Chapter 27

1. **THINK** The circuit consists of two batteries and two resistors. We apply Kirchhoff's loop rule to solve for the current.

EXPRESS Let i be the current in the circuit and take it to be positive if it is to the left in R_1. Kirchhoff's loop rule gives

$$\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0.$$

For parts (b) and (c), we note that if i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2 R$.

ANALYZE (a) We solve for i:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \text{A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

(b) For R_1, the dissipation rate is $P_1 = i^2 R_1 = (0.50\ \text{A})^2(4.0\ \Omega) = 1.0\ \text{W}$.

(c) For R_2, the rate is $P_2 = i^2 R_2 = (0.50\ \text{A})^2\ (8.0\ \Omega) = 2.0\ \text{W}$.

If i is the current in a battery with emf ε, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. On the other hand, the battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For ε_1, $P_1 = i\varepsilon_1 = (0.50\ \text{A})(12\ \text{V}) = 6.0\ \text{W}$.

(e) For ε_2, $P_2 = i\varepsilon_2 = (0.50\ \text{A})(6.0\ \text{V}) = 3.0\ \text{W}$.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

LEARN Multiplying the equation obtained from Kirchhoff's loop rule by idt leads to the "energy-method" equation discussed in Section 27-4:

$$i\varepsilon_1 dt - i^2 R_1 dt - i^2 R_2 dt - i\varepsilon_2 dt = 0.$$

The first term represents the rate of work done by battery 1, the second and third terms the thermal energies that appear in resistors R_1 and R_2, and the last term the work done *on* battery 2.

2. The current in the circuit is

$$i = (150 \text{ V} - 50 \text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20 \text{ A}.$$

So from $V_Q + 150 \text{ V} - (2.0\ \Omega)i = V_P$, we get

$$V_Q = 100 \text{ V} + (2.0\ \Omega)(20 \text{ A}) - 150 \text{ V} = -10 \text{ V}.$$

3. (a) The potential difference is $V = \varepsilon + ir = 12 \text{ V} + (50 \text{ A})(0.040\ \Omega) = 14 \text{ V}$.

(b) $P = i^2 r = (50 \text{ A})^2(0.040\ \Omega) = 1.0\times10^2 \text{ W}$.

(c) $P' = iV = (50 \text{ A})(12 \text{ V}) = 6.0\times10^2 \text{ W}$.

(d) In this case $V = \varepsilon - ir = 12 \text{ V} - (50 \text{ A})(0.040\ \Omega) = 10 \text{ V}$.

(e) $P_r = i^2 r = (50 \text{ A})^2(0.040\ \Omega) = 1.0\times10^2 \text{ W}$.

4. (a) The loop rule leads to a voltage-drop across resistor 3 equal to 5.0 V (since the total drop along the upper branch must be 12 V). The current there is consequently $i = (5.0 \text{ V})/(200\ \Omega) = 25 \text{ mA}$. Then the resistance of resistor 1 must be $(2.0 \text{ V})/i = 80\ \Omega$.

(b) Resistor 2 has the same voltage-drop as resistor 3; its resistance is 200 Ω.

5. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where q is the charge that passes through in time $\Delta t = 6.0$ min, and ε is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and

$$\Delta E = i\varepsilon\,\Delta t = (5.0 \text{ A})(6.0 \text{ V})\ (6.0 \text{ min})\ (60 \text{ s/min}) = 1.1 \times 10^4 \text{ J}.$$

We note the conversion of time from minutes to seconds.

6. (a) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/2.0 \text{ W} \cdot \text{h})\ (\$0.80) = \$3.2 \times 10^2$.

(b) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/10^3 \text{ W} \cdot \text{h})\ (\$0.06) = \$0.048 = 4.8$ cents.

7. (a) The energy transferred is

$$U = Pt = \frac{\varepsilon^2 t}{r+R} = \frac{(2.0\text{ V})^2(2.0\,\text{min})(60\,\text{s}/\text{min})}{1.0\,\Omega + 5.0\,\Omega} = 80\text{ J}.$$

(b) The amount of thermal energy generated is

$$U' = i^2 Rt = \left(\frac{\varepsilon}{r+R}\right)^2 Rt = \left(\frac{2.0\text{ V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\text{ min})(60\text{ s}/\text{min}) = 67\text{ J}.$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

8. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and ε is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\varepsilon}{P} = \frac{(120\,\text{A}\cdot\text{h})(12.0\,\text{V})}{100\,\text{W}} = 14.4\text{ h}.$$

9. (a) The work done by the battery relates to the potential energy change:

$$q\Delta V = eV = e\left(12.0\,\text{V}\right) = 12.0\text{ eV}.$$

(b) $P = iV = neV = (3.40 \times 10^{18}/\text{s})(1.60 \times 10^{-19}\text{ C})(12.0\text{ V}) = 6.53\text{ W}$.

10. (a) We solve $i = (\varepsilon_2 - \varepsilon_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{i} - r_1 - r_2 = \frac{3.0\text{ V} - 2.0\text{ V}}{1.0 \times 10^{-3}\text{ A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9 \times 10^2\,\Omega.$$

(b) $P = i^2R = (1.0 \times 10^{-3}\text{ A})^2(9.9 \times 10^2\,\Omega) = 9.9 \times 10^{-4}\text{ W}$.

11. **THINK** As shown in Fig. 27-29, the circuit contains an emf device X. How it is connected to the rest of the circuit can be deduced from the power dissipated and the potential drop across it.

EXPRESS The power absorbed by a circuit element is given by $P = i\Delta V$, where i is the current and ΔV is the potential difference across the element. The end-to-end potential difference is given by

$$V_A - V_B = +iR + \varepsilon,$$

where ε is the emf of device X and is taken to be positive if it is to the left in the diagram.

ANALYZE (a) The potential difference between A and B is

$$\Delta V = \frac{P}{i} = \frac{50\ \text{W}}{1.0\ \text{A}} = 50\ \text{V}.$$

Since the energy of the charge decreases, point A is at a higher potential than point B; that is, $V_A - V_B = 50$ V.

(b) From the equation above, we find the emf of device X to be

$$\varepsilon = V_A - V_B - iR = 50\ \text{V} - (1.0\ \text{A})(2.0\ \Omega) = 48\ \text{V}.$$

(c) A positive value was obtained for ε, so it is toward the left. The negative terminal is at B.

LEARN Writing the potential difference as $V_A - iR - \varepsilon = V_B$, we see that our result is consistent with the resistance and emf rules. Namely, starting at point A, the change in potential is $-iR$ for a move through a resistance R in the direction of the current, and the change in potential is $-\varepsilon$ for a move through an emf device in the opposite direction of the emf arrow (which points from negative to positive terminals).

12. (a) For each wire, $R_{\text{wire}} = \rho L/A$ where $A = \pi r^2$. Consequently, we have

$$R_{\text{wire}} = (1.69 \times 10^{-8}\ \Omega\cdot\text{m})(0.200\ \text{m})/\pi(0.00100\ \text{m})^2 = 0.0011\ \Omega.$$

The total resistive load on the battery is therefore

$$R_{\text{tot}} = 2R_{\text{wire}} + R = 2(0.0011\ \Omega) + 6.00\ \Omega = 6.0022\ \ \Omega.$$

Dividing this into the battery emf gives the current

$$i = \frac{\varepsilon}{R_{\text{tot}}} = \frac{12.0\ \text{V}}{6.0022\Omega} = 1.9993\ \text{A}.$$

The voltage across the $R = 6.00\ \Omega$ resistor is therefore

$$V = iR = (1.9993\ \text{A})(6.00\ \Omega) = 11.996\ \text{V} \approx 12.0\ \text{V}.$$

(b) Similarly, we find the voltage-drop across each wire to be

$$V_{\text{wire}} = iR_{\text{wire}} = (1.9993\ \text{A})(0.0011\ \Omega) = 2.15\ \text{mV}.$$

(c) $P = i^2R = (1.9993\ \text{A})(6.00\ \Omega)^2 = 23.98\ \text{W} \approx 24.0\ \text{W}.$

(d) Similarly, we find the power dissipated in each wire to be 4.30 mW.

13. (a) We denote L = 10 km and α = 13 Ω/km. Measured from the east end we have

$$R_1 = 100\ \Omega = 2\alpha(L - x) + R,$$

and measured from the west end $R_2 = 200\ \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\,\Omega\,.$$

14. (a) Here we denote the battery emf's as V_1 and V_2. The loop rule gives

$$V_2 - ir_2 + V_1 - ir_1 - iR = 0 \quad \Rightarrow \quad i = \frac{V_2 + V_1}{r_1 + r_2 + R} \quad .$$

The terminal voltage of battery 1 is V_{1T} and (see Fig. 27-4(a)) is easily seen to be equal to $V_1 - ir_1$; similarly for battery 2. Thus,

$$V_{1T} = V_1 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R}, \quad V_{2T} = V_2 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R} \quad .$$

The problem tells us that V_1 and V_2 each equal 1.20 V. From the graph in Fig. 27-32(b) we see that $V_{2T} = 0$ and $V_{1T} = 0.40$ V for R = 0.10 Ω. This supplies us (in view of the above relations for terminal voltages) with simultaneous equations, which, when solved, lead to r_1 = 0.20 Ω.

(b) The simultaneous solution also gives r_2 = 0.30 Ω.

15. Let the emf be V. Then $V = iR = i'(R + R')$, where i = 5.0 A, i' = 4.0 A, and R' = 2.0 Ω. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4.0\text{ A})(2.0\,\Omega)}{5.0\text{ A} - 4.0\text{ A}} = 8.0\ \Omega.$$

16. (a) Let the emf of the solar cell be ε and the output voltage be V. Thus,

$$V = \varepsilon - ir = \varepsilon - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get

$$0.10\text{ V} = \varepsilon - (0.10\text{ V}/500\ \Omega)r$$
$$0.15\text{ V} = \varepsilon - (0.15\text{ V}/1000\ \Omega)r.$$

We solve for ε and r.

(a) $r = 1.0\times10^3\ \Omega$.

(b) $\varepsilon = 0.30$ V.

(c) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\text{V}}{(1000\Omega)\left(5.0\,\text{cm}^2\right)\left(2.0\times10^{-3}\,\text{W/cm}^2\right)} = 2.3\times10^{-3} = 0.23\%.$$

17. **THINK** A zero terminal-to-terminal potential difference implies that the emf of the battery is equal to the voltage drop across its internal resistance, that is, $\varepsilon = ir$.

EXPRESS To be as general as possible, we refer to the individual emf's as ε_1 and ε_2 and wait until the latter steps to equate them ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\varepsilon_1 + \varepsilon_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

ANALYZE (a) The current in the circuit is

$$i = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R},$$

and the requirement of zero terminal voltage leads to $\varepsilon_1 = ir_1$, or

$$R = \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{\varepsilon_1} = \frac{(12.0\text{ V})(0.016\,\Omega) - (12.0\text{ V})(0.012\,\Omega)}{12.0\text{ V}} = 0.0040\ \Omega.$$

Note that $R = r_1 - r_2$ when we set $\varepsilon_1 = \varepsilon_2$.

(b) As mentioned above, this occurs in battery 1.

LEARN If we assume the potential difference across battery 2 to be zero and repeat the calculation above, we would find $R = r_2 - r_1 < 0$, which is physically impossible. Thus, only the potential difference across the battery with the larger internal resistance can be made zero with suitable choice of R.

