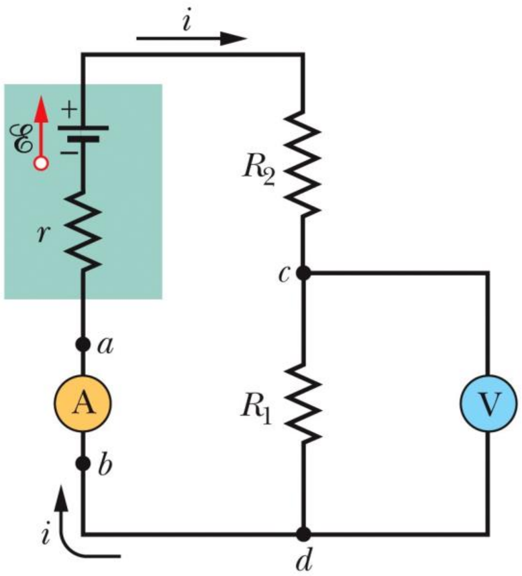
27-2 Multi-Loop Circuits

Resistance and capacitors

Table 27-1 Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

27-3 The Ammeter and The Voltmeter



An instrument used to measure currents is called an **ammeter**. To measure the current in a wire, you usually have to break or cut the wire and insert the ammeter so that the current to be measured passes through the meter. In the figure, ammeter A is set up to measure current *i*. It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit. Otherwise, the very presence of the meter will change the current to be measured.

A meter used to measure potential differences is called a **voltmeter**. To find the potential difference between any two points in the circuit, the voltmeter terminals are connected between those points without breaking or cutting the wire. In the Figure, voltmeter V is set up to measure the voltage across R_1. It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected. This is to insure that only a negligible current passes through the voltmeter, otherwise, the meter alters the potential difference that is to be measured.

27-3

RC Circuits

27-4 *RC* Circuits

Charging a capacitor: The capacitor of capacitance *C* in the figure is initially uncharged. To charge it, we close switch S on point a. This completes an *RC* series circuit consisting of the capacitor, an ideal battery of *emf* $\mathscr{E}$, and a resistance *R*.
The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

in which $C = q_0$ is the equilibrium (final) charge and $RC=\tau$ is the capacitive time constant of the circuit. During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \quad \text{(charging a capacitor).}$$

And the voltage is:

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

The product *RC* is called the **capacitive time constant** of the circuit and is represented with the symbol *τ*.

$$\tau = RC \quad \text{(time constant).}$$

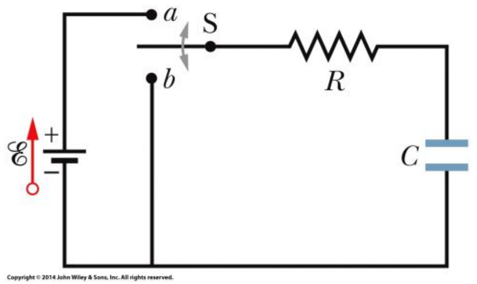
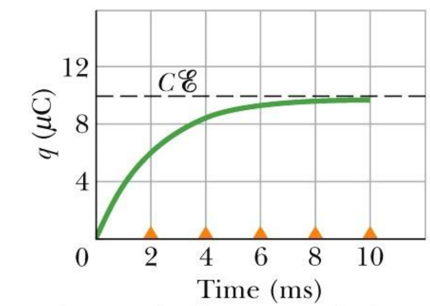


Figure: RC circuit



The plot shows the buildup of charge on the capacitor of the above figure.

27-4 *RC* Circuits

Discharging a capacitor: Assume now that the capacitor of the figure is fully charged to a potential V_0 equal to the emf $\mathscr{E}$ of the battery. At a new time *t=0*, switch S is thrown from a to b so that the capacitor can discharge through resistance *R*.
When a capacitor discharges through a resistance R, the charge on the capacitor decays according to

$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor)},$$

where q_0 (=CV_0) is the initial charge on the capacitor.

