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CHAPTER 24

Electric Potential

24-1

Electric Potential

24-1 Electric Potential

Particle 1 is located at point P in the electric field of particle 2.

- If we release particle 1 at P, it begins to move and thus has kinetic energy.
- Energy cannot appear by magic, so from where does it come?

It comes from the electric potential energy U associated with the force between the two particles

- To account for **the potential energy U** (which is a scalar quantity), we define an **electric potential V** (also a scalar quantity) that is set up at P by particle 2.
- The electric potential exists regardless of whether particle 1 is at P.

24-1 Electric Potential

- By analogy to the gravitational potential energy, we have for the electric potential energy a 1/r relationship.

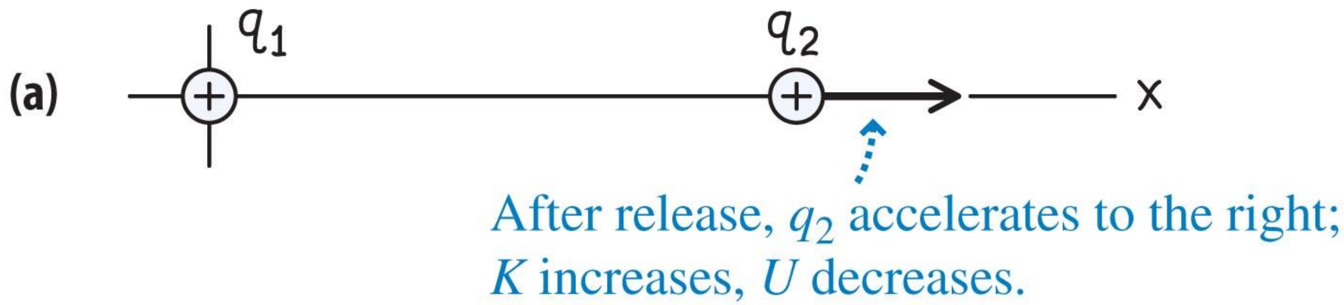
$$U = \frac{kq_1q_2}{r} \qquad \text{(Potential energy of two point charges; SI unit: J)}$$

- Where the zero of potential energy is at very large separation r (r >>1)

- Potential energy can be positive or negative. It is positive for two like charges and negative or two unlike charges.

Attraction or repulsion

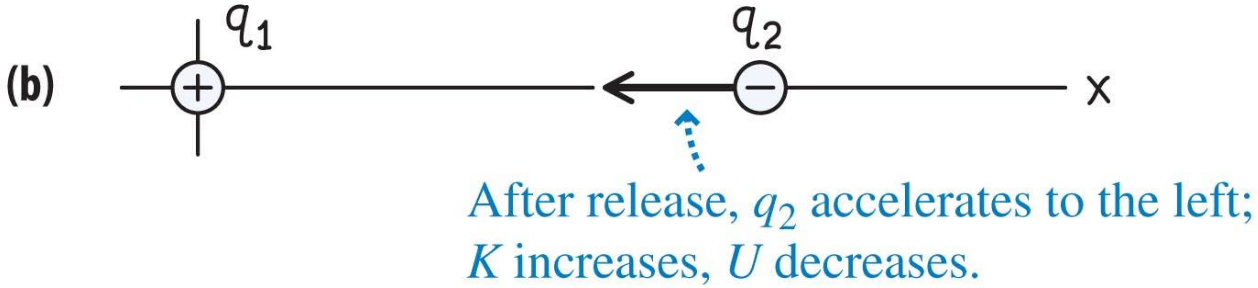
- **Two positive charges**: q_1 is fixed at the origin, and q_2 is free to move.
- If charge q_2 is released from rest on the +x-axis, describe its subsequent motion. Tell what happens to the kinetic energy K, potential energy U, and total energy E



- After release, q_2 accelerates to the right
- K increases, U decreases
- E = K + U is conserved and stays constant.

Attraction or repulsion

- **Two opposite charges**: q_1 is fixed at the origin, and q_2 is free to move.
- If charge q_2 is released from rest on the +x-axis, describe its subsequent motion. Tell what happens to the kinetic energy K, potential energy U, and total energy E



- After release, q_2 accelerates to the left, what about K, U & E?;
- K increases, U decreases (it becomes even more negative)
- E = K + U is conserved and stays constant.

24-1 Electric Potential

- The electric potential V at a point P in the electric field of a charged object is

$$V = \frac{U}{q_0}$$

- The electric potential is the amount of electric potential energy per unit charge

Two Cautions. (1) The (now very old) decision to call V a *potential* was unfortunate because the term is easily confused with *potential energy*. Yes, the two quantities are related (that is the point here) but they are very different and not interchangeable. (2) Electric potential is a scalar, not a vector. (When you come to the homework problems, you will rejoice on this point.)

- The SI unit for potential that follows is the joule per coulomb.

This combination occurs so often that a special unit, the volt (abbreviated V), is used to represent it.

1 volt = 1 joule per coulomb.