18. The currents i_1, i_2 and i_3 are obtained from Eqs. 27-18 through 27-20:

$$i_1 = \frac{\varepsilon_1(R_2+R_3)-\varepsilon_2 R_3}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\text{V})(10\ \Omega+5.0\,\Omega)-(1.0\,\text{V})(5.0\ \ \Omega)}{(10\ \Omega)(10\ \Omega)+(10\ \Omega)(5.0\ \Omega)+(10\,\Omega)(5.0\ \ \Omega)} = 0.275\ \text{A},$$

$$i_2 = \frac{\varepsilon_1 R_3-\varepsilon_2(R_1+R_2)}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\ \text{V})(5.0\,\Omega)-(1.0\ \ \text{V})(10\,\Omega+5.0\ \ \Omega)}{(10\,\Omega)(10\ \ \Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\ \ \Omega)} = 0.025\ \text{A},$$

$$i_3 = i_2 - i_1 = 0.025\text{A} - 0.275\text{A} = -0.250\text{A}\ .$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get

$$V_d - V_c = i_2R_2 = (0.0250\ \text{A})\ (10\ \Omega) = +0.25\ \text{V};$$

from $V_d + i_3R_3 + \varepsilon_2 = V_c$ we get

$$V_d - V_c = i_3R_3 - \varepsilon_2 = -(-\,0.250\ \text{A})\ (5.0\ \Omega) - 1.0\ \text{V} = +0.25\ \text{V}.$$

19. (a) Since $R_{\text{eq}} < R$, the two resistors (R = 12.0 Ω and R_x) must be connected in parallel:

$$R_{\text{eq}} = 3.00\,\Omega = \frac{R_x R}{R+R_x} = \frac{R_x\,(12.0\,\Omega)}{12.0\,\Omega + R_x}.$$

We solve for R_x: $R_x = R_{\text{eq}}R/(R - R_{\text{eq}})$ = (3.00 Ω)(12.0 Ω)/(12.0 Ω – 3.00 Ω) = 4.00 Ω.

(b) As stated above, the resistors must be connected in parallel.

20. Let the resistances of the two resistors be R_1 and R_2, with $R_1 < R_2$. From the statements of the problem, we have

$$R_1R_2/(R_1 + R_2) = 3.0\ \Omega \text{ and } R_1 + R_2 = 16\ \Omega.$$

So R_1 and R_2 must be 4.0 Ω and 12 Ω, respectively.

(a) The smaller resistance is R_1 = 4.0 Ω.

(b) The larger resistance is R_2 = 12 Ω.

21. The potential difference across each resistor is V = 25.0 V. Since the resistors are identical, the current in each one is

$$i = V/R = (25.0\ \text{V})/(18.0\ \Omega) = 1.39\ \text{A}.$$

The total current through the battery is then i_{total} = 4(1.39 A) = 5.56 A. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{eq}} = \sum \frac{1}{R} = \frac{4}{R}.$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus

$$i_{total} = V/R_{eq} = 4V/R = 4(25.0 \text{ V})/(18.0\ \Omega) = 5.56 \text{ A}.$$

22. (a) R_{eq} (FH) = (10.0 Ω)(10.0 Ω)(5.00 Ω)/[(10.0 Ω)(10.0 Ω) + 2(10.0 Ω)(5.00 Ω)] = 2.50 Ω.

(b) R_{eq} (FG) = (5.00 Ω) R/(R + 5.00 Ω), where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So R_{eq} (FG) = (5.00 Ω)(8.33 Ω)/(5.00 Ω + 8.33 Ω) = 3.13 Ω.

23. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0\,.$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0 \text{ V}}{100\,\Omega} = 0.050 \text{ A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0\ .$$

The equation gives

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0 \text{ V} - 5.0 \text{ V} - 4.0 \text{ V}}{50\,\Omega} = -0.060 \text{ A}\,,$$

or $|\,i_2\,| = 0.060$ A. The negative sign indicates that the current in R_2 is actually downward.

(c) If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so

$$V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0 \text{ V} + 5.0 \text{ V} = 9.0 \text{ V}.$$

24. We note that two resistors in parallel, R_1 and R_2, are equivalent to

$$\frac{1}{R_{12}}=\frac{1}{R_1}+\frac{1}{R_2} \quad \Rightarrow \; R_{12}=\frac{R_1R_2}{R_1+R_2}.$$

This situation consists of a parallel pair that are then in series with a single R_3 = 2.50 Ω resistor. Thus, the situation has an equivalent resistance of

$$R_{\text{eq}}=R_3+R_{12}=2.50\Omega+\frac{(4.00\Omega)(4.00\Omega)}{4.00\Omega+4.00\Omega}=4.50\Omega.$$

25. **THINK** The resistance of a copper wire varies with its cross-sectional area, or its diameter.

EXPRESS Let r be the resistance of each of the narrow wires. Since they are in parallel the equivalent resistance R_{eq} of the composite is given by

$$\frac{1}{R_{\text{eq}}}=\frac{9}{r},$$

or $R_{\text{eq}} = r/9$. Now each thin wire has a resistance $r=4\rho\ell/\pi d^2$, where ρ is the resistivity of copper, and $A = \pi d^2/4$ is the cross-sectional area of a single thin wire. On the other hand, the resistance of the thick wire of diameter D is $R=4\rho\ell/\pi D^2$, where the cross-sectional area is $\pi D^2/4$.

ANALYZE If the single thick wire is to have the same resistance as the composite of 9 thin wires, $R=R_{\text{eq}}$, then

$$\frac{4\rho\ell}{\pi D^2}=\frac{4\rho\ell}{9\pi d^2}.$$

Solving for D, we obtain $D = 3d$.

LEARN The equivalent resistance R_{eq} is smaller than r by a factor of 9. Since $r\sim 1/A\sim 1/d^2$, increasing the diameter of the wire threefold will also reduce the resistance by a factor of 9.

26. The part of R_0 connected in parallel with R is given by $R_1 = R_0x/L$, where L = 10 cm. The voltage difference across R is then $V_R = \varepsilon R'/R_{\text{eq}}$, where $R' = RR_1/(R + R_1)$ and

$$R_{\text{eq}} = R_0(1 - x/L) + R'.$$

Thus,

$$P_R=\frac{V_R^2}{R}=\frac{1}{R}\left(\frac{\varepsilon RR_1/(R+R_1)}{R_0(1-x/L)+RR_1/(R+R_1)}\right)^2=\frac{100R(\varepsilon x/R_0)^2}{\left(100\,R/R_0+10x-x^2\right)^2},$$

where x is measured in cm.

27. Since the potential differences across the two paths are the same, $V_1 = V_2$ (V_1 for the left path, and V_2 for the right path), we have $i_1 R_1 = i_2 R_2$, where $i = i_1 + i_2 = 5000$ A. With $R = \rho L / A$ (see Eq. 26-16), the above equation can be rewritten as

$$i_1 d = i_2 h \quad \Rightarrow \quad i_2 = i_1 (d/h).$$

With $d/h = 0.400$, we get $i_1 = 3571\,\text{A}$ and $i_2 = 1429\,\text{A}$. Thus, the current through the person is $i_1 = 3571\,\text{A}$, or approximately $3.6\,\text{kA}$.

28. Line 1 has slope R_1 = 6.0 kΩ. Line 2 has slope R_2 = 4.0 kΩ. Line 3 has slope R_3 = 2.0 kΩ. The parallel pair equivalence is $R_{12} = R_1R_2/(R_1+R_2)$ = 2.4 kΩ. That in series with R_3 gives an equivalence of

$$R_{123} = R_{12} + R_3 = 2.4\ \text{k}\Omega + 2.0\ \text{k}\Omega = 4.4\ \text{k}\Omega.$$

The current through the battery is therefore $i = \varepsilon / R_{123} =$ (6 V)/(4.4 kΩ) and the voltage drop across R_3 is (6 V)(2 kΩ)/(4.4 kΩ) = 2.73 V. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across R_2. Then Ohm's law gives the current through R_2: (6 V – 2.73 V)/(4 kΩ) = 0.82 mA.

29. (a) The parallel set of three identical R_2 = 18 Ω resistors reduce to R = 6.0 Ω, which is now in series with the R_1 = 6.0 Ω resistor at the top right, so that the total resistive load across the battery is $R' = R_1 + R$ = 12 Ω. Thus, the current through R' is (12V)/R' = 1.0 A, which is the current through R. By symmetry, we see one-third of that passes through any one of those 18 Ω resistors; therefore, i_1 = 0.333 A.

(b) The direction of i_1 is clearly rightward.

(c) We use Eq. 26-27: $P = i^2R'$ = (1.0 A)^2(12 Ω) = 12 W. Thus, in 60 s, the energy dissipated is (12 J/s)(60 s) = 720 J.

30. Using the junction rule ($i_3 = i_1 + i_2$) we write two loop rule equations:

$$10.0\ \text{V} - i_1R_1 - (i_1 + i_2)\,R_3 = 0$$

$$5.00\ \text{V} - i_2R_2 - (i_1 + i_2)\,R_3 = 0.$$

(a) Solving, we find i_2 = 0, and

(b) $i_3 = i_1 + i_2$ = 1.25 A (downward, as was assumed in writing the equations as we did).

31. **THINK** This problem involves a multi-loop circuit. We first simplify the circuit by finding the equivalent resistance. We then apply Kirchhoff's loop rule to calculate the current in the loop, and the potentials at various points in the circuit.

EXPRESS We first reduce the parallel pair of identical 2.0-Ω resistors (on the right side) to $R' = 1.0\ \Omega$, and we reduce the series pair of identical 2.0-Ω resistors (on the upper left side) to $R'' = 4.0\ \Omega$. With R denoting the 2.0-Ω resistor at the bottom (between V_2 and V_1), we now have three resistors in series which are equivalent to

$$R_{\text{eq}} = R + R' + R'' = 7.0\ \Omega$$

across which the voltage is $\varepsilon_2 - \varepsilon_1 = 7.0$ V (by the loop rule, this is 12 V – 5.0 V), implying that the current is

$$i = \frac{\varepsilon_2 - \varepsilon_1}{R_{\text{eq}}} = \frac{7.0\ \text{V}}{7.0\ \Omega} = 1.0\ \text{A}.$$

The direction of i is upward in the right-hand emf device. Knowing i allows us to solve for V_1 and V_2.

ANALYZE (a) The voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and V_1 is 12 V – 1.0 V = 11 V. Noting the orientation of the battery, we conclude that $V_1 = -11\ \text{V}$.

(b) The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and V_2 is 5.0 V + 4.0 V = 9.0 V. Noting the orientation of the battery, we conclude V_2 = –9.0 V.

LEARN The potential difference between points 1 and 2 is

$$V_2 - V_1 = -9.0\ \text{V} - (-11.0\ \text{V}) = 2.0\ \text{V},$$

which is equal to $iR = (1.0\ \text{A})(2.0\ \Omega) = 2.0\ \text{V}$.

32. (a) For typing convenience, we denote the emf of battery 2 as V_2 and the emf of battery 1 as V_1. The loop rule (examining the left-hand loop) gives $V_2 + i_1 R_1 - V_1 = 0$. Since V_1 is held constant while V_2 and i_1 vary, we see that this expression (for large enough V_2) will result in a negative value for i_1, so the downward sloping line (the line that is dashed in Fig. 27-43(b)) must represent i_1. It appears to be zero when V_2 = 6 V. With $i_1 = 0$, our loop rule gives $V_1 = V_2$, which implies that V_1 = 6.0 V.

