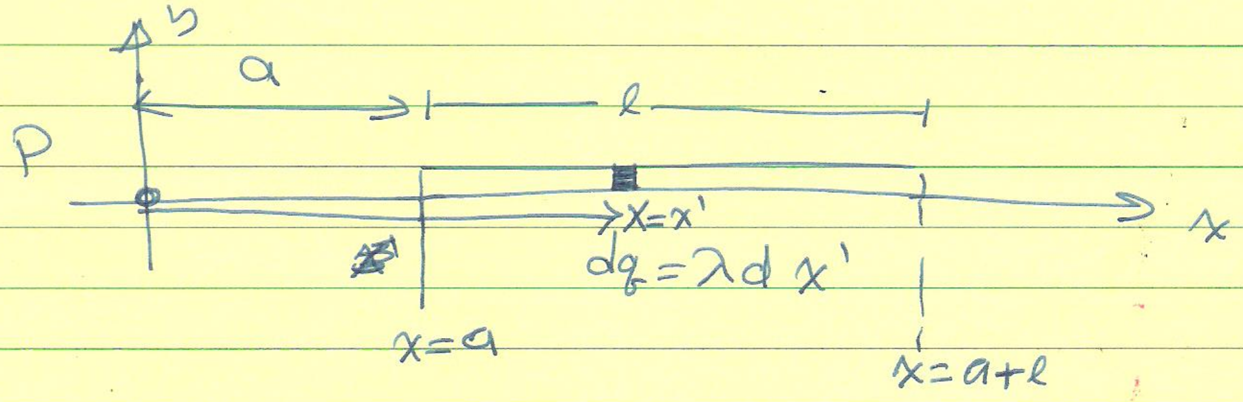
problem 31

find E at P



$$dE = k_e \frac{dq}{x'^2}$$

$$E = k_e \int_{x'=a}^{x'=l+a} \frac{\lambda dx'}{x'^2} = k_e \lambda \left(-\frac{1}{x'}\right]_{x'=a}^{x'=a+l}$$

$$= k_e \lambda \left[\frac{1}{a} - \frac{1}{a+l}\right]$$

$$= k_e \lambda \left[\frac{a+l-a}{a(a+l)}\right]$$

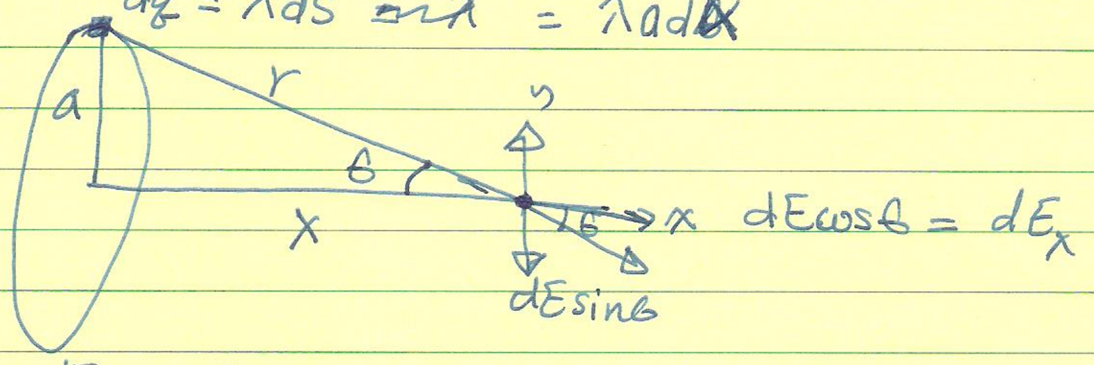
$$= k_e \frac{\lambda a}{a(a+l)}$$

$$= \frac{k_e Q}{a(a+l)}$$

as $a \rightarrow \infty$ l becomes very small.

$$\Rightarrow \boxed{E = \frac{k_e Q}{a^2}}$$

problem 24



$dE_y = dE\sin\theta$ will vanish since there will be always a component that will cancle with it.

so $dE_x = dE\cos\theta$

$$= \frac{k_e dq}{r^2}\cos\theta = k_e \frac{dq}{x^2+a^2} \cdot \frac{x}{(x^2+a^2)^{\frac{1}{2}}}$$