With two unit conversions, we can now switch the unit for electric field from newtons per coulomb to a more conventional unit:

$$1\ \mathrm{N/C} = \left(1\frac{\mathrm{N}}{\mathrm{C}}\right)\left(\frac{1\ \mathrm{V}}{1\ \mathrm{J/C}}\right)\left(\frac{1\ \mathrm{J}}{1\ \mathrm{N \cdot m}}\right) = 1\ \mathrm{V/m}.$$

24-1 Electric Potential

Change in Electric Potential. If the particle moves through a potential difference *ΔV*, the change in the electric potential energy is

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$

Work by the Field. The work W done by the electric force as the particle moves from *i* to *f:*

$$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$

Conservation of Energy. If a particle moves through a change *ΔV* in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

$$U_i + K_i = U_f + K_f,$$

$$\Delta K = -\Delta U.$$

$$\Delta K = -q\,\Delta V = -q(V_f - V_i).$$

Work by an Applied Force. If some force in addition to the electric force acts on the particle, we account for that work

$$\text{(initial energy)} + \text{(work by applied force)} = \text{(final energy)}$$

$$U_i + K_i + W_{\text{app}} = U_f + K_f.$$

$$\Delta K = -\Delta U + W_{\text{app}} = -q\,\Delta V + W_{\text{app}}.$$

24-1 Electric Potential

Electron-volts. In atomic and subatomic physics, energy measures in the SI unit of joules often require awkward powers of ten. A more convenient (but non-SI unit) is the *electron-volt* (eV), which is defined to be equal to the work required to move a single elementary charge e (such as that of an electron or proton) through a potential difference ΔV of exactly one volt.

$$1\text{ eV} = e(1\text{ V})$$

$$= (1.602 \times 10^{-19}\text{ C})(1\text{ J/C}) = 1.602 \times 10^{-19}\text{ J}.$$

24-1 Electric Potential

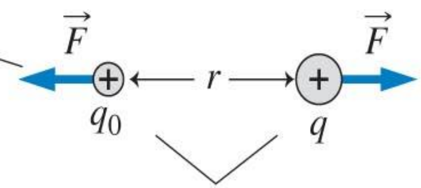
- **Definition:** The **electric potential**, V, is the **potential energy per unit charge**.

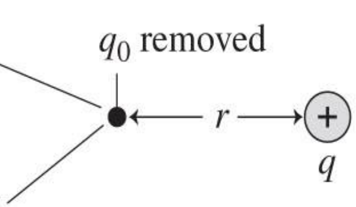
- Thus, the electric potential a distance r for a point charge q is:

$$V = \frac{kq}{r} \quad \text{(Electric potential of a point charge; SI unit: V)}$$

TABLE 16.2 Electric potential defined by analogy with electric field

	Electric force:	$F = \frac{kqq_0}{r^2}$	The force the charges exert on each other
	Potential energy:	$U = \frac{kqq_0}{r}$	The potential energy associated with the pair of charges
Force and potential energy per unit charge q_0	Electric field:	$E = \frac{F}{q_0} = \frac{kq}{r^2}$	The field at this point
	Potential:	$V = \frac{U}{q_0} = \frac{kq}{r}$	The potential at this point





24-2

Equipotential Surfaces and the Electric Field

24-2 Equipotential Surfaces and the Electric Field

Adjacent points that have the same electric potential form an **equipotential surface**, which can be either an imaginary surface or a real, physical surface.

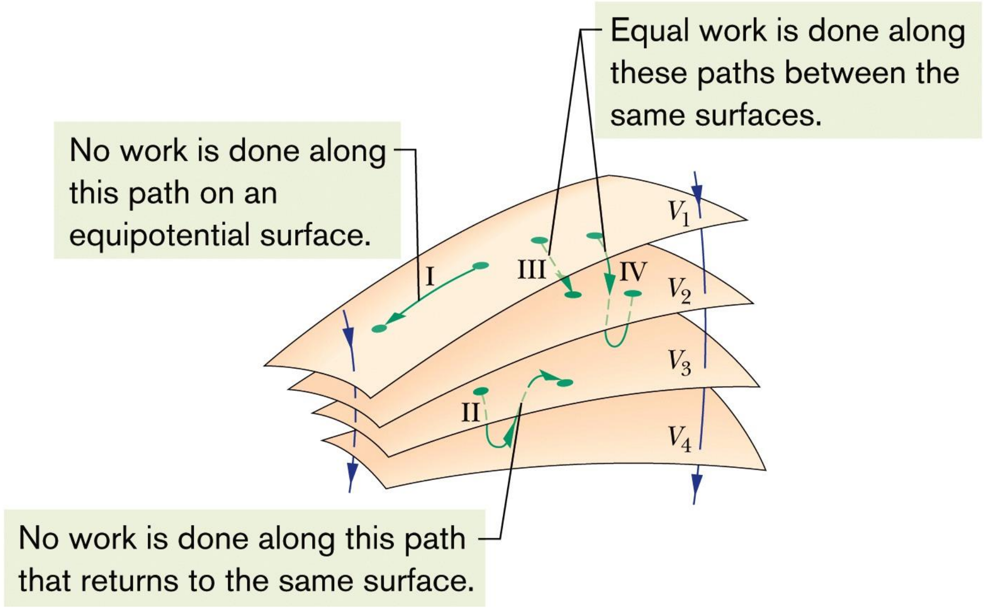


Figure shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. **The work done by the electric field on a charged particle as the particle moves from one end to the other of paths I and II is zero** because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential. **The work done as the charged particle moves from one end to the other of paths III and IV is not zero but has the same value** for both these paths because the initial and final potentials are identical for the two paths; that is, paths **III** and **IV** connect the same pair of equipotential surfaces.