(b) At V_2 = 2 V (in the graph) it appears that i_1 = 0.2 A. Now our loop rule equation (with the conclusion about V_1 found in part (a)) gives $R_1 = 20\ \Omega$.

(c) Looking at the point where the upward-sloping i_2 line crosses the axis (at V_2 = 4 V), we note that i_1 = 0.1 A there and that the loop rule around the right-hand loop should give

$$V_1 - i_1 R_1 = i_1 R_2$$

when i_1 = 0.1 A and i_2 = 0. This leads directly to R_2 = 40 Ω.

33. First, we note in V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (1.40 \text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8 \text{ V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4$ = 16.8 V/(16.0 Ω) = 1.05 A.

By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 1.05 \text{ A} + 1.40 \text{ A} = 2.45 \text{ A},$$

so its voltage is V_2 = (2.00 Ω)(2.45 A) = 4.90 V.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4$ = 21.7 V (implying that the current through it is $i_3 = V_3$/(2.00 Ω) = 10.85 A).

The junction rule now gives the current in R_1 as

$$i_1 = i_2 + i_3 = 2.45 \text{ A} + 10.85 \text{ A} = 13.3 \text{ A},$$

implying that the voltage across it is V_1 = (13.3 A)(2.00 Ω) = 26.6 V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6 \text{ V} + 21.7 \text{ V} = 48.3 \text{ V}.$$

34. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of "voltage going through" a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm's law) the voltages across R_1 and R_3 (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery, which means more current is in R_3, implying its voltage drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across R_1 has decreased a corresponding amount. When the switch was open, the voltage across R_1 was 6.0 V (easily seen from symmetry considerations). With the switch closed, R_1 and R_2 are equivalent (by Eq. 27-24) to 3.0 Ω, which means the total load on the battery is 9.0 Ω. The current therefore is 1.33 A, which implies that the voltage drop across R_3 is 8.0 V. The loop rule then tells us that the voltage drop across R_1 is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

35. (a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\varepsilon - i_2R_2 - i_1R_1 = 0$$
$$\varepsilon - 2i_1R_1 - (i_1 - i_2)R_3 = 0$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1 = 0.002625$ A, $i_2 = 0.00225$ A and $i_3 = i_1 - i_2 = 0.000375$ A. Therefore,

$$V_A - V_B = i_1R_1 = 5.25 \text{ V}.$$

(b) It follows also that $V_B - V_C = i_3R_3 = 1.50$ V.

(c) We find $V_C - V_D = i_1R_1 = 5.25$ V.

(d) Finally, $V_A - V_C = i_2R_2 = 6.75$ V.

36. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2R_2 - (i_2 + i_3)R_1 = 0$$
$$\varepsilon_2 - i_3R_3 - (i_2 + i_3)R_1 = 0.$$

Solving, we find $i_2 = 0.0109$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.0273$ A (leftward), and $i_1 = i_2 + i_3 = 0.0382$ A (downward).

(b) The direction is downward. See the results in part (a).

(c) $i_2 = 0.0109$ A . See the results in part (a).

(d) The direction is rightward. See the results in part (a).

(e) $i_3 = 0.0273$ A. See the results in part (a).

(f) The direction is leftward. See the results in part (a).

(g) The voltage across R_1 equals V_A: (0.0382 A)(100 Ω) = +3.82 V.

37. The voltage difference across R_3 is $V_3 = \varepsilon R'/(R' + 2.00\ \Omega)$, where

$$R' = (5.00\ \Omega R)/(5.00\ \Omega + R_3).$$

Thus,

$$P_3 = \frac{V_3^2}{R_3} = \frac{1}{R_3}\left(\frac{\varepsilon R'}{R' + 2.00\,\Omega}\right)^2 = \frac{1}{R_3}\left(\frac{\varepsilon}{1 + 2.00\,\Omega/R'}\right)^2 = \frac{\varepsilon^2}{R_3}\left[1 + \frac{(2.00\,\Omega)(5.00\,\Omega + R)}{(5.00\,\Omega)R_3}\right]^{-2}$$
$$\equiv \frac{\varepsilon^2}{f(R_3)}$$

where we use the equivalence symbol ≡ to define the expression $f(R_3)$. To maximize P_3 we need to minimize the expression $f(R_3)$. We set

$$\frac{df(R_3)}{dR_3} = -\frac{4.00\,\Omega^2}{R_3^{\,2}} + \frac{49}{25} = 0$$

to obtain $R_3 = \sqrt{(4.00\,\Omega^2)(25)/49} = 1.43\,\Omega.$

38. (a) The voltage across $R_3 = 6.0\ \Omega$ is $V_3 = iR_3 = (6.0\text{ A})(6.0\ \Omega) = 36$ V. Now, the voltage across $R_1 = 2.0\ \Omega$ is

$$(V_A - V_B) - V_3 = 78 - 36 = 42\text{ V},$$

which implies the current is $i_1 = (42\text{ V})/(2.0\ \Omega) = 21$ A. By the junction rule, then, the current in $R_2 = 4.0\ \Omega$ is

$$i_2 = i_1 - i = 21\text{ A} - 6.0\text{ A} = 15\text{ A}.$$

The total power dissipated by the resistors is (using Eq. 26-27)

$$i_1^2\,(2.0\ \Omega) + i_2^2\,(4.0\ \Omega) + i^2\,(6.0\ \Omega) = 1998\text{ W} \approx 2.0\text{ kW}.$$

By contrast, the power supplied (externally) to this section is $P_A = i_A\,(V_A - V_B)$ where $i_A = i_1 = 21$ A. Thus, $P_A = 1638$ W. Therefore, the "Box" must be providing energy.

(b) The rate of supplying energy is (1998 – 1638)W = 3.6×10^2 W.

39. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields

$$\varepsilon - ir - 2iR = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{r + 2R}.$$

The power dissipated in R is

$$P=(2i)^2R=\frac{4\varepsilon^2R}{(r+2R)^2}.$$

We find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR}=\frac{4\varepsilon^2}{(r+2R)^3}-\frac{16\varepsilon^2R}{(r+2R)^3}=\frac{4\varepsilon^2(r-2R)}{(r+2R)^3}.$$

The derivative vanishes (and P is a maximum) if $R = r/2$. With $r = 0.300\ \Omega$, we have $R=0.150\ \Omega$.

(b) We substitute $R = r/2$ into $P = 4\varepsilon^2R/(r + 2R)^2$ to obtain

$$P_{\max}=\frac{4\varepsilon^2(r/2)}{[r+2(r/2)]^2}=\frac{\varepsilon^2}{2r}=\frac{(12.0\text{ V})^2}{2(0.300\ \Omega)}=240\text{ W}.$$

40. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\varepsilon = ir + (2i)R$ with $r = 0.200\ \Omega$ and $R = 2.00r$, we get

$$i_R=2i=\frac{2\varepsilon}{r+2R}=\frac{2(12.0\text{V})}{0.200\Omega+2(0.400\Omega)}=24.0\text{ A}.$$

(b) When connected in series $2\varepsilon - i_Rr - i_Rr - i_RR = 0$, or $i_R = 2\varepsilon/(2r + R)$. The result is

$$i_R=2i=\frac{2\varepsilon}{2r+R}=\frac{2(12.0\text{V})}{2(0.200\Omega)+0.400\Omega}=30.0\text{ A}.$$

(c) They are in series arrangement, since $R > r$.

(d) If $R = r/2.00$, then for parallel connection,

$$i_R=2i=\frac{2\varepsilon}{r+2R}=\frac{2(12.0\text{V})}{0.200\Omega+2(0.100\Omega)}=60.0\text{ A}.$$

(e) For series connection, we have

$$i_R=2i=\frac{2\varepsilon}{2r+R}=\frac{2(12.0\text{V})}{2(0.200\Omega)+0.100\Omega}=48.0\text{ A}.$$

(f) They are in parallel arrangement, since $R < r$.

41. We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is upward. The junction rule produces

$$i_1 + i_2 + i_3 = 0 .$$

The loop rule applied to the left-hand loop produces

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0$$

and applied to the right-hand loop produces

$$\varepsilon_2 - i_2 R_2 + i_3 R_3 = 0 .$$

We substitute $i_3 = -i_2 - i_1$, from the first equation, into the other two to obtain

$$\varepsilon_1 - i_1 R_1 - i_2 R_3 - i_1 R_3 = 0$$

and

$$\varepsilon_2 - i_2 R_2 - i_2 R_3 - i_1 R_3 = 0 .$$

Solving the above equations yield

$$i_1 = \frac{\varepsilon_1 (R_2 + R_3) - \varepsilon_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(3.00\text{ V})(2.00\,\Omega + 5.00\,\Omega) - (1.00\text{ V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = 0.421\text{ A.}$$

$$i_2 = \frac{\varepsilon_2 (R_1 + R_3) - \varepsilon_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(1.00\text{ V})(4.00\,\Omega + 5.00\,\Omega) - (3.00\text{ V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.158\text{ A.}$$

$$i_3 = -\frac{\varepsilon_2 R_1 + \varepsilon_1 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} = -\frac{(1.00\text{ V})(4.00\,\Omega) + (3.00\text{ V})(2.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.263\text{ A.}$$

Note that the current i_3 in R_3 is actually downward and the current i_2 in R_2 is to the right. The current i_1 in R_1 is to the right.

(a) The power dissipated in R_1 is $P_1 = i_1^2 R_1 = \left(0.421\,\text{A}\right)^2 \left(4.00\,\Omega\right) = 0.709\,\text{W}$.

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\,\text{A})^2 (2.00\,\Omega) = 0.0499\,\text{W} \approx 0.050$ W.

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = \left(-0.263\,\text{A}\right)^2 \left(5.00\,\Omega\right) = 0.346\,\text{W}$.

(d) The power supplied by ε_1 is $i_3\varepsilon_1$ = (0.421 A)(3.00 V) = 1.26 W.

(e) The power "supplied" by ε_2 is $i_2\varepsilon_2$ = (–0.158 A)(1.00 V) = –0.158 W. The negative sign indicates that ε_2 is actually absorbing energy from the circuit.

42. The equivalent resistance in Fig. 27-52 (with n parallel resistors) is

$$R_{\text{eq}} = R + \frac{R}{n} = \left(\frac{n+1}{n}\right)R \ .$$

The current in the battery in this case should be

$$i_n = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n}{n+1}\frac{V_{\text{battery}}}{R} \ .$$

If there were n +1 parallel resistors, then

$$i_{n+1} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n+1}{n+2}\frac{V_{\text{battery}}}{R} \ .$$

For the relative increase to be 0.0125 (= 1/80), we require

$$\frac{i_{n+1} - i_n}{i_n} = \frac{i_{n+1}}{i_n} - 1 = \frac{(n+1)/(n+2)}{n/(n+1)} - 1 = \frac{1}{80} \ .$$

This leads to the second-degree equation $n^2 + 2n - 80 = (n + 10)(n - 8) = 0$.

Clearly the only physically interesting solution to this is n = 8. Thus, there are eight resistors in parallel (as well as that resistor in series shown toward the bottom) in Fig. 27-52.