During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \quad \text{(discharging a capacitor)}.$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

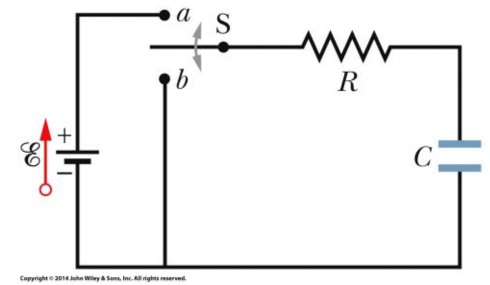
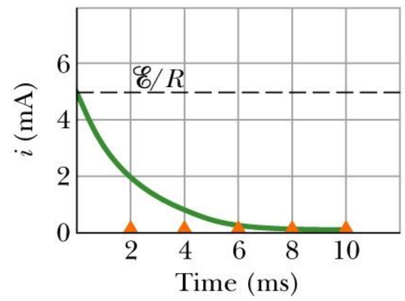


Figure: RC circuit



A plot shows the decline of the charging current in the circuit of the above figure.

27-3 RC Circuits

A first order non-homogeneous differential equation: $a\frac{dq(t)}{dt} + bq(t) + c = 0$ where a,b,c are constants	
General solution: $q(t) = Ke^{-\frac{b}{a}t} + \frac{c}{b}[e^{-\frac{b}{a}t} - 1]$	
Charging a capacitor	**Capacitor Discharge**
Example: $R\frac{dq(t)}{dt} + \frac{q(t)}{C} - \mathcal{E} = 0$ $i = +\frac{dq}{dt}$	Example: $R\frac{dq(t)}{dt} + \frac{q(t)}{C} = 0$ $i = -\frac{dq}{dt}$
$R = a\,; b = \frac{1}{C}\,; c = -\mathcal{E}$	$R = a\,; b = \frac{1}{C}\,; c = 0$
Solution: $q(t) = Ke^{-\frac{1}{RC}t} - C\mathcal{E}[e^{-\frac{1}{RC}t} - 1]$	Solution: $q(t) = Ke^{-\frac{1}{RC}t}$
$at\ t = 0\ \ q = 0\ \rightarrow\ \ K = 0$	$at\ t = 0\ \ q = C\mathcal{E}\ \rightarrow\ \ K = C\mathcal{E}$
$q(t) = C\mathcal{E}[1 -\ e^{-\frac{1}{RC}t}]$	$q(t) = C\mathcal{E}\ e^{-\frac{1}{RC}t}$

27-4

Summary

27 Summary

Emf

- The **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{)}. \qquad \textbf{Eq. 27-1}$$

Single-Loop Circuits

- Current in a single-loop circuit:

$$i = \frac{\mathscr{E}}{R + r}, \qquad \textbf{Eq. 27-4}$$

Power

- The rate P of energy transfer to the charge carriers is

$$P = iV, \qquad \textbf{Eq. 27-14}$$

- The rate P_r at which energy is dissipated as thermal energy in the battery is

$$P_r = i^2 r. \qquad \textbf{Eq. 27-16}$$

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}. \qquad \textbf{Eq. 27-17}$$

Series Resistance

- When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j \qquad \textbf{Eq. 27-7}$$

Parallel Resistance

- When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j} \qquad \textbf{Eq. 27-24}$$

RC Circuits

- The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC}) \qquad \textbf{Eq. 27-33}$$

- During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \qquad \textbf{Eq. 27-34}$$

- During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \qquad \textbf{Eq. 27-40}$$

27+

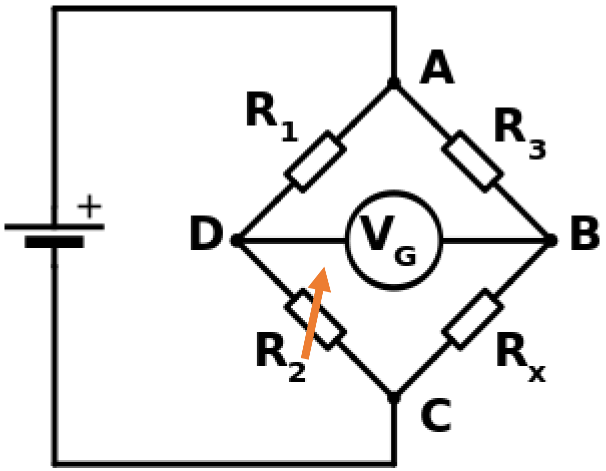
Wheatstone Bridge

History

- The Wheatstone Bridge was invented in 1833 by Samuel Hunter Christie
- Later named after Sir Charles Wheatstone for his many applications of the circuit through the 1840s
- The most common procedure for the bridge remains the testing of unknown electrical resistance

How Does it Work?