24-2 Equipotential Surfaces and the Electric Field

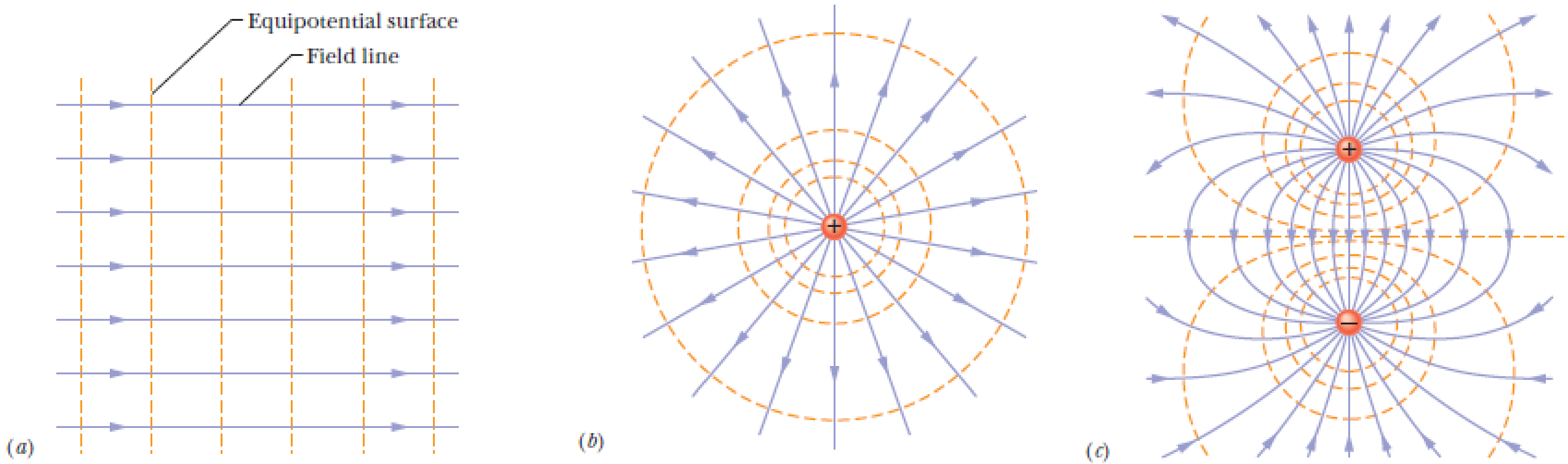


Figure 24-5 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (*a*) a uniform electric field, (*b*) the field due to a charged particle, and (*c*) the field due to an electric dipole.

24-2 Equipotential Surfaces and the Electric Field

The electric potential difference between two points *i* and *f* is

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

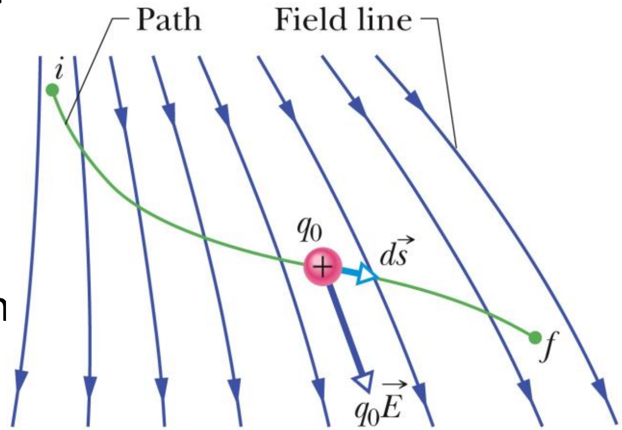
where the integral is taken over any path connecting the points. If the integration is difficult along any particular path we can choose a different path along which the integration might be easier.

If we choose $V_i = 0$, we have, for the potential at a particular point,

$$V = -\int_i^f \vec{E} \cdot d\vec{s}.$$

In a uniform field of magnitude E, the change in potential from a higher equipotential surface to a lower one, separated by distance Δx, is

$$\Delta V = -E\,\Delta x.$$



A test charge q_0 moves from point *i* to point *f* along the path shown in a non-uniform electric field. During a displacement ***ds***, an electric force q_0***E*** acts on the test charge. This force points in the direction of the field line at the location of the test charge.

24-3

Potential due to a Charged Particles

24-3 Potential due to a Charged Particle

Potential due to a group of Charged Particles

The potential due to a collection of charged particles is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Thus, the potential is the algebraic sum of the individual potentials, with no consideration of directions.

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

24-4

Potential due to a Electric Dipole

24-4 Potential due to a Electric Dipole

The net potential at P is given by

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

We can approximate the two lines to P as being parallel and their length difference as being the leg of a right triangle with hypotenuse d (Fig. b). Also, that difference is so small that the product of the lengths is approximately r^2.

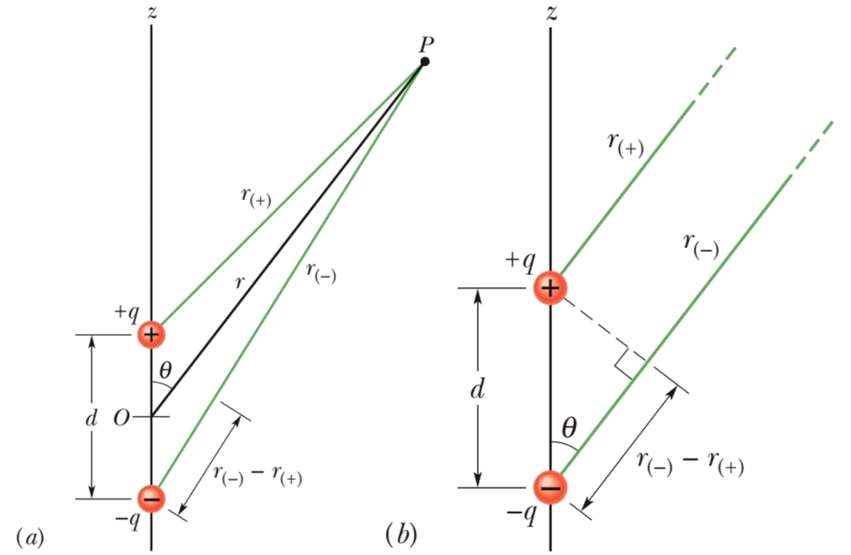
$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

We can approximate *V* to be

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2},$$

where ϑ is measured from the dipole axis as shown in Fig. a. And since *p=qd,* we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)}$$



(a) Point P is a distance *r* from the midpoint O of a dipole. The line OP makes an angle ϑ with the dipole axis.

(b) If P is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length *r*, and the dashed black line is approximately perpendicular to the line of length $r_{(-)}$.

