Note for the Final Exam: by Hagop Harfoushian

* R(T) = Span{T(v1)…T(vn)}
* dimN(T)+dimR(T)=n (T: V🡪W and dimV=n)
* nullity(A) + rank(A)=number of columns of A
* 1-1 => N(T)={0}
* Onto=> R(T)=W (T: V🡪W)
* T is isomorphism if:
  + T is linear
  + T is 1-1
  + T is Onto
* If T=AX then T is an isomorphism if A-1 exists => det(A)≠0
* [v]B’=[I]B B’[v]B; B={v1…vn} [I]B B’={[v1]B’, [v2]B’…[vn]B’}
* [T(v)]**B’ =** A[v]**B;** B={v1…vn} A={T(v1)B’…T(vn)B’}
* T is a linear operator on V🡺[Tn]B=([T]B)n
* [T]B2=P-1[T]B1P where P=[I]B2 B1
* A is similar to B 🡺 A=C-1BC
* **Eigenvalues and Eigenvectors**
* To find eigenvalues: Det(A-λI)=0
* To find eigenvectors: (A-λI)=0 for each λ
* Av=λv
* # of eigenvectors=geometric multiplicity
* Geometric multiplicity ≤ Algebraic Multiplicity
* If geometric multiplicity=Algebraic multiplicity then A is diagonalizable.
* A:nxn, A is diagonalizable iff A has n linearly independent eigenvectors.
* A=PDP-1: The diagonal entries of D are the eigenvalues, and the columns of P are their corresponding eigenvectors.
* If A has n distinct eigenvalues, then A is diagonalizable
* 2x2 symmetric matrices are diagonalizable.
* [T]B and [T]B’ have the same eigenvalues.
* **Inner Product:**
* Conditions to be an inner product:
  + <u,u> ≥ 0
  + <u,v>=<v,u>
  + <u,v+w>=<u,v> + <u,w>
  + <u,kv>=k<u,v>
* ǁuǁ≥0
* ǁkuǁ=ǁkǁ.ǁuǁ where k is a constant
* ǁu+vǁ≤ǁuǁ+ǁvǁ (triangular inequality)
* ǁ<u,v>ǁ≤ǁuǁ.ǁvǁ (Cauchy–Schwarz inequality)
* Cosϴ=<u,v>/(ǁuǁ.ǁvǁ)
* ǁuǁ=√(<u,u>)
* D(u,v)= ǁu-vǁ (distance between u and v)
* Col(A): 🡪 put A in REF, then take the pivot columns from the original matrix (not from the REF)
* Row(A):🡪 put A in REF, then take the pivot rows from the REF matrix
* Ci=<vi , v>
* Proju v= (<u,v>/ǁvǁ2). V
* U- Proju v and V are orthogonal
* Gran Schmidt process:
  + W1=V1
  + W2=V2- Projv2 w1
  + W3=V3= Projv3w1- Projv3 w2
  + …
* *W*⊥ is a subspace
* *W*⊥∩W={0}
* *dimV= dimW*⊥+ dimW
* N(At)= y/Aty=0 (definition)
* N(A)= x/Ax=0
* N(A)=(rowA) ⊥
* N(At)=(colA) ⊥
* Projv **W=**Σ(<b,wi>/ǁwiǁ2). Wi Where W is an **Orthogonal** basis
* Ax=b is inconsistent if b ∉ col(A)
* Ax= Projv **Col(A)** is the closest vector for the system to be consistent
* **Least mean square solutions**
* **ẍ** is the least mean square solution
* Procedure to find **ẍ**
  + Find a basis for col(A) (by placing in REF and taking the pivot columns from the original matrix)
  + Use Gran Schmidt to find orthogonal matrix
  + Find Projv **W=**Σ(<b,wi>/ǁwiǁ2  where W=col(A)
  + Solve A **ẍ=** Projv **W**
* Procedure 2 to find **ẍ**
  + Solve AtAx=Atb
* A **ẍ=** Projv **Col(A)**

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