43. Let the resistors be divided into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{\text{eq}}} = \sum_{1}^{m}\frac{1}{nR} = \frac{m}{nR}.$$

Since the problem requires R_{eq} = 10 Ω = R, we must select n = m. Next we make use of Eq. 27-16. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{\text{total}} = n^2P$, where $P = 1.0$ W is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{total} \geq 5.0P$, so n^2 must be at least as large as 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

44. (a) Resistors R_2, R_3, and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50.0\Omega)(50.0\Omega)(75.0\Omega)}{(50.0\Omega)(50.0\Omega) + (50.0\Omega)(75.0\Omega) + (50.0\Omega)(75.0\Omega)}$$
$$= 18.8\Omega.$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{eq} = R_1 + R = 100\ \Omega + 18.8\ \Omega = 118.8\ \Omega \approx 119\ \Omega$.

(b) $i_1 = \varepsilon/R_{eq} = 6.0\text{ V}/(118.8\ \Omega) = 5.05 \times 10^{-2}$ A.

(c) $i_2 = (\varepsilon - V_1)/R_2 = (\varepsilon - i_1R_1)/R_2 = [6.0\text{V} - (5.05 \times 10^{-2}\text{ A})(100\Omega)]/50\ \Omega = 1.90 \times 10^{-2}$ A.

(d) $i_3 = (\varepsilon - V_1)/R_3 = i_2R_2/R_3 = (1.90 \times 10^{-2}\text{ A})(50.0\ \Omega/50.0\ \Omega) = 1.90 \times 10^{-2}$ A.

(e) $i_4 = i_1 - i_2 - i_3 = 5.05 \times 10^{-2}\text{ A} - 2(1.90 \times 10^{-2}\text{ A}) = 1.25 \times 10^{-2}$ A.

45. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\text{ V} - 2.0\text{ V}}{4(1.0\ \Omega) + 2.0\ \Omega} = 0.33\text{A}.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A.

(b) The direction of i_1 is downward.

(c) The current through ε_2 is $i_2 = 0.33$ A.

(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\text{ A})(2.0\ \Omega) + 4.0\text{ V} = 3.3$ V.

46. (a) When $R_3 = 0$ all the current passes through R_1 and R_3 and avoids R_2 altogether. Since that value of the current (through the battery) is 0.006 A (see Fig. 27-55(b)) for $R_3 = 0$ then (using Ohm's law)

$$R_1 = (12\text{ V})/(0.006\text{ A}) = 2.0\times10^3\,\Omega.$$

(b) When $R_3 = \infty$ all the current passes through R_1 and R_2 and avoids R_3 altogether. Since that value of the current (through the battery) is 0.002 A (stated in problem) for $R_3 = \infty$ then (using Ohm's law)

$$R_2 = (12\text{ V})/(0.002\text{ A}) - R_1 = 4.0\times10^3\,\Omega.$$

47. **THINK** The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them.

EXPRESS Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2}.$$

We solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. We find

$$i_C = \frac{r_C^2 \rho_C i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}$$

and

$$i_A = \frac{\left(r_A^2 - r_C^2\right)\rho_C\, i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}.$$

ANALYZE (a) The denominators are the same and each has the value

$$\begin{aligned}\left(b^2 - a^2\right)\rho_C + a^2 \rho_A &= \left[\left(0.380\times10^{-3}\text{ m}\right)^2 - \left(0.250\times10^{-3}\text{ m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\\ &\quad + \left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\,\Omega\cdot\text{m}\right)\\ &= 3.10\times10^{-15}\,\Omega\cdot\text{m}^3.\end{aligned}$$

Thus,

$$i_C = \frac{\left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\text{ A}\right)}{3.10\times10^{-15}\Omega\cdot\text{m}^3} = 1.11\text{ A}\,.$$

(b) Similarly,

$$i_A = \frac{\left[\left(0.380\times10^{-3}\text{m}\right)^2 - \left(0.250\times10^{-3}\text{m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(2.00\,\text{A}\right)}{3.10\times10^{-15}\Omega\cdot\text{m}^3} = 0.893\text{A}.$$

(c) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi a^2$, so the length of the composite wire is

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{(\pi)\left(0.250\times10^{-3}\text{ m}\right)^2(12.0\text{ V})}{(1.11\text{ A})\left(1.69\times10^{-8}\Omega\cdot\text{m}\right)} = 126\text{ m}.$$

LEARN The potential difference can also be written as $V = i_A R_A = i_A \rho_A L/\pi(b^2 - a^2)$. Thus,

$$L = \frac{\pi(b^2 - a^2)V}{i_A \rho_A} = \frac{\pi\left[(0.380\times10^{-3}\text{ m})^2 - (0.250\times10^{-3}\text{ m})^2\right](12.0\text{ V})}{(0.893\text{ A})\left(2.75\times10^{-8}\Omega\cdot\text{m}\right)} = 126\text{ m},$$

in agreement with the result found in (c).

48. (a) We use $P = \varepsilon^2/R_{eq}$, where

$$R_{eq} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega) + (12.0\,\Omega)R + (4.00\,\Omega)R}.$$

Put $P = 60.0$ W and $\varepsilon = 24.0$ V and solve for R: $R = 19.5\ \Omega$.

(b) Since $P \propto R_{eq}$, we must minimize R_{eq}, which means $R = 0$.

(c) Now we must maximize R_{eq}, or set $R = \infty$.

(d) Since $R_{eq,\,min} = 7.00\ \Omega$, $P_{max} = \varepsilon^2/R_{eq,\,min} = (24.0\text{ V})^2/7.00\ \Omega = 82.3$ W.

(e) Since $R_{eq,\,max} = 7.00\ \Omega + (12.0\ \Omega)(4.00\ \Omega)/(12.0\ \Omega + 4.00\ \Omega) = 10.0\ \Omega$,

$$P_{min} = \varepsilon^2/R_{eq,\,max} = (24.0\text{ V})^2/10.0\ \Omega = 57.6\text{ W}.$$

49. (a) The current in R_1 is given by

$$i_1 = \frac{\varepsilon}{R_1 + R_2 R_3 / (R_2 + R_3)} = \frac{5.0 \text{ V}}{2.0\Omega + (4.0\Omega)(6.0\Omega)/(4.0\Omega + 6.0\Omega)} = 1.14 \text{ A}.$$

Thus,

$$i_3 = \frac{\varepsilon - V_1}{R_3} = \frac{\varepsilon - i_1 R_1}{R_3} = \frac{5.0 \text{ V} - (1.14 \text{ A})(2.0\Omega)}{6.0\Omega} = 0.45 \text{ A}.$$

(b) We simply interchange subscripts 1 and 3 in the equation above. Now

$$i_3 = \frac{\varepsilon}{R_3 + \left(R_2 R_1 / (R_2 + R_1)\right)} = \frac{5.0\text{V}}{6.0\Omega + \left((2.0\Omega)(4.0\Omega)/(2.0\Omega + 4.0\Omega)\right)} = 0.6818\text{A}$$

and

$$i_1 = \frac{5.0 \text{ V} - (0.6818 \text{ A})(6.0\Omega)}{2.0\Omega} = 0.45 \text{ A},$$

the same as before.

50. Note that there is no voltage drop across the ammeter. Thus, the currents in the bottom resistors are the same, which we call i (so the current through the battery is $2i$ and the voltage drop across each of the bottom resistors is iR). The resistor network can be reduced to an equivalence of

$$R_{\text{eq}} = \frac{(2R)(R)}{2R + R} + \frac{(R)(R)}{R + R} = \frac{7}{6} R$$

which means that we can determine the current through the battery (and also through each of the bottom resistors):

$$2i = \frac{\varepsilon}{R_{\text{eq}}} \Rightarrow i = \frac{\varepsilon}{2R_{\text{eq}}} = \frac{\varepsilon}{2(7R/6)} = \frac{3\varepsilon}{7R}.$$

By the loop rule (going around the left loop, which includes the battery, resistor $2R$, and one of the bottom resistors), we have

$$\varepsilon - i_{2R}(2R) - iR = 0 \Rightarrow i_{2R} = \frac{\varepsilon - iR}{2R}.$$

Substituting $i = 3\varepsilon/7R$, this gives $i_{2R} = 2\varepsilon/7R$. The difference between i_{2R} and i is the current through the ammeter. Thus,

$$i_{\text{ammeter}} = i - i_{2R} = \frac{3\varepsilon}{7R} - \frac{2\varepsilon}{7R} = \frac{\varepsilon}{7R} \Rightarrow \frac{i_{\text{ammeter}}}{\varepsilon / R} = \frac{1}{7} = 0.143.$$

51. Since the current in the ammeter is i, the voltmeter reading is

$$V' = V + i\, R_A = i\,(R + R_A),$$

or $R = V'/i - R_A = R' - R_A$, where $R' = V'/i$ is the apparent reading of the resistance. Now, from the lower loop of the circuit diagram, the current through the voltmeter is $i_V = \varepsilon/(R_{eq} + R_0)$, where

$$\frac{1}{R_{eq}} = \frac{1}{R_V} + \frac{1}{R_A + R} \quad \Rightarrow \quad R_{eq} = \frac{R_V\left(R + R_A\right)}{R_V + R + R_A} = \frac{\left(300\,\Omega\right)\left(85.0\,\Omega + 3.00\,\Omega\right)}{300\,\Omega + 85.0\,\Omega + 3.00\,\Omega} = 68.0\,\Omega.$$

The voltmeter reading is then

$$V' = i_V R_{eq} = \frac{\varepsilon R_{eq}}{R_{eq} + R_0} = \frac{(12.0\text{ V})(68.0\ \Omega)}{68.0\ \Omega + 100\Omega} = 4.86\text{ V}.$$

(a) The ammeter reading is

$$i = \frac{V'}{R + R_A} = \frac{4.86\text{ V}}{85.0\ \Omega + 3.00\Omega} = 0.0552\text{ A}.$$

(b) As shown above, the voltmeter reading is $V' = 4.86$ V.

(c) $R' = V'/i$ = 4.86 V/(5.52 × 10^{-2} A) = 88.0 Ω.

(d) Since $R = R' - R_A$, if R_A is decreased, the difference between R' and R decreases. In fact, when $R_A = 0$, $R' = R$.

52. (a) Since $i = \varepsilon/(r + R_{ext})$ and $i_{max} = \varepsilon/r$, we have $R_{ext} = R(i_{max}/i - 1)$ where r = 1.50 V/1.00 mA = 1.50 × 10^3 Ω. Thus,

$$R_{ext} = (1.5 \times 10^3\ \Omega)(1/0.100 - 1) = 1.35 \times 10^4\ \Omega\,.$$

(b) $R_{ext} = (1.5 \times 10^3\ \Omega)(1/0.500 - 1) = 1.5 \times 10^3\ \Omega$.

(c) $R_{ext} = (1.5 \times 10^3\ \Omega)(1/0.900 - 1) = 167\ \Omega$.

(d) Since r = 20.0 Ω + R, R = 1.50 × 10^3 Ω – 20.0 Ω = 1.48 × 10^3 Ω.

53. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop:

$$\varepsilon - iR_2 - i_1 R_1 - ir = 0.$$

Similarly, applying the loop rule to the right-hand loop gives

$$i_1R_1 - (i - i_1)R_V = 0.$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V} i_1.$$

We substitute this into the first equation to obtain

$$\varepsilon - \frac{(R_2 + r)(R_1 + R_V)}{R_V} i_1 + R_1 i_1 = 0.$$

This has the solution

$$i_1 = \frac{\varepsilon R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}.$$

The reading on the voltmeter is

$$i_1 R_1 = \frac{\varepsilon R_V R_1}{(R_2 + r)\,(R_1 + R_V) + R_1 R_V} = \frac{(3.0\text{V})\,(5.0\times10^3\,\Omega)\,(250\Omega)}{(300\Omega + 100\Omega)(250\Omega + 5.0\times10^3\,\Omega) + (250\Omega)(5.0\times10^3\,\Omega)}$$
$$= 1.12 \text{ V}.$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1 R_1 = \frac{\varepsilon R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\ \Omega + 300\ \Omega + 100\,\Omega} = 1.15\,\text{V}.$$

The fractional error is (1.12 – 1.15)/(1.15) = –0.030, or –3.0%.

54. (a) $\varepsilon = V + ir$ = 12 V + (10.0 A) (0.0500 Ω) = 12.5 V.

(b) Now $\varepsilon = V' + (i_{\text{motor}} + 8.00\text{ A})r$, where

$$V' = i'_A R_{\text{light}} = (8.00\text{ A})\,(12.0\text{ V}/10\text{ A}) = 9.60\text{ V}.$$

Therefore,

$$i_{\text{motor}} = \frac{\varepsilon - V'}{r} - 8.00\,\text{A} = \frac{12.5\,\text{V} - 9.60\,\text{V}}{0.0500\,\Omega} - 8.00\,\text{A} = 50.0\,\text{A}.$$

55. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point *a* in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward *b* in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points *a* and *b* are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$\left(R_1+R_2\right)i_1=\left(R_x+R_s\right)\frac{R_1}{R_s}i_1 \quad\Rightarrow\quad R_x=\frac{R_2R_s}{R_1}.$$

56. The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R}=\frac{1}{R'}-\frac{1}{R_V} \quad\Rightarrow R'=\frac{RR_V}{R+R_V}.$$

The equivalent resistance of the circuit is $R_{\text{eq}}=R_A+R_0+R'=R_A+R_0+\dfrac{RR_V}{R+R_V}$.

(a) The ammeter reading is

$$i'=\frac{\varepsilon}{R_{\text{eq}}}=\frac{\varepsilon}{R_A+R_0+R_V\,R/\left(R+R_V\right)}=\frac{12.0\,\text{V}}{3.00\,\Omega+100\,\Omega+\left(300\,\Omega\right)\left(85.0\,\Omega\right)/\left(300\,\Omega+85.0\,\Omega\right)}$$
$$=7.09\times10^{-2}\,\text{A}.$$

(b) The voltmeter reading is

$$V=\varepsilon-i'\,(R_A+R_0)=12.0\text{ V}-(0.0709\text{ A})\,(103.00\ \Omega)=4.70\text{ V}.$$

(c) The apparent resistance is $R' = V/i' = 4.70\text{ V}/(7.09\times10^{-2}\text{ A}) = 66.3\ \Omega$.

(d) If R_V is increased, the difference between R and R' decreases. In fact, $R'\to R$ as $R_V\to\infty$.

57. Here we denote the battery emf as V. Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{\text{cap}}$, or

$$Ve^{-t/RC} = V(1-e^{-t/RC})$$

where Eqs. 27-34 and 27-35 have been used. This leads to $t = RC\ln 2$, or $t =$ 0.208 ms.

58. (a) $\tau = RC = (1.40\times10^{6}\ \Omega)(1.80\times10^{-6}\text{ F}) = 2.52$ s.

(b) $q_o = \varepsilon C = (12.0\text{ V})(1.80\ \mu\text{F}) = 21.6\ \mu\text{C}$.

(c) The time t satisfies $q = q_0(1-e^{-t/RC})$, or

$$t=RC\ln\left(\frac{q_0}{q_0-q}\right)=\left(2.52\,\text{s}\right)\ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C}-16.0\,\mu\text{C}}\right)=3.40\,\text{s}.$$

59. **THINK** We have an RC circuit that is being charged. When fully charged, the charge on the capacitor is equal to $C\varepsilon$.

EXPRESS During charging, the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where C is the capacitance, ε is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{eq} = C\varepsilon$, so we require $q = 0.99q_{eq} = 0.99C\varepsilon$.

ANALYZE The time required to reach 99% of its final charge is given by

$$0.99 = 1 - e^{-t/\tau}.$$

Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\tau$.

LEARN The corresponding current in a charging capacitor is given by

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/\tau}.$$

The current has an initial value ε / R but decays exponentially to zero as the capacitor becomes fully charged. The plots of $q(t)$ and $i(t)$ are shown in Fig. 27-16 of the text.

60. (a) We use $q = q_0 e^{-t/\tau}$, or $t = \tau \ln (q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus,

$$t_{1/3} = \tau \ln\left(\frac{q_0}{2q_0/3}\right) = \tau \ln\left(\frac{3}{2}\right) = 0.41\tau \quad \Rightarrow \quad \frac{t_{1/3}}{\tau} = 0.41.$$

(b) $t_{2/3} = \tau \ln\left(\dfrac{q_0}{q_0/3}\right) = \tau \ln 3 = 1.1\tau \quad \Rightarrow \quad \dfrac{t_{2/3}}{\tau} = 1.1.$

61. (a) The voltage difference V across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At $t = 1.30\ \mu s$ we have $V(t) = 5.00$ V, so $5.00\text{ V} = (12.0\text{ V})(1 - e^{-1.30\ \mu s/RC})$, which gives

$$\tau = (1.30\ \mu\text{s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) The capacitance is $C = \tau/R = (2.41\ \mu\text{s})/(15.0\text{ k}\Omega) = 161$ pF.

62. The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \varepsilon\left(1 - e^{-t/RC}\right)$. We solve for R:

$$R = \frac{t}{C \ln \varepsilon/(\varepsilon - V_L)} = \frac{0.500\,\text{s}}{(0.150\times10^{-6}\,\text{F})\ln 95.0\,\text{V}/(95.0\,\text{V} - 72.0\,\text{V})} = 2.35\times10^{6}\,\Omega$$

where we used t = 0.500 s given (implicitly) in the problem.

63. **THINK** We have a multi-loop circuit with a capacitor that's being charged. Since at t = 0 the capacitor is completely uncharged, the current in the capacitor branch is as it would be if the capacitor were replaced by a wire.

EXPRESS Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0.$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

ANALYZE (a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2\times10^{3}\,\text{V})}{3(0.73\times10^{6}\,\Omega)} = 1.1\times10^{-3}\,\text{A}\,,$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^{3}\,\text{V}}{3(0.73\times10^{6}\,\Omega)} = 5.5\times10^{-4}\,\text{A},$

(c) and $i_3 = i_2 = 5.5\times10^{-4}\,\text{A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon - i_1 R_1 - i_1 R_2 = 0.$

(d) The solution is $i_1 = \dfrac{\varepsilon}{2R} = \dfrac{1.2\times10^{3}\,\text{V}}{2(0.73\times10^{6}\,\Omega)} = 8.2\times10^{-4}\,\text{A}$

(e) and $i_2 = i_1 = 8.2\times10^{-4}\,\text{A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0.$$

We use the first equation to substitute for i_1 in the second and obtain

$$\varepsilon - 2i_2 R - i_3 R = 0.$$

Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC}.$$

The current in the center branch is

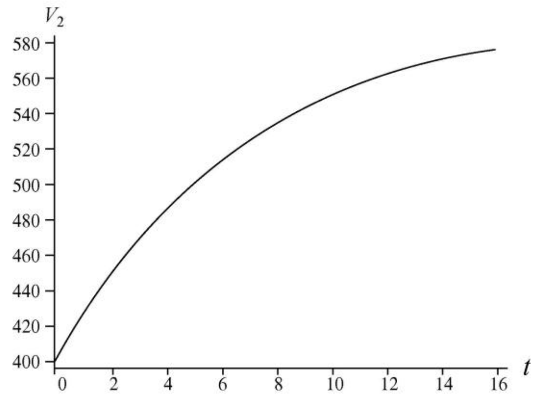
$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

and the potential difference across R_2 is $V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right)$.

(g) For $t = 0$, $e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\,\text{V}\right)/3 = 4.0\times10^2\,\text{V}$.

(h) For $t = \infty$, $e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\,\text{V}\right)/2 = 6.0\times10^2\,\text{V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.



LEARN A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current. However, a long time later after it's fully charged, it acts like a broken wire.

64. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0 e^{-t/\tau}$, this means $V = V_0 e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\,\text{s}}{\ln\,(1.00\,\text{V})/(100\,\text{V})} = 2.17\,\text{s}.$$

(b) At $t = 17.0$ s, $t/\tau = (17.0 \text{ s})/(2.17 \text{ s}) = 7.83$, so

$$V = V_0 e^{-t/\tau} = (100\,\text{V})e^{-7.83} = 3.96 \times 10^{-2}\ \text{V}\ .$$

65. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2 \frac{\varepsilon}{R_1 + R_2} = (15.0\ \text{k}\Omega)\left(\frac{20.0\,\text{V}}{10.0\ \text{k}\Omega + 15.0\ \text{k}\Omega}\right) = 12.0\,\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 6.16/15000 = 4.11×10^{-4} A.

66. We apply Eq. 27-39 to each capacitor, demand their initial charges are in a ratio of 3:2 as described in the problem, and solve for the time. With

$$\tau_1 = R_1C_1 = (20.0\ \Omega)(5.00\times10^{-6}\ \text{F}) = 1.00\times10^{-4}\ \text{s}$$
$$\tau_2 = R_2C_2 = (10.0\ \Omega)(8.00\times10^{-6}\ \text{F}) = 8.00\times10^{-5}\ \text{s}\ ,$$

we obtain

$$t = \frac{\ln(3/2)}{\tau_2^{-1} - \tau_1^{-1}} = \frac{\ln(3/2)}{1.25\times10^4\ \text{s}^{-1} - 1.00\times10^4\ \text{s}^{-1}} = 1.62\times10^{-4}\text{s}\ .$$

67. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0$ s, we find

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\text{s}}{(2.0\times10^{-6}\ \text{F})\ln 4} = 7.2\times10^5\ \Omega.$$

68. (a) The initial energy stored in a capacitor is given by $U_C = q_0^2/2C$, where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2(1.0\times10^{-6}\ \text{F})(0.50\,\text{J})} = 1.0\times10^{-3}\ \text{C}\ .$$

(b) The charge as a function of time is given by $q = q_0 e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}e^{-t/\tau}\ ,$$

and the initial current is $i_0 = q_0/\tau$. The time constant is

$$\tau = RC = (1.0\times10^{-6}\ \text{F})(1.0\times10^6\ \Omega) = 1.0\ \text{s}\ .$$

Thus $i_0 = (1.0\times10^{-3}\ \text{C})\big/(1.0\,\text{s}) = 1.0\times10^{-3}\ \text{A}$.

(c) We substitute $q = q_0\, e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}e^{-t/\tau} = \left(\frac{1.0\times10^{-3}\ \text{C}}{1.0\times10^{-6}\ \text{F}}\right)e^{-t/1.0\ \text{s}} = (1.0\times10^3\ \text{V})e^{-1.0t}\ ,$$

where t is measured in seconds.