24-5

Potential due to a Continuous Charge Distribution

24-5 Potential due to a Continuous Charge Distribution

For a continuous distribution of charge (over an extended object), the potential is found by
(1) dividing the distribution into charge elements dq that can be treated as particles and then
(2) summing the potential due to each element by integrating over the full distribution:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

We now examine two continuous charge distributions, a line and a disk.

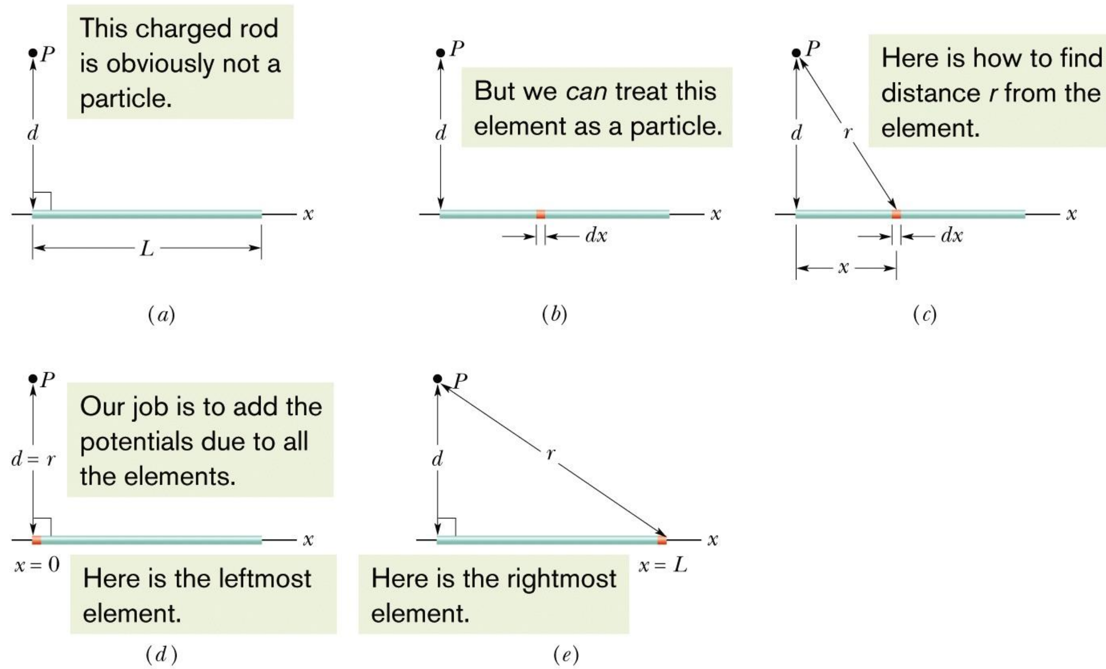
Line of Charge

Fig. a has a thin conducting rod of length *L*. As shown in fig. b the element of the rod has a differential charge of

$$dq = \lambda\, dx.$$

This element produces an electric potential *dV* at point P (fig c) given by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}.$$



We now find the total potential V produced by the rod at point P by integrating *dV* along the length of the rod, from *x = 0* to *x = L* (Figs.d and e)

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx$$

Simplified to,

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$

24-6

Calculating the Field from the Potential

24-6 Calculating the Field from the Potential

- The component of $\vec{E}$ in any direction is the negative of the rate at which the potential changes with distance in that direction:

$$E_s = -\frac{\partial V}{\partial s}.$$

- The x, y, and z components of $\vec{E}$ may be found from

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}.$$

When $\vec{E}$ is uniform, all this reduces to

$$E = -\frac{\Delta V}{\Delta s},$$

where s is perpendicular to the equipotential surfaces.

- The electric field is zero parallel to an equipotential surface.

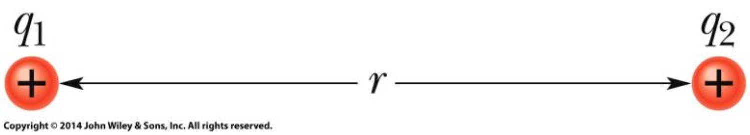
24-7

Electric Potential Energy of a System of Charged Particles

24-7 Electric Potential Energy of a System of Charged Particles

> The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other. For two particles at separation *r*,



Two charges held a fixed distance *r* apart.

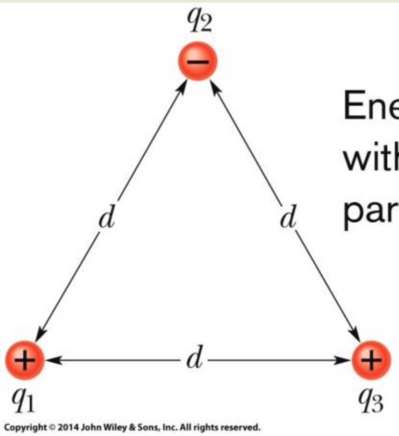
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \quad \text{(two-particle system).}$$

Sample Problem 24.06 **Potential energy of a system of three charged particles**

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.



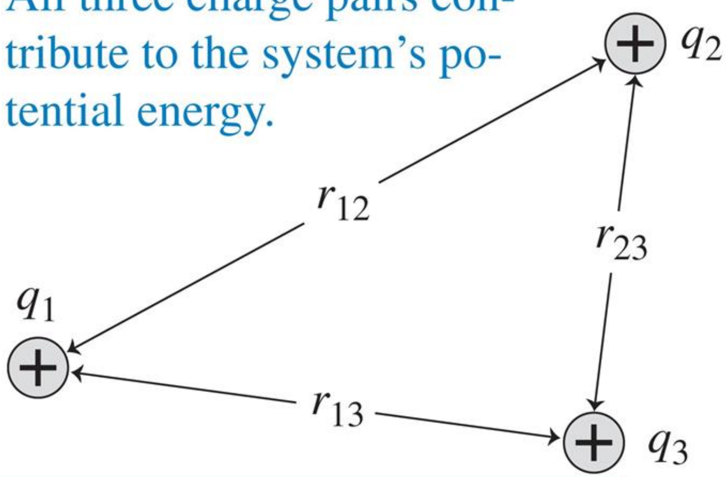
Energy is associated with each pair of particles.

Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

Electric Potential Energy

Multiple charges

- For 3 charges and more, each pair will have a potential energy. The total potential energy is the sum over all three pairs.



$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$ (Potential energy of multiple charges; SI unit: J)

- The more charges there are the bigger the number of charge pairs *(for 4 charges, there exist 6 pairs for 5 charges, 10 pairs, and so on…)* The bigger the computation above becomes.
- It becomes necessary to describe energy in terms of the **electric potential.**

Summary

24 Summary

Electric Potential

- The electric potential *V* at point *P* in the electric field of a charged object:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0},$$ **Eq. 24-2**

Electric Potential Energy

- Electric potential energy *U* of the particle-object system:

$$U = qV.$$ **Eq. 24-3**

- If the particle moves through potential *ΔV*:

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$ **Eq. 24-4**

Mechanical Energy

- Applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q\,\Delta V.$$ **Eq. 24-9**

- In case of an applied force in a particle

$$\Delta K = -q\,\Delta V + W_{app}.$$ **Eq. 24-11**

- In a special case when *ΔK=0:*

$$W_{app} = q\,\Delta V \quad (\text{for } K_i = K_f).$$ **Eq. 24-12**

Finding V from *E*

- The electric potential difference between two point *I* and *f* is:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$ **Eq. 24-18**

24 Summary

Potential due to a Charged Particle

- due to a single charged particle at a distance r from that particle :

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$ **Eq. 24-26**

- due to a collection of charged particles

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}.$$ **Eq. 24-27**

Potential due to an Electric Dipole

- The electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$ **Eq. 24-30**

Potential due to a Continuous Charge Distribution

- For a continuous distribution of charge:

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$ **Eq. 24-32**

Calculating *E* from V

- The component of ***E*** in any direction is:

$$E_s = -\frac{\partial V}{\partial s}.$$ **Eq. 24-40**

Electric Potential Energy of a System of Charged Particle

- For two particles at separation *r*:

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$ **Eq. 24-46**

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CHAPTER 25

Capacitance

Introduction

- The capacitor: a device in which electrical energy can be stored

The batteries can supply energy at only a modest rate, too slowly for the photoflash unit to emit a flash of light. However, once the capacitor is charged, it can supply energy at a much greater rate when the photoflash unit is triggered—enough energy to allow the unit to emit a burst of bright light.

- The first step in our discussion of capacitors is to determine how much charge can be stored. This "how much" is called capacitance.

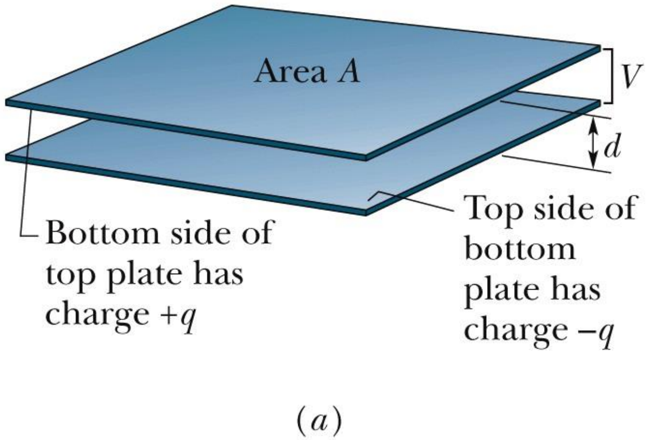
25-1 Capacitance

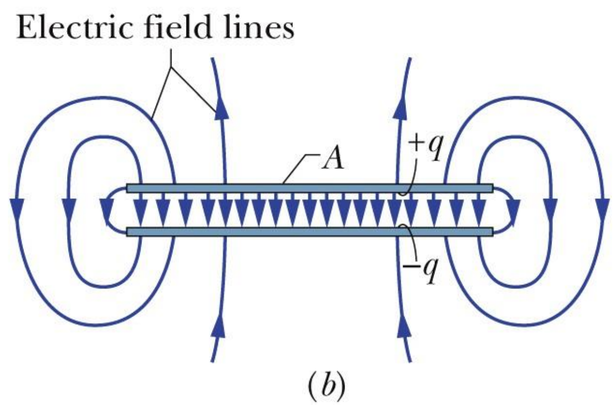
A capacitor consists of two isolated conductors (the plates) with charges *+q* and *-q*. Its **capacitance** *C* is defined from

$$q = CV.$$

1 farad = 1 F = 1 coulomb per volt = 1 C/V.

where V is the potential difference between the plates.





A parallel-plate capacitor, made up of two plates of area A separated by a distance *d*. The charges on the facing plate surfaces have the same magnitude *q* but opposite signs

As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

- Electric Circuit:

- Battery:

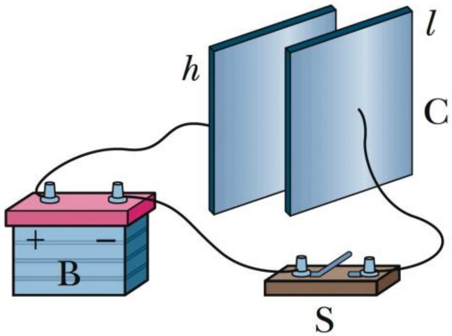
- Electric Circuit: An electric circuit is a path through which charge can flow.

