Final Material

I

(a) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^6}{x^3} = \lim_{(x,y)\to(0,0)} 1+\frac{y^6}{x^3}$

Take $y=\sqrt{x}$ $\lim_{x\to 0} \frac{x^3}{x^3} = 1$

Take $y=x$ $\lim_{x\to 0} \frac{x^6}{x^3} = \lim_{x\to 0} x^3 = 0$

$1\neq 0 \Rightarrow$ limit D.N.E

(b) $\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y+y^2}$

Take $y=mx$ $\lim_{x\to 0} \frac{x+mx}{x^2+mx+m^2x^2} = \frac{\lim x(m+1)}{x(x+m+m^2x)}$

Since limit depends on m, it does NOT exist. $= \frac{m+1}{m}$

→ Show that following limits exist:

a) $\lim_{(x,y)\to(1,2)} \frac{xy-y}{x^2-x+2xy-2y} = \lim \frac{y(x-1)}{x(x-1)+2y(x-1)}$

$= \lim \frac{y}{x+2y} = 2/5$

b) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\ln(x^2+y^2)} = 0\cdot\frac{1}{-\infty} = 0$

~> Show that $\lim\limits_{(x,y)\to(-1,1)} \dfrac{x^2-1}{y-1}$ DNE by considering

$y = x^2 \Rightarrow \lim\limits_{x\to -1} \dfrac{x^2-1}{x^2-1} = 1$

& $\qquad\qquad\qquad\qquad \neq$

$y = -x \Rightarrow \lim\limits_{x\to -1} \dfrac{x^2-1}{-x-1} = \lim\limits_{x\to -1} -(x-1) = 2$

~> Use polar to show that

$\lim\limits_{(x,y)\to(0,0)} \dfrac{3y}{(x^2+y^2)^2 + xy + y}$ exists

$y = r\sin\theta$, $x^2+y^2 = r^2$
$x = r\cos\theta$

$\lim\limits_{r\to 0} \dfrac{3r\sin\theta}{r^4 + r^2\sin\theta\cos\theta + r\sin\theta} = \lim\limits_{r\to 0} \dfrac{3\sin\theta}{r^3 + r\sin\theta\cos\theta + \sin\theta}$

θ could be anything!

$= \lim\limits_{r\to 0} \dfrac{3\sin\theta}{\sin\theta} = 3$

~> Evaluate the following, or show it DNE

$\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+2y^2}$

Take $y = mx$, $\lim\limits_{x\to 0} \dfrac{2x \cdot mx}{x^2 + 2m^2x^2} = \lim\limits_{x\to 0} \dfrac{2m}{1+2m^2}$

depends on m
$\Rightarrow$ lim does NOT exist

II Partial derivatives:

$\rightsquigarrow$ $f(x,y) = \tan^{-1}(xy^2)$

$f_x = \dfrac{1}{1+x^2y^4} \cdot y^2$ $\qquad$ $f_y = \dfrac{1}{1+x^2y^4} \cdot 2xy$

$f_{xx} = y^2 \dfrac{\partial}{\partial x}(1+x^2y^4)^{-1}$
$= -y^2(1+x^2y^4)^{-2} \cdot 2xy^4$

$f_{yx} =$

Same for all points in the domain where f_{xy}, f_{yx} are continuous

$f_{xy} = \dfrac{\partial}{\partial y}\left\{(1+x^2y^4)^{-1}y^2\right\}$
$= -(1+x^2y^4)^{-2} \cdot 4x^2y^3 \cdot y^2 +$
$2y(1+x^2y^4)^{-1}$

$f_{yy} = \dfrac{\partial}{\partial y}\left\{(1+x^2y^4)^{-1}2xy\right\}$
$= -(1+x^2y^4)^{-2}4x^2y^3 \cdot 2xy +$
$2x(1+x^2y^4)^{-1}$

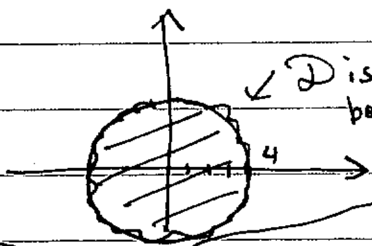
$\rightsquigarrow$ $f(x,y) = \cos(x-2y) + \ln(x-2y)$

$f_x = -\sin(x-2y) + (x-2y)^{-1}$
$f_{xx} = -\cos(x-2y) - (x-2y)^{-2}$
$\dfrac{\partial^2 f}{\partial y \partial x} = f_{xy} = 2\cos(x-2y) + 2(x-2y)^{-2}$

III

Find the domain & level curves of $f(x,y) = \sqrt{16-x^2-y^2}$

Domain: $16-x^2-y^2 \geq 0 \Rightarrow 16 \geq x^2+y^2$



Disk including boundary

Level curves are circles centered at $(0,0)$ of radius $\sqrt{16-c^2}$ where $|c|<4$.

Level Curves: $c = \text{Constant} = \sqrt{16-x^2-y^2}$

: $c^2 = 16-x^2-y^2 \Rightarrow x^2+y^2 = 16-c^2 \geq 0$ must have $c^2 < 16 \Rightarrow |c| < 4$