(d) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau} e^{-t/\tau} = \frac{(1.0\times10^{-3}\,\mathrm{C})(1.0\times10^{6}\,\Omega)}{1.0\,\mathrm{s}} e^{-t/1.0\,\mathrm{s}} = (1.0\times10^{3}\,\mathrm{V})e^{-1.0t} \;,$$

where t is measured in seconds.

(e) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $P = i^2 R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2} e^{-2t/\tau} = \frac{(1.0\times10^{-3}\,\mathrm{C})^2(1.0\times10^{6}\,\Omega)}{(1.0\,\mathrm{s})^2} e^{-2t/1.0\,\mathrm{s}} = (1.0\,\mathrm{W})e^{-2.0t} \;,$$

where t is again measured in seconds.

69. (a) The charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where ε is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is

$$\tau = RC = (3.00 \times 10^6\ \Omega)(1.00 \times 10^{-6}\ \mathrm{F}) = 3.00\ \mathrm{s}.$$

At t = 1.00 s, t/τ = (1.00 s)/(3.00 s) = 0.333 and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\varepsilon}{\tau} e^{-t/\tau} = \frac{(1.00\times10^{-6}\ \mathrm{F})(4.00\,\mathrm{V})}{3.00\,\mathrm{s}} e^{-0.333} = 9.55\times10^{-7}\ \mathrm{C/s}.$$

(b) The energy stored in the capacitor is given by $U_C = \dfrac{q^2}{2C}$, and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt}.$$

Now

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right) = (1.00\times10^{-6})(4.00\,\mathrm{V})\left(1 - e^{-0.333}\right) = 1.13\times10^{-6}\ \mathrm{C},$$

so

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt} = \left(\frac{1.13\times10^{-6}\ \mathrm{C}}{1.00\times10^{-6}\ \mathrm{F}}\right)(9.55\times10^{-7}\ \mathrm{C/s}) = 1.08\times10^{-6}\ \mathrm{W}.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is 9.55×10^{-7} A, so

$$P = \left(9.55\times10^{-7}\ \mathrm{A}\right)^2\left(3.00\times10^{6}\ \Omega\right) = 2.74\times10^{-6}\ \mathrm{W}.$$

(d) The rate at which energy is delivered by the battery is

$$i\varepsilon = \left(9.55\times10^{-7}\ \mathrm{A}\right)\left(4.00\ \mathrm{V}\right) = 3.82\times10^{-6}\ \mathrm{W}.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\varepsilon = (q/C)\ (dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

70. (a) From symmetry we see that the current through the top set of batteries (i) is the same as the current through the second set. This implies that the current through the R = 4.0 Ω resistor at the bottom is $i_R = 2i$. Thus, with r denoting the internal resistance of each battery (equal to 4.0 Ω) and ε denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3\left(\varepsilon - ir\right) - \left(2i\right)R = 0.$$

This yields i = 3.0 A. Consequently, i_R = 6.0 A.

(b) The terminal voltage of each battery is $\varepsilon - ir$ = 8.0 V.

(c) Using Eq. 27-17, we obtain $P = i\varepsilon$ = (3)(20) = 60 W.

(d) Using Eq. 26-27, we have $P = i^2r$ = 36 W.

71. (a) If S_1 is closed, and S_2 and S_3 are open, then $i_a = \varepsilon/2R_1$ = 120 V/40.0 Ω = 3.00 A.

(b) If S_3 is open while S_1 and S_2 remain closed, then

$$R_{\mathrm{eq}} = R_1 + R_1\,(R_1 + R_2)\,/(2R_1 + R_2) = 20.0\ \Omega + (20.0\ \Omega) \times (30.0\ \Omega)/(50.0\ \Omega) = 32.0\ \Omega,$$

so $i_a = \varepsilon/R_{\mathrm{eq}}$ = 120 V/32.0 Ω = 3.75 A.

(c) If all three switches S_1, S_2, and S_3 are closed, then $R_{\mathrm{eq}} = R_1 + R_1 R'/(R_1 + R')$ where

$$R' = R_2 + R_1\,(R_1 + R_2)/(2R_1 + R_2) = 22.0\ \Omega,$$

that is,

$$R_{\mathrm{eq}} = 20.0\ \Omega + (20.0\ \Omega)\,(22.0\ \Omega)/(20.0\ \Omega + 22.0\ \Omega) = 30.5\ \Omega,$$

so $i_a = \varepsilon/R_{\mathrm{eq}}$ = 120 V/30.5 Ω = 3.94 A.

72. (a) The four resistors R_1, R_2, R_3, and R_4 on the left reduce to

$$R_{eq} = R_{12} + R_{34} = \frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4} = 7.0\,\Omega + 3.0\,\Omega = 10\,\Omega .$$

With $\varepsilon = 30$ V across R_{eq} the current there is $i_2 = 3.0$ A.

(b) The three resistors on the right reduce to

$$R'_{eq} = R_{56} + R_7 = \frac{R_5R_6}{R_5+R_6} + R_7 = \frac{(6.0\,\Omega)(2.0\,\Omega)}{6.0\,\Omega + 2.0\,\Omega} + 1.5\,\Omega = 3.0\,\Omega .$$

With $\varepsilon = 30$ V across R'_{eq} the current there is $i_4 = 10$ A.

(c) By the junction rule, $i_1 = i_2 + i_4 = 13$ A.

(d) By symmetry, $i_3 = \frac{1}{2} i_2 = 1.5$ A.

(e) By the loop rule (proceeding clockwise),

$$30V - i_4(1.5\ \Omega) - i_5(2.0\ \Omega) = 0$$

readily yields $i_5 = 7.5$ A.

73. **THINK** Since the wires are connected in series, the current is the same in both wires.

EXPRESS Let i be the current in the wires and V be the applied potential difference. Using Kirchhoff's loop rule, we have $V - iR_A - iR_B = 0$. Thus, the current is $i = V/(R_A + R_B)$, and the corresponding current density is

$$J = \frac{i}{A} = \frac{V}{R_A + R_B}.$$

ANALYZE (a) For wire A, the magnitude of the current density vector is

$$J_A = \frac{i}{A} = \frac{V}{(R_A+R_B)A} = \frac{4V}{(R_1+R_2)\pi D^2} = \frac{4(60.0\text{V})}{\pi(0.127\Omega + 0.729\Omega)(2.60\times10^{-3}\text{m})^2}$$
$$= 1.32\times10^7\ \text{A/m}^2 .$$

(b) The potential difference across wire A is

$$V_A = iR_A = V R_A/(R_A + R_B) = (60.0 \text{ V})(0.127\ \Omega)/(0.127\ \Omega + 0.729\ \Omega) = 8.90 \text{ V}.$$

(c) The resistivity of wire A is

$$\rho_A = \frac{R_A A}{L_A} = \frac{\pi R_A D^2}{4L_A} = \frac{\pi(0.127\,\Omega)(2.60\times10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 1.69\times10^{-8}\,\Omega\cdot\text{m}.$$

So wire A is made of copper.

(d) Since wire B has the same length and diameter as wire A, and the currents are the same, we have $J_B = J_A = 1.32\times10^7 \text{ A/m}^2$.

(e) The potential difference across wire B is $V_B = V - V_A = 60.0 \text{ V} - 8.9 \text{ V} = 51.1 \text{ V}$.

(f) The resistivity of wire B is

$$\rho_B = \frac{R_B A}{L_B} = \frac{\pi R_B D^2}{4L_B} = \frac{\pi(0.729\ \Omega)(2.60\times10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 9.68\times10^{-8}\,\Omega\cdot\text{m},$$

so wire B is made of iron.

LEARN Resistance R is the property of an object (depending on quantities such as L and A), while resistivity is a property of the material itself. Knowing the value of ρ allows us to deduce what material the wire is made of.

74. The resistor by the letter i is above three other resistors; together, these four resistors are equivalent to a resistor $R = 10\ \Omega$ (with current i). As if we were presented with a maze, we find a path through R that passes through any number of batteries (10, it turns out) but no other resistors, which — as in any good maze — winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\varepsilon = 40$ V.

(a) The current through R is then $i = \varepsilon/R = 4.0$ A.

(b) The direction is upward in the figure.

75. (a) In the process described in the problem, no charge is gained or lost. Thus, q = constant. Hence,

$$q = C_1V_1 = C_2V_2 \quad\Rightarrow\quad V_2 = V_1\frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3.0\times10^3\,\text{V}.$$

(b) Equation 27-39, with $\tau = RC$, describes not only the discharging of q but also of V. Thus,

$$V = V_0 e^{-t/\tau} \;\Rightarrow\; t = RC\ln\left(\frac{V_0}{V}\right) = \left(300\times10^{9}\,\Omega\right)\left(10\times10^{-12}\text{ F}\right)\ln\left(\frac{3000}{100}\right)$$

which yields t = 10 s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0 e^{-t/RC}$ for R with the new values V_0 = 1400 V and t = 0.30 s. Thus,

$$R = \frac{t}{C\ln(V_0/V)} = \frac{0.30\,\text{s}}{\left(10\times10^{-12}\text{ F}\right)\ln(1400/100)} = 1.1\times10^{10}\,\Omega\ .$$

76. (a) We reduce the parallel pair of resistors (at the bottom of the figure) to a single R' =1.00 Ω resistor and then reduce it with its series 'partner' (at the lower left of the figure) to obtain an equivalence of R'' = 2.00 Ω +1.00Ω =3.00 Ω. It is clear that the current through R'' is the i_1 we are solving for. Now, we employ the loop rule, choose a path that includes R'' and all the batteries (proceeding clockwise). Thus, assuming i_1 goes leftward through R'', we have

$$5.00\text{ V} + 20.0\text{ V} - 10.0\text{ V} - i_1R'' = 0$$

which yields i_1 = 5.00 A.

(b) Since i_1 is positive, our assumption regarding its direction (leftward) was correct.

(c) Since the current through the ε_1 = 20.0 V battery is "forward", battery 1 is supplying energy.

(d) The rate is P_1 = (5.00 A)(20.0 V) = 100 W.

(e) Reducing the parallel pair (which are in parallel to the ε_2 = 10.0 V battery) to a single R' = 1.00 Ω resistor (and thus with current i' = (10.0 V)/(1.00 Ω) = 10.0 A downward through it), we see that the current through the battery (by the junction rule) must be $i = i' - i_1$ = 5.00 A *upward* (which is the "forward" direction for that battery). Thus, battery 2 is supplying energy.

(f) Using Eq. 27-17, we obtain P_2 = 50.0 W.

(g) The set of resistors that are in parallel with the ε_3 = 5 V battery is reduced to R''' = 0.800 Ω (accounting for the fact that two of those resistors are actually reduced in series, first, before the parallel reduction is made), which has current i''' = (5.00 V)/(0.800 Ω) = 6.25 A downward through it. Thus, the current through the battery (by the junction rule) must be $i = i''' + i_1$ = 11.25 A *upward* (which is the "forward" direction for that battery). Thus, battery 3 is supplying energy.

(h) Equation 27-17 leads to P_3 = 56.3 W.

77. **THINK** The silicon resistor and the iron resistor are connected in series. Both resistors are temperature-dependent, but we want the combination to be independent of temperature.