- Battery: A battery is a device that maintains a certain potential difference between its terminals

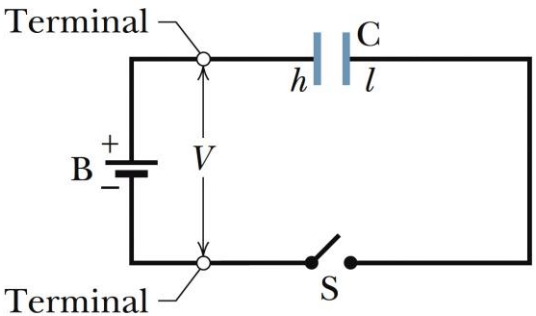
25-1 Capacitance

Charging Capacitor

When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.



(a)



(b)

In Fig. a, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit. The same circuit is shown in the schematic diagram of Fig. b, in which the symbols for a battery, a switch, and a capacitor represent those devices. The battery maintains potential difference V between its terminals. The terminal of higher potential is labeled + and is often called the positive terminal; the terminal of lower potential is labeled - and is often called the negative terminal.

25-2 Calculating the Capacitance

Calculating electric field and potential difference

To relate the electric field ***E*** between the plates of a capacitor to the charge *q* on either plate, we shall use Gauss' law:

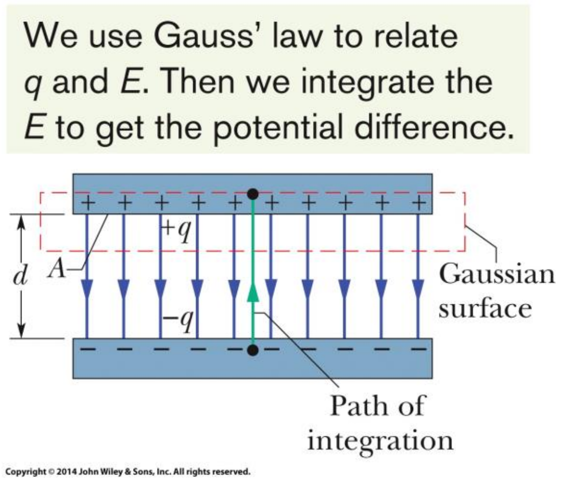
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

the potential difference between the plates of a capacitor is related to the field ***E*** by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

Letting V represent the difference V_f - V_i , we can then recast the above equation as:

$$V = \int_-^+ E\, ds$$



A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration is taken along a path extending directly from the negative plate to the positive plate.

25-2 Calculating the Capacitance

Parallel-Plate Capacitor

We assume, as Figure suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric field at the edges of the plates, taking ***E*** to be constant throughout the region between the plates.

We draw a Gaussian surface that encloses just the charge *q* on the positive plate

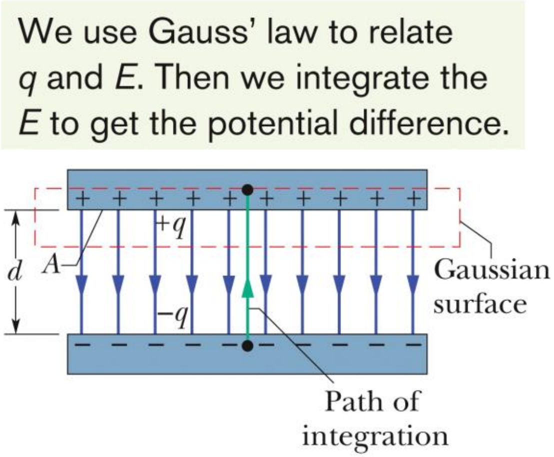
$$q = \varepsilon_0 E A$$

where A is the area of the plate. And therefore,

$$V = \int_{-}^{+} E\, ds = E \int_{0}^{d} ds = Ed.$$

Now if we substitute *q* in the above relations to *q=CV*, we get,

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).}$$



A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration is taken along a path extending directly from the negative plate to the positive plate.

25-2 Calculating the Capacitance

Cylindrical Capacitor

Figure shows, in cross section, a cylindrical capacitor of length *L* formed by two coaxial cylinders of radii *a* and *b*. We assume that *L* >> *b* so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude *q*. Here, charge and the field magnitude *E* is related as follows,

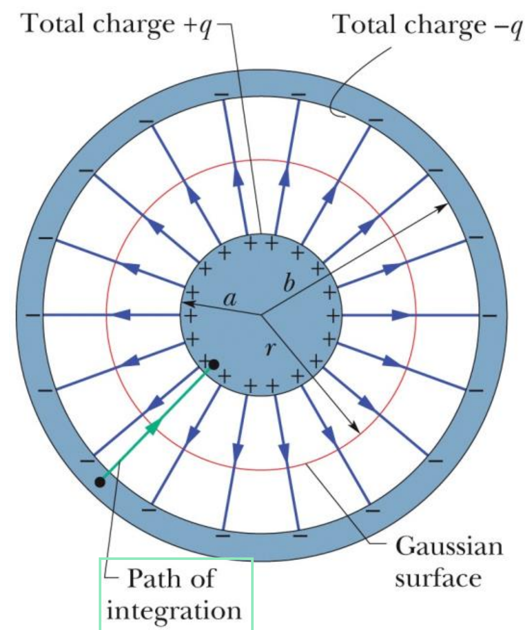
$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

Solving for *E* field:

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

From the relation *C= q/V*, we then have

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

25-2 Calculating the Capacitance

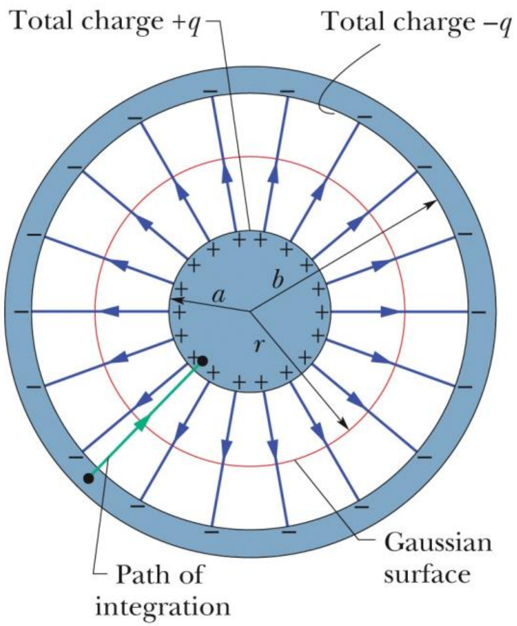
Others…

For **spherical capacitor** the capacitance is:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).}$$