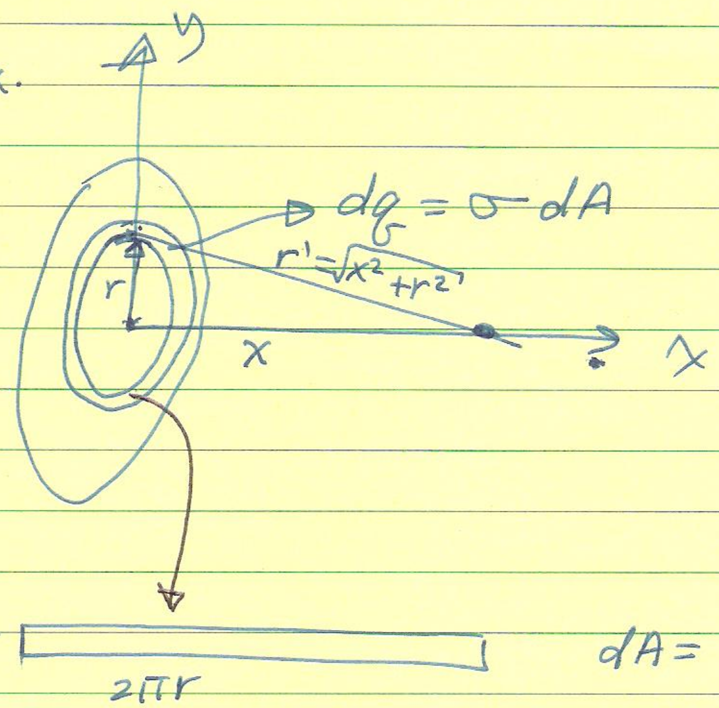
$$= \frac{k_e \lambda a d\theta \, x}{(x^2+a^2)^{\frac{3}{2}}}$$

$$E = \frac{k_e \lambda a x}{(x^2+a^2)^{\frac{3}{2}}} \int_{\theta=0}^{2\pi} d\theta = \frac{k_e \lambda a 2\pi x}{(x^2+a^2)^{\frac{3}{2}}}$$

$$\boxed{E = k_e Q \frac{x}{(x^2+a^2)^{\frac{3}{2}}}}$$

as $x \to \infty$ $a \ll x$ $\Rightarrow$ $\boxed{E = \frac{k_e Q}{x^2}}$

Ex.



$dA = 2\pi r\, dr$

$\therefore \quad dq = \sigma\, 2\pi r\, dr$

$$dE = \frac{k_e\, dq}{(x^2+r^2)^{3/2}}\, x$$

$$dE = \frac{k_e\, \sigma\, 2\pi r\, dr}{(x^2+r^2)^{3/2}}\, x$$

$$E = k_e\, 2\pi x\, \sigma \int_{r=0}^{r=R} \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

let $x^2+r^2 = u$ so if $r=0$ $u = x^2$

$\Rightarrow 2r\,dr = du$ if $r=R$ $u = R^2+x^2$

$$\therefore \quad E = \frac{\sigma k_e\, 2\pi\, x}{2} \int_{u=x^2}^{u=R^2+x^2} \frac{du}{u^{3/2}}$$

$$E = -2k_e \pi x \sigma u^{-\frac{1}{2}} \Big]_{x^2}^{x^2+R^2}$$

$$E = 2k_e \pi x \left[\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}} \right]$$

$$= 2k_e \pi \sigma \left[1 - \frac{x}{\sqrt{x^2+R^2}} \right]$$

$$= 2k_e \pi \sigma \left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \right]$$

for $x >> R$

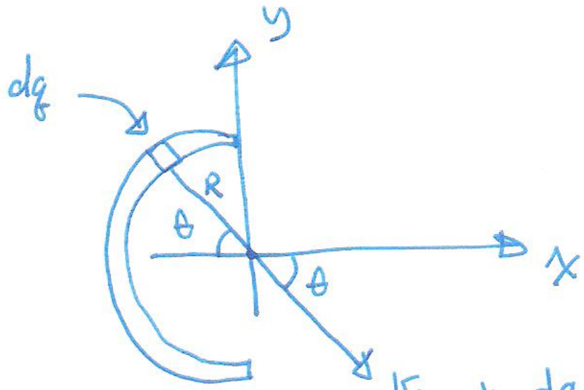
$$\left[1 + \frac{R^2}{x^2}\right]^{-\frac{1}{2}} = 1 - \frac{1}{2}\frac{R^2}{x^2} + \ldots$$

$$E = 2k_e \pi \sigma \left[1 - 1 + \frac{1}{2}\frac{R^2}{x^2} \right]$$

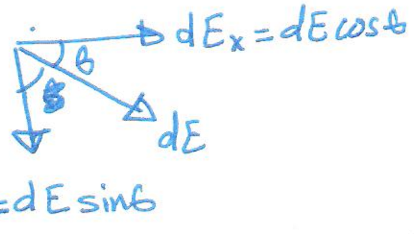
$$= \frac{k_e \pi \sigma R^2}{x^2}$$ But $\pi R^2 \sigma = Q$