EXPRESS We denote silicon with subscript *s* and iron with *i*. Let T_0 = 20°. The resistances of the two resistors can be written as

$$R_s(T) = R_s(T_0)\left[1+\alpha_s(T-T_0)\right], \qquad R_i(T) = R_i(T_0)\left[1+\alpha_i(T-T_0)\right].$$

The resistors are in series connection so

$$\begin{aligned} R(T) &= R_s(T) + R_i(T) = R_s(T_0)\left[1+\alpha_s(T-T_0)\right] + R_i(T_0)\left[1+\alpha_i(T-T_0)\right] \\ &= R_s(T_0) + R_i(T_0) + \left[R_s(T_0)\alpha_s + R_i(T_0)\alpha_i\right](T-T_0). \end{aligned}$$

Now, if $R(T)$ is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R = 1000\ \Omega$.

ANALYZE (a) We solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0) = \frac{R\alpha_i}{\alpha_i - \alpha_s} = \frac{(1000\Omega)\left(6.5\times10^{-3}/\mathrm{K}\right)}{(6.5\times10^{-3}/\mathrm{K}) - (-70\times10^{-3}/\mathrm{K})} = 85.0\Omega.$$

(b) Similarly, $R_i(T_0) = 1000\ \Omega - 85.0\ \Omega = 915\ \Omega$.

LEARN The temperature independence of the combined resistor was possible because α_i and α_s, the temperature coefficients of resistivity of the two materials have opposite signs, so their temperature dependences can cancel.

78. The current in the ammeter is given by

$$i_A = \varepsilon/(r + R_1 + R_2 + R_A).$$

The current in R_1 and R_2 without the ammeter is $i = \varepsilon/(r + R_1 + R_2)$. The percent error is then

$$\begin{aligned} \frac{\Delta i}{i} &= \frac{i - i_A}{i} = 1 - \frac{r + R_1 + R_2}{r + R_1 + R_2 + R_A} = \frac{R_A}{r + R_1 + R_2 + R_A} = \frac{0.10\Omega}{2.0\Omega + 5.0\Omega + 4.0\Omega + 0.10\Omega} \\ &= 0.90\%. \end{aligned}$$

79. **THINK** As the capacitor in an *RC* circuit is being charged, some energy supplied by the emf device also goes to the resistor as thermal energy.

EXPRESS The charge q on the capacitor as a function of time is $q(t) = (\varepsilon C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\varepsilon/R)e^{-t/RC}$. The rate at which the emf device supplies energy is $P_\varepsilon = i\varepsilon dt$.

ANALYZE (a) The energy supplied by the emf is then

$$U = \int_0^\infty P_\varepsilon dt = \int_0^\infty \varepsilon i\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC}dt = C\varepsilon^2 = 2U_C$$

where $U_C = \dfrac{1}{2}C\varepsilon^2$ is the energy stored in the capacitor.

(b) By directly integrating i^2R we obtain

$$U_R = \int_0^\infty i^2 R dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}dt = \frac{1}{2}C\varepsilon^2.$$

LEARN Half of the energy supplied by the emf device is stored in the capacitor as electrical energy, while the other half is dissipated in the resistor as thermal energy.

80. In the steady state situation, there is no current going to the capacitors, so the resistors all have the same current. By the loop rule,

$$20.0 \text{ V} = (5.00\ \Omega)i + (10.0\ \Omega)i + (15.0\ \Omega)i$$

which yields $i = \frac{2}{3}$ A. Consequently, the voltage across the R_1 = 5.00 Ω resistor is (5.00 Ω)(2/3 A) = 10/3 V, and is equal to the voltage V_1 across the C_1 = 5.00 μF capacitor. Using Eq. 26-22, we find the stored energy on that capacitor:

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(5.00\times10^{-6}\text{ F})\left(\frac{10}{3}\text{V}\right)^2 = 2.78\times10^{-5}\text{ J}.$$

Similarly, the voltage across the R_2 = 10.0 Ω resistor is (10.0 Ω)(2/3 A) = 20/3 V and is equal to the voltage V_2 across the C_2 = 10.0 μF capacitor. Hence,

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(10.0\times10^{-6}\text{ F})\left(\frac{20}{3}\text{V}\right)^2 = 2.22\times10^{-5}\text{ J}$$

Therefore, the total capacitor energy is $U_1 + U_2 = 2.50 \times 10^{-4}$ J.

81. The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\varepsilon R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\,\text{V}.$$

82. From $V_a - \varepsilon_1 = V_c - ir_1 - iR$ and $i = (\varepsilon_1 - \varepsilon_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c &= \varepsilon_1 - i(r_1 + R) = \varepsilon_1 - \left(\frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2}\right)(r_1 + R) \\ &= 4.4\,\text{V} - \left(\frac{4.4\,\text{V} - 2.1\,\text{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\ &= 2.5\,\text{V}. \end{aligned}$$

83. **THINK** The time constant in an RC circuit is $\tau = RC$, where R is the resistance and C is the capacitance. A greater value of τ means a longer discharging time.

EXPRESS The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/\tau}$, where $\tau = RC$. Thus, $R = \dfrac{t}{C\ln(V_0/V)}$.

ANALYZE (a) Then, for the smaller time interval $t_{\min} = 10.0\ \mu\text{s}$

$$R_{\min} = \frac{10.0\,\mu\text{s}}{(0.220\,\mu\text{F})\ln(5.00/0.800)} = 24.8\,\Omega.$$

(b) Similarly, for the larger time interval $t_{\max} = 6.00$ ms,

$$R_{\max} = \frac{6.00\times10^{-3}\,\text{s}}{(0.220\,\mu\text{F})\ln(5.00\,\text{V}/0.800\,\text{V})} = 1.49\times10^{4}\,\Omega.$$

LEARN The two extrema of the resistances are related by

$$\frac{R_{\max}}{R_{\min}} = \frac{t_{\max}}{t_{\min}}.$$

The larger the value of R for a given capacitance, the longer the discharging time.

84. (a) Since $R_{\text{tank}} = 140\,\Omega, i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0\times10^{-2}\,\text{A}$.

(b) Now, $R_{\text{tank}} = (140\ \Omega + 20\ \Omega)/2 = 80\ \Omega$, so $i = 12\ \text{V}/(10\ \Omega + 80\ \Omega) = 0.13$ A.

(c) When full, $R_{\text{tank}} = 20\ \Omega$ so $i = 12\ \text{V}/(10\ \Omega + 20\ \Omega) = 0.40$ A.

85. **THINK** One of the three parts could be defective: the battery, the motor, or the cable.

EXPRESS All three circuit elements are connected in series, so the current is the same in all of them. The battery is discharging, so the potential drop across the terminals is $V_{\text{battery}} = \varepsilon - ir,$ where ε is the emf and r is the internal resistance. On the other hand, the resistances in the cable and the motor are $R_{\text{cable}} = V_{\text{cable}}/i$ and $R_{\text{motor}} = V_{\text{motor}}/i,$ respectively.

ANALYZE The internal resistance of the battery is

$$r = \frac{\varepsilon - V_{\text{battery}}}{i} = \frac{12\ \text{V} - 11.4\ \text{V}}{50\ \text{A}} = 0.012\ \Omega$$

which is less than 0.020 Ω. So the battery is OK. For the motor, we have

$$R_{\text{motor}} = \frac{V_{\text{motor}}}{i} = \frac{11.4\ \text{V} - 3.0\ \text{V}}{50\ \text{A}} = 0.17\ \Omega$$

which is less than 0.20 Ω. So the motor is OK. Now, the resistance of the cable is

$$R_{\text{cable}} = \frac{V_{\text{cable}}}{i} = \frac{3.0\ \text{V}}{50\ \text{A}} = 0.060\ \Omega$$

which is greater than 0.040 Ω. So the cable is defective.

LEARN In this exercise, we see that a defective component has a resistance outside its the range of acceptance.

86. When connected in series, the rate at which electric energy dissipates is $P_s = \varepsilon^2/(R_1 + R_2)$. When connected in parallel, the corresponding rate is $P_p = \varepsilon^2(R_1 + R_2)/R_1R_2$. Letting $P_p/P_s = 5$, we get $(R_1 + R_2)^2/R_1R_2 = 5$, where $R_1 = 100\ \Omega$. We solve for R_2: $R_2 = 38\ \Omega$ or $260\ \Omega$.

(a) Thus, the smaller value of R_2 is 38 Ω.

(b) The larger value of R_2 is 260 Ω.

87. When S is open for a long time, the charge on C is $q_i = \varepsilon_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is

$$i = (\varepsilon_2 - \varepsilon_1)/(R_1 + R_2) = (3.0\ \text{V} - 1.0\ \text{V})/(0.20\ \Omega + 0.40\ \Omega) = 3.33\ \text{A}.$$

The voltage difference V across the capacitor is then

$$V = \varepsilon_2 - iR_2 = 3.0 \text{ V} - (3.33 \text{ A})(0.40\ \Omega) = 1.67 \text{ V}.$$

Thus the final charge on C is $q_f = VC$. So the change in the charge on the capacitor is

$$\Delta q = q_f - q_i = (V - \varepsilon_2)C = (1.67 \text{ V} - 3.0 \text{ V})(10\ \mu\text{F}) = -13\ \mu\text{C}.$$

88. Using the junction and the loop rules, we have

$$\begin{aligned} 20.0 - i_1R_1 - i_3R_3 &= 0 \\ 20.0 - i_1R_1 - i_2R_2 - 50 &= 0 \\ i_2 + i_3 &= i_1 \end{aligned}$$

Requiring no current through the battery 1 means that $i_1 = 0$, or $i_2 = i_3$. Solving the above equations with $R_1 = 10.0\Omega$ and $R_2 = 20.0\Omega$, we obtain

$$i_1 = \frac{40 - 3R_3}{20 + 3R_3} = 0 \quad \Rightarrow \quad R_3 = \frac{40}{3} = 13.3\,\Omega\,.$$

89. The bottom two resistors are in parallel, equivalent to a $2.0R$ resistance. This, then, is in series with resistor R on the right, so that their equivalence is $R' = 3.0R$. Now, near the top left are two resistors ($2.0R$ and $4.0R$) that are in series, equivalent to $R'' = 6.0R$. Finally, R' and R'' are in parallel, so the net equivalence is

$$R_{\text{eq}} = \frac{(R')\,(R'')}{R' + R''} = 2.0R = 20\ \Omega$$

where in the final step we use the fact that $R = 10\ \Omega$.

90. (a) Using Eq. 27-4, we take the derivative of the power $P = i^2R$ with respect to R and set the result equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\varepsilon^2 R}{(R + r)^2}\right) = \frac{\varepsilon^2 (r - R)}{(R + r)^3} = 0$$

which clearly has the solution $R = r$.

(b) When $R = r$, the power dissipated in the external resistor equals

$$P_{\max} = \left.\frac{\varepsilon^2 R}{(R + r)^2}\right|_{R=r} = \frac{\varepsilon^2}{4r}.$$

91. (a) We analyze the lower left loop and find

$$i_1 = \varepsilon_1/R = (12.0\text{ V})/(4.00\ \Omega) = 3.00\text{ A}.$$

(b) The direction of i_1 is downward.

(c) Letting R = 4.00 Ω, we apply the loop rule to the tall rectangular loop in the center of the figure (proceeding clockwise):

$$\varepsilon_2 + (+i_1R) + (-i_2R) + \left(-\frac{i_2}{2}R\right) + (-i_2R) = 0\,.$$

Using the result from part (a), we find i_2 = 1.60 A.

(d) The direction of i_2 is downward (as was assumed in writing the equation as we did).

(e) Battery 1 is supplying this power since the current is in the "forward" direction through the battery.