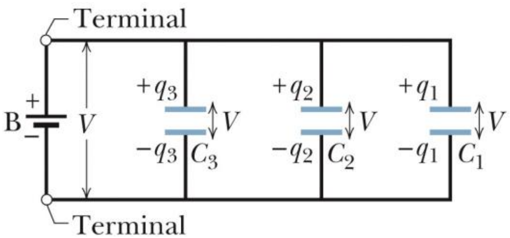
Capacitance of an **isolated sphere:**

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

Capacitors in Parallel



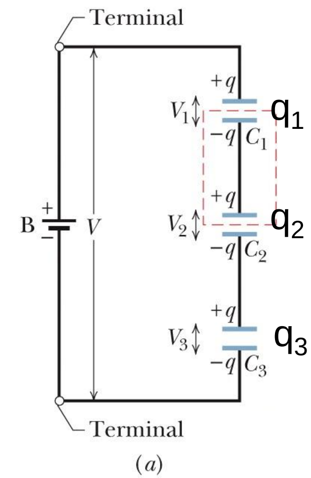
$$V = V_1 = V_2 = V_3$$

$$q = q_1 + q_2 + q_3$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}).$$

Capacitors in Series



$$q = q_1 = q_2 = q_3$$

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$

Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

The total charge on the parallel combination of Fig. 25-8*a* is then

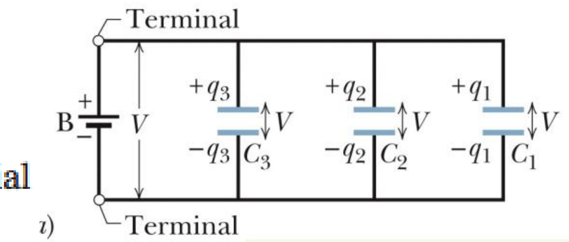
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

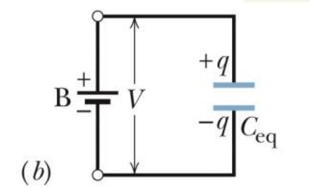
$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}).$$



Parallel capacitors and their equivalent have the same V (“par-V”).



Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

Capacitors in Series

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

The equivalent capacitance is then

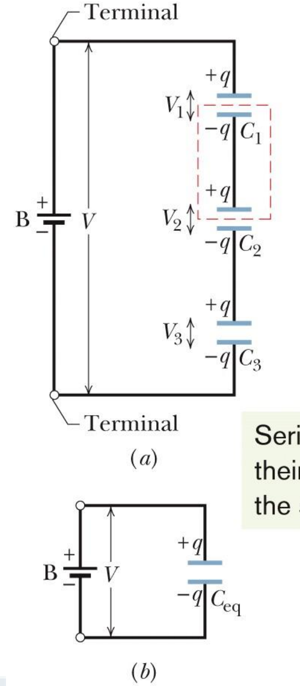
$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.



Series capacitors and their equivalent have the same q (“seri-q”).

25-4 Energy Stored in an Electric Field

The **electric potential energy** *U* of a charged capacitor,

$$U = \frac{q^2}{2C} \quad \text{(potential energy).}$$

and,

$$U = \tfrac{1}{2}CV^2 \quad \text{(potential energy).}$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field ***E***.

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Every electric field, in a capacitor or from any other source, has an associated stored energy. In vacuum, the **energy density** *u* (potential energy per unit volume) in a field of magnitude *E* is

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \quad \text{(energy density).}$$

25-5 Capacitor with a Dielectric

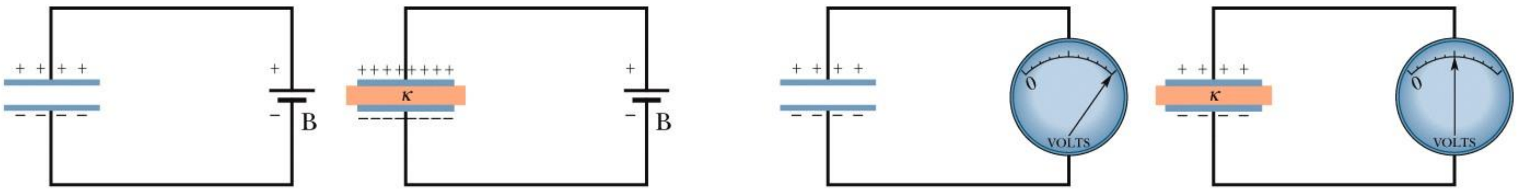
- A dielectric material (dielectric for short) is an electrical insulator that can be polarized by an applied electric field.

25-5 Capacitor with a Dielectric

If the space between the plates of a capacitor is completely filled with a **dielectric material**, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's **dielectric constant** κ, (Greek kappa) which is a number greater than 1.

$$C = \kappa C_{air}$$

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.



V = a constant

(a)

q = a constant

(b)

(a)If the potential difference between the plates of a capacitor is maintained, as by the presence of battery B, the effect of a dielectric is to increase the charge on the plates.

(b) If the charge on the capacitor plates is maintained, as in this case by isolating the capacitor, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a potentiometer, a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

25-5 Capacitor with a Dielectric

Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8
For a vacuum, κ = unity.		

[a]Measured at room temperature, except for the water.