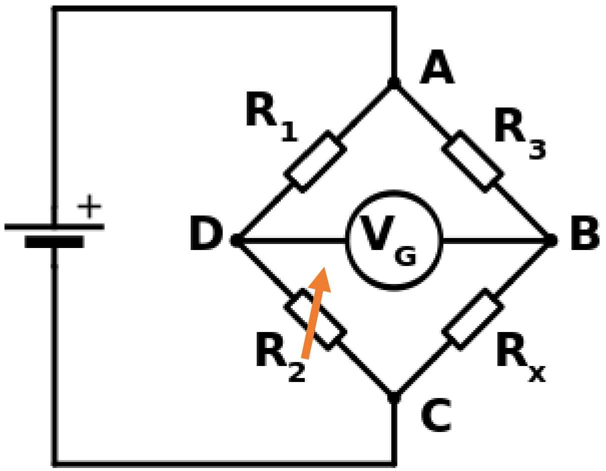
- Uses ratio of 3 known resistors
- Measures fourth unknown resistance
- Balanced voltage between point 1 and battery's negative, and between point 2 and battery's negative allows the measurement



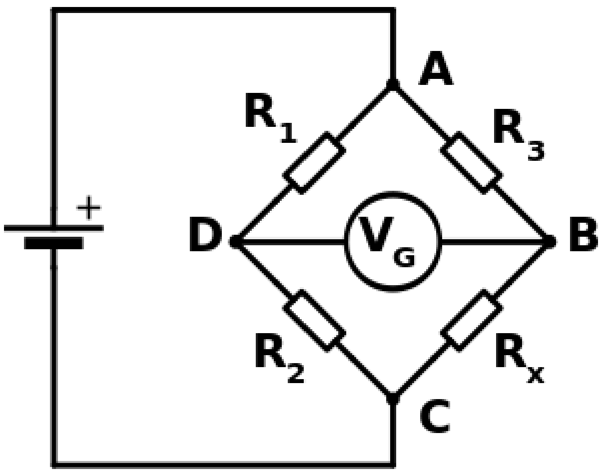
How Does it Work?

If the ratio of the two resistances (R2 / R1) is equal to the ratio of the two (Rx / R3), then the voltage between the two midpoints (B and D) will be zero and no current will flow through the galvanometer

$$\frac{R_2}{R_1} = \frac{R_x}{R_3}$$
$$\Rightarrow R_x = \frac{R_2}{R_1} \cdot R_3$$



How Does it Work?



$$I_3 - I_x + I_G = 0$$
$$I_1 - I_2 - I_G = 0$$

Kirchhoff's rule is used for finding the voltage in the loops ABD and BCD

$$(I_3 \cdot R_3) - (I_G \cdot R_G) - (I_1 \cdot R_1) = 0$$
$$(I_x \cdot R_x) - (I_2 \cdot R_2) + (I_G \cdot R_G) = 0$$

When the bridge is balanced, then $I_G = 0$, so the second set of equations can be rewritten as:

$$I_3 \cdot R_3 = I_1 \cdot R_1$$
$$I_x \cdot R_x = I_2 \cdot R_2$$

$I_3 = I_x$ and $I_1 = I_2$

$$R_x = \frac{R_2 \cdot I_2 \cdot I_3 \cdot R_3}{R_1 \cdot I_1 \cdot I_x}$$

$$R_x = \frac{R_3 \cdot R_2}{R_1}$$

How is it used?

- Main focus of the Wheatstone Bridge = applications using electricity
- The circuit monitors sensor devices like strain gauges – it reads the level of the strain in the system
- The galvanometer measures whether the gauges are balanced or not
- Electrical power distributors use the Wheatstone Bridge to locate breaks in the power lines