(f) We apply Eq. 27-17: The current through the ε_1 = 12.0 V battery is, by the junction rule, 3.00 A + 1.60 A = 4.60 A and

$$P = (4.60\text{ A})(12.0\text{ V}) = 55.2\text{ W}.$$

(g) Battery 2 is supplying this power since the current is in the "forward" direction through the battery.

(h) $P = i_2(4.00\text{ V}) = 6.40$ W.

92. The equivalent resistance of the series pair of $R_3 = R_4$ = 2.0 Ω is R_{34}= 4.0 Ω, and the equivalent resistance of the parallel pair of $R_1 = R_2$ = 4.0 Ω is R_{12}= 2.0 Ω. Since the voltage across R_{34} must equal that across R_{12}:

$$V_{34} = V_{12} \quad \Rightarrow \quad i_{34}R_{34} = i_{12}R_{12} \quad \Rightarrow \quad i_{34} = \frac{1}{2}i_{12}$$

This relation, plus the junction rule condition $I = i_{12} + i_{34} = 6.00$ A, leads to the solution $i_{12} = 4.0$ A. It is clear by symmetry that $i_1 = i_{12}/2 = 2.00$ A.

93. (a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\text{W})(0.10\Omega)} = 1.0\text{V}$.

(b) From $i = V/R = (\varepsilon - V)/r$ we find

$$r = R\left(\frac{\varepsilon - V}{V}\right) = (0.10\ \Omega)\left(\frac{1.5\text{ V} - 1.0\text{ V}}{1.0\text{ V}}\right) = 0.050\ \Omega.$$

94. (a) $R_{eq}(AB)$ = 20.0 Ω/3 = 6.67 Ω (three 20.0 Ω resistors in parallel).

(b) $R_{eq}(AC)$ = 20.0 Ω/3 = 6.67 Ω (three 20.0 Ω resistors in parallel).

(c) $R_{eq}(BC)$ = 0 (as B and C are connected by a conducting wire).

95. The maximum power output is (120 V)(15 A) = 1800 W. Since 1800 W/500 W = 3.6, the maximum number of 500 W lamps allowed is 3.

96. Here we denote the battery emf as V. Eq. 27-30 leads to

$$i = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{12\text{ V}}{4.0\,\Omega} - \frac{8.0\times10^{-6}\text{ C}}{(4.0\,\Omega)(4.0\times10^{-6}\text{F})} = 2.50\text{ A.}$$

97. **THINK** To calculate the current in the resistor R, we first find the equivalent resistance of the N batteries.

EXPRESS When all the batteries are connected in parallel, the emf is ε and the equivalent resistance is $R_{\text{parallel}} = R + r/N,$ so the current is

$$i_{\text{parallel}} = \frac{\varepsilon}{R_{\text{parallel}}} = \frac{\varepsilon}{R + r/N} = \frac{N\varepsilon}{NR + r}.$$

Similarly, when all the batteries are connected in series, the total emf is $N\varepsilon$ and the equivalent resistance is $R_{\text{series}} = R + Nr.$ Therefore,

$$i_{\text{series}} = \frac{N\varepsilon}{R_{\text{series}}} = \frac{N\varepsilon}{R + Nr}.$$

ANALYZE Comparing the two expressions, we see that the two currents i_{parallel} and i_{series} are equal if $R = r,$ with

$$i_{\text{parallel}} = i_{\text{series}} = \frac{N\varepsilon}{(N+1)r}.$$

LEARN In general, the current difference is

$$i_{\text{parallel}} - i_{\text{series}} = \frac{N\varepsilon}{NR + r} - \frac{N\varepsilon}{R + Nr} = \frac{N\varepsilon(N-1)(r-R)}{(NR + r)(R + Nr)}.$$

If $R > r$, then $i_{\text{parallel}} < i_{\text{series}}$.

98. **THINK** The rate of energy supplied by the battery is $i\varepsilon$. So we first calculate the current in the circuit.

EXPRESS With R_2 and R_3 in parallel, and the combination in series with R_1, the equivalent resistance for the circuit is

$$R_{\text{eq}} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

and the current is

$$i = \frac{\varepsilon}{R_{\text{eq}}} = \frac{(R_2 + R_3)\varepsilon}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

The rate at which the battery supplies energy is

$$P = i\varepsilon = \frac{(R_2 + R_3)\varepsilon^2}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

To find the value of R_3 that maximizes P, we differentiate P with respect to R_3.

ANALYZE (a) With a little algebra, we find

$$\frac{dP}{dR_3} = -\frac{R_2^2 \varepsilon^2}{(R_1 R_2 + R_1 R_3 + R_2 R_3)^2}.$$

The derivative is negative for all positive value of R_3. Thus, we see that P is maximized when $R_3 = 0$.

(b) With the value of R_3 set to zero, we obtain $P = \dfrac{\varepsilon^2}{R_1} = \dfrac{(12.0\text{ V})^2}{10.0\,\Omega} = 14.4\text{ W}.$

LEARN Mathematically speaking, the function P is a monotonically decreasing function of R_3 (as well as R_2 and R_1), so P is a maximum at $R_3 = 0$.

99. **THINK** A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current.

EXPRESS The capacitor is *initially* uncharged. So immediately after the switch is closed, by the Kirchhoff's loop rule, there is zero voltage (at $t = 0$) across the $R_2 = 10\text{ k}\Omega$ resistor, and that $\varepsilon = 30$ V is across the $R_1 = 20\text{ k}\Omega$ resistor.

ANALYZE (a) By Ohm's law, the initial current in R_1 is

$$i_{10} = \varepsilon / R_1 = (30\text{ V})/(20\text{ k}\Omega) = 1.5 \times 10^{-3}\text{ A}.$$

(b) Similarly, the initial current in R_2 is $i_{20} = 0$.

(c) As $t \to \infty$ the current to the capacitor reduces to zero and the R_1 = 20 kΩ and R_2 = 10 kΩ resistors behave more like a series pair (having the same current), equivalent to

$$R_{eq} = R_1 + R_2 = 30\ \text{k}\Omega.$$

The current through them, then, at long times, is

$$i = \varepsilon / R_{eq} = (30\ \text{V})/(30\ \text{k}\Omega) = 1.0 \times 10^{-3}\ \text{A}.$$

LEARN A long time later after a capacitor is being fully charged, it acts like a broken wire.

100. (a) Reducing the bottom two series resistors to a single R' = 4.00 Ω (with current i_1 through it), we see we can make a path (for use with the loop rule) that passes through R, the ε_4 = 5.00 V battery, the ε_1 = 20.0 V battery, and the ε_3 = 5.00 V. This leads to

$$i_1 = \frac{\varepsilon_1 + \varepsilon_3 + \varepsilon_4}{R'} = \frac{20.0\ \text{V} + 5.00\ \text{V} + 5.00\ \text{V}}{40.0\ \Omega} = \frac{30.0\ \text{V}}{4.0\ \Omega} = 7.50\ \text{A}.$$

(b) The direction of i_1 is leftward.

(c) The voltage across the bottom series pair is $i_1 R'$ = 30.0 V. This must be the same as the voltage across the two resistors directly above them, one of which has current i_2 through it and the other (by symmetry) has current $\frac{1}{2} i_2$ through it. Therefore,

$$30.0\ \text{V} = i_2\,(2.00\ \Omega) + \tfrac{1}{2} i_2\,(2.00\ \Omega)$$

which leads to i_2 = (30.0 V)/(3.00 Ω) = 10.0 A.

(d) The direction of i_2 is also leftward.

(e) We use Eq. 27-17: $P_4 = (i_1 + i_2)\varepsilon_4 = (7.50\ \text{A} + 10.0\ \text{A})(5.00\ \text{V}) = 87.5\ \text{W}$.

(f) The energy is being supplied to the circuit since the current is in the "forward" direction through the battery.

101. Consider the lowest branch with the two resistors R_4 = 3.00 Ω and R_5 = 5.00 Ω. The voltage difference across R_5 is

$$V = i_5 R_5 = \frac{\varepsilon R_5}{R_4 + R_5} = \frac{(120\text{V})(5.00\Omega)}{3.00\Omega + 5.00\Omega} = 7.50\text{V}.$$

102. (a) Here we denote the battery emf as V. See Fig. 27-4(a): $V_T = V - ir$.

(b) Doing a least squares fit for the V_T versus i values listed, we obtain

$$V_T = 13.61 - 0.0599i$$

which implies $V = 13.6$ V.

(c) It also implies the internal resistance is 0.060 Ω.

103. (a) The loop rule (proceeding counterclockwise around the right loop) leads to $\varepsilon_2 - i_1R_1 = 0$ (where i_1 was assumed downward). This yields $i_1 = 0.0600$ A.

(b) The direction of i_1 is downward.

(c) The loop rule (counterclockwise around the left loop) gives

$$\left(+\varepsilon_1\right)+\left(+i_1R_1\right)+\left(-i_2R_2\right)=0$$

where i_2 has been assumed leftward. This yields $i_3 = 0.180$ A.

(d) A positive value of i_3 implies that our assumption on the direction is correct, i.e., it flows leftward.

(e) The junction rule tells us that the current through the 12 V battery is 0.180 + 0.0600 = 0.240 A.

(f) The direction is upward.

104. (a) Since $P = \varepsilon^2/R_{eq}$, the higher the power rating the smaller the value of R_{eq}. To achieve this, we can let the low position connect to the larger resistance (R_1), middle position connect to the smaller resistance (R_2), and the high position connect to both of them in parallel.

(b) For $P = 300$ W, $R_{eq} = R_1R_2/(R_1 + R_2) = (144\ \Omega)R_2/(144\ \Omega + R_2) = (120\ \text{V})^2/(300\ \text{W})$. We obtain $R_2 = 72\ \Omega$.

(c) For $P = 100$ W, $R_{eq} = R_1 = \varepsilon^2/P = (120\ \text{V})^2/100\ \text{W} = 144\ \Omega$;

105. (a) The six resistors to the left of $\varepsilon_1 = 16$ V battery can be reduced to a single resistor $R = 8.0\ \Omega$, through which the current must be $i_R = \varepsilon_1/R = 2.0$ A. Now, by the loop rule, the current through the 3.0 Ω and 1.0 Ω resistors at the upper right corner is

$$i' = \frac{16.0\,\text{V} - 8.0\,\text{V}}{3.0\,\Omega + 1.0\,\Omega} = 2.0\,\text{A}$$

in a direction that is "backward" relative to the ε_2 = 8.0 V battery. Thus, by the junction rule, $i_1 = i_R + i' = 4.0\,\text{A}$.

(b) The direction of i_1 is upward (that is, in the "forward" direction relative to ε_1).

(c) The current i_2 derives from a succession of symmetric splittings of i_R (reversing the procedure of reducing those six resistors to find R in part (a)). We find

$$i_2 = \frac{1}{2}\left(\frac{1}{2} i_R\right) = 0.50\,\text{A}.$$

(d) The direction of i_2 is clearly downward.

(e) Using our conclusion from part (a) in Eq. 27-17, we have

$$P = i_1\varepsilon_1 = (4.0\text{ A})(16\text{ V}) = 64\text{ W}.$$

(f) Using results from part (a) in Eq. 27-17, we obtain $P = i'\varepsilon_2 = (2.0\text{ A})(8.0\text{ V}) = 16\text{ W}$.

(g) Energy is being supplied in battery 1.

(h) Energy is being absorbed in battery 2.