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

- The magnitude of the electric field produced by a point charge inside a dielectric is given by this modified form of:

$$E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}$$

- Because k is always greater than unity, both these equations show that for a fixed distribution of charges, the effect of a dielectric is to weaken the electric field that would otherwise be present.

An Atomic View

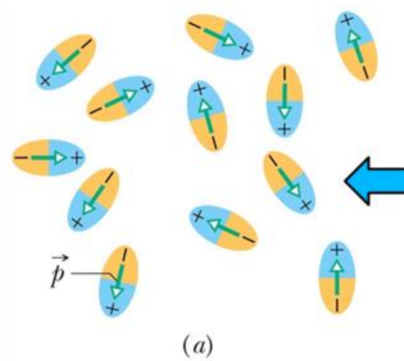
- **Polar dielectrics**: The molecules of some dielectrics, like water, have permanent electric dipole moments. In such materials (called polar dielectrics), the electric dipoles tend to line up with an external electric field

- **Nonpolar dielectrics:** Regardless of whether they have permanent electric dipole moments, molecules acquire dipole moments by induction when placed in an external electric field.

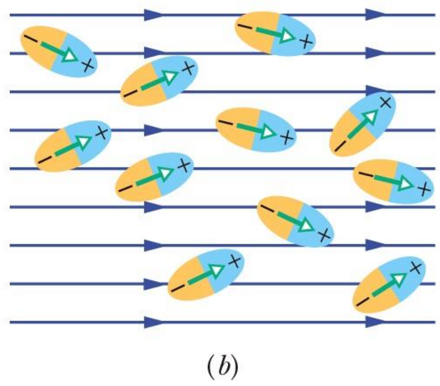
Thus, the effect of both polar and nonpolar dielectrics is to weaken any applied field within them, as between the plates of a capacitor.

25-5 Capacitor with a Dielectric

An Atomic View



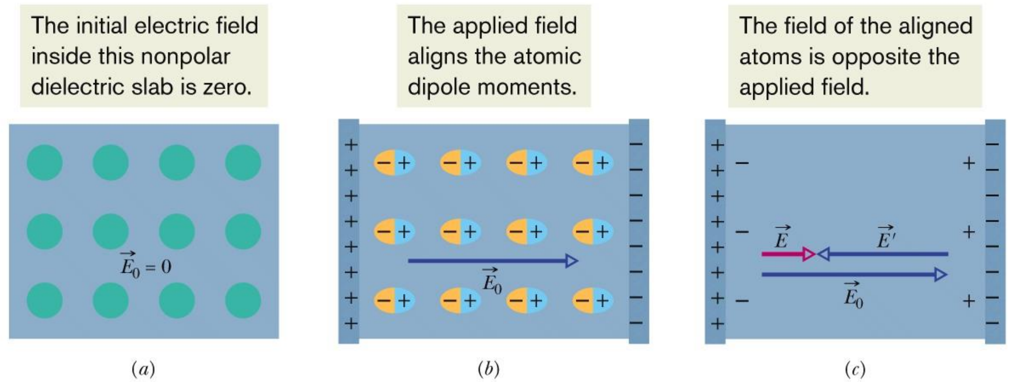
Polar Dielectrics



(a) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field.

(b) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

Nonpolar Dielectrics



25-6 Dielectrics and Gauss' Law

- Inserting a dielectric into a capacitor causes induced charge to appear on the faces of the dielectric and weakens the electric field between the plates.
- The induced charge is less than the free charge on the plates.

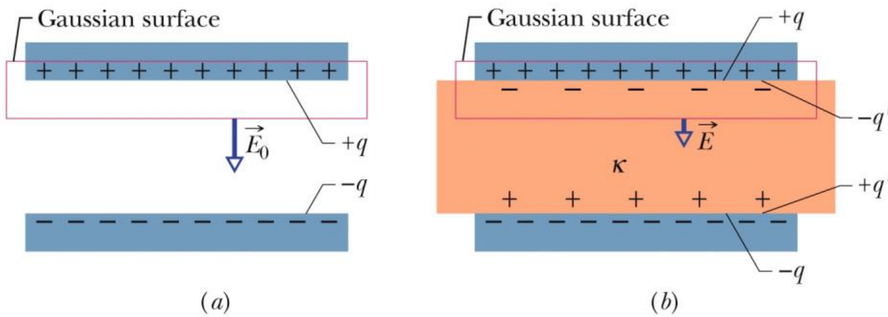
When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric).}$$

where q is the free charge. Any induced surface charge is accounted for by including the dielectric constant k inside the integral.

Note:
The flux integral now involves $\kappa\boldsymbol{E}$, not just $\boldsymbol{E}$. The vector $\varepsilon_0\kappa\boldsymbol{E}$ is sometimes called the electric displacement $\boldsymbol{D}$, so that the above equation can be written in the form

$$\oint \vec{D} \cdot d\vec{A} = q.$$



A parallel-plate capacitor (a) without and (b) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.

25 Summary

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$q = CV$$ **Eq. 25-1**

Determining Capacitance

- Parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{d}.$$ **Eq. 25-9**

- Cylindrical Capacitor:

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}.$$ **Eq. 25-14**

- Spherical Capacitor:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}.$$ **Eq. 25-17**

- Isolated sphere:

$$C = 4\pi\varepsilon_0 R.$$ **Eq. 25-18**

Capacitor in parallel and series

- In parallel:

$$C_{eq} = \sum_{j=1}^{n} C_j$$ **Eq. 25-19**

- In series

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$ **Eq. 25-20**

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$$ **Eq. 25-21&22**

- Energy density (*u*)

$$u = \frac{1}{2}\varepsilon_0 E^2.$$ **Eq. 25-25**

25 Summary

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance *C* is increased by a factor *κ*, called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q. \qquad \textbf{Eq. 25-36}$$