$$\boxed{E = \frac{k_e Q}{x^2}}$$

problem 29

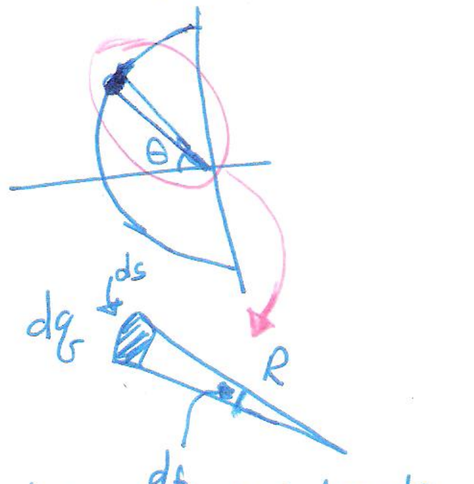


$$dE = k_e \frac{dq}{R^2}$$



$\boxed{\text{The } dE_y \text{ will be cancled!}}$

$$dq = \lambda\, ds = \lambda R\, d\theta$$

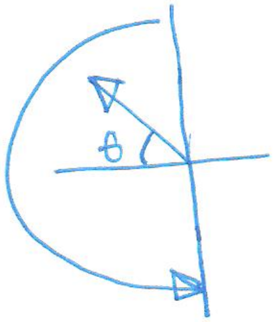


ds is the arc that has dq

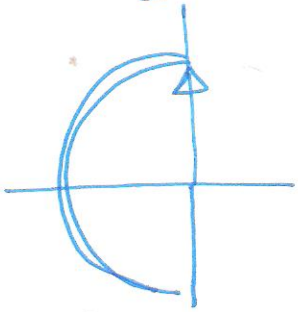
So

$$dE_x = k_e \frac{dq}{R^2}\cos\theta$$

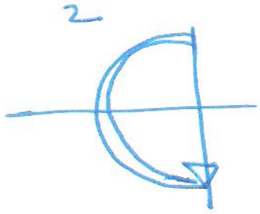
θ goes from $\frac{\pi}{2}$ to $-\frac{\pi}{2}$



the rod starts at $\frac{\pi}{2}$



and ends at $\frac{-\pi}{2}$



so

(2)

$$E_x = k_e \frac{\lambda R}{R^2} \int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} \cos\theta \, d\theta$$

$$= k_e \frac{\lambda R}{R^2} \sin\theta \Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= k_e \frac{\lambda R}{R^2} [2]$$

$$= \frac{2 k_e \lambda}{R}$$ But $\lambda = \frac{Q}{\pi R}$

$$\therefore \quad E_x = \frac{2}{\pi} \frac{k_e Q}{R^2}$$

for the second part when all the charge is concentrated in a point the

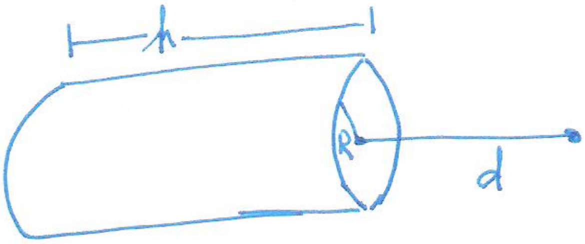
$$E_{point} = \frac{k_e Q}{R^2}$$

therfore $\frac{E_x}{E_{point}} = \frac{\frac{2}{\pi} \cancel{k_e Q / R^2}}{\cancel{k_e Q / R^2}} = \frac{2}{\pi}$

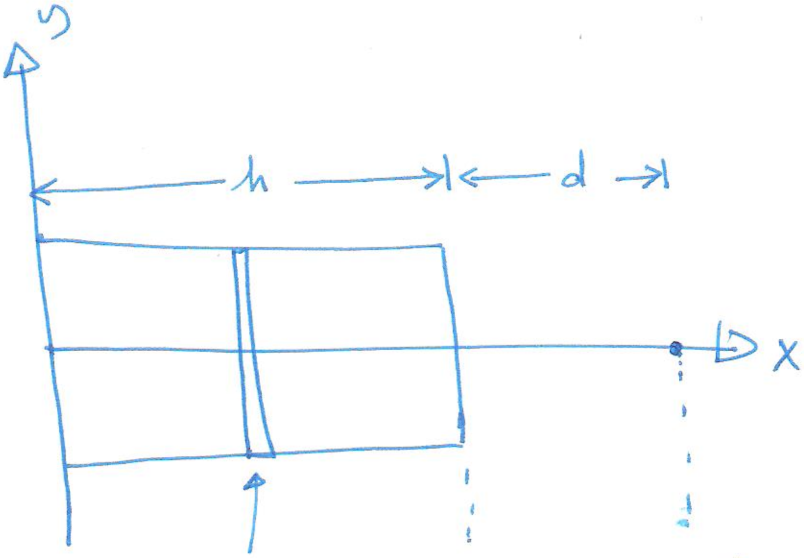
So $E_{point} = \frac{\pi}{2} E_x$

$$= 1.57 \, E_x$$

the electric field due to a cylinder with charge Q at a point d from the cylinder.



→ establish a coordanite system.



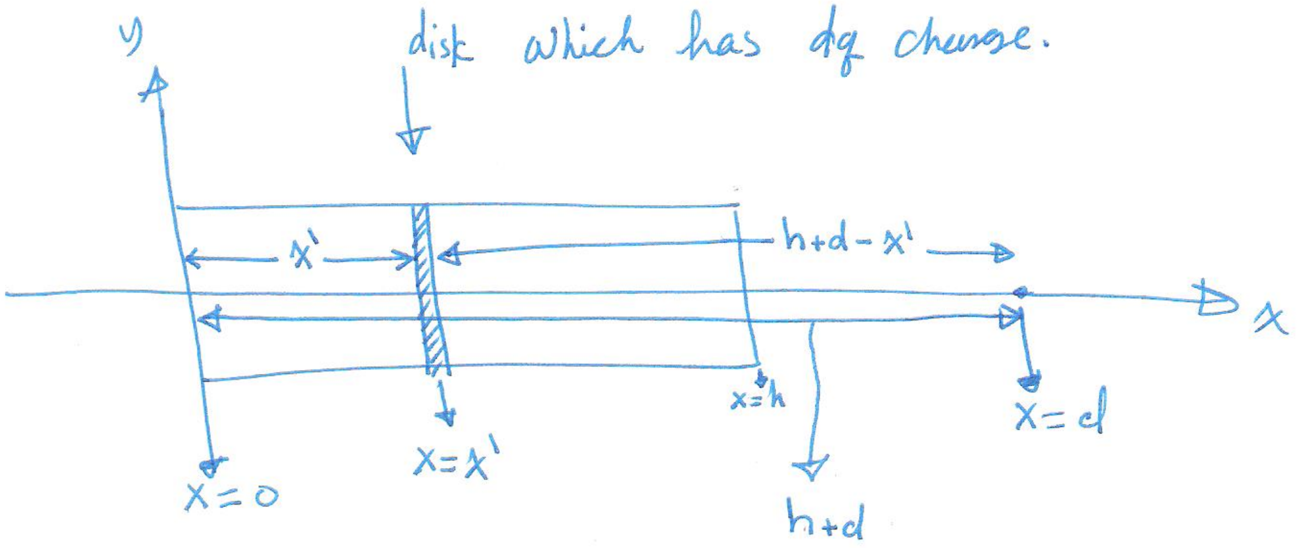
The electric field for a disk

$$E(x) = 2\sigma\pi k_e\left[1 - \frac{x}{(x^2+R^2)^{1/2}}\right]$$

$$= 2ke\frac{Q}{R^2}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

you should know that x in the fourmla for the disk is the distance from the disk to the point where you want to find the electric field.

so if you divied the cylinder into disks you should be carfull in finding this distance



according to the coordainte system and the disk fourmla you should substitute $(h+d-x')$ for the x in the equation

so $$E(x) = 2k_e \frac{Q}{R^2}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

becomes

$$dE(d) = 2k_e \frac{dq}{R^2}\left[1 - \frac{(h+d-x')}{\sqrt{(h+d-x')^2+R^2}}\right]$$

because we want to find the electric field at "d".

now we look for dq

we know that the change is distributed in a Volume of "$\pi R^2 h$" so

$\rho = \frac{Q}{\pi R^2 h}$ however we are looking for dq which is $dq = \rho dV$

now $dV = \pi R^2 dx'$ and again I used the prime to indecate that the location of dq is at x'.

now $dq = \frac{Q}{\pi R^2 h} \cdot \pi R^2 dx'$

$$\boxed{dq = Q \frac{dx'}{h}}$$

So $\Rightarrow$

$$dE(d) = 2k_e Q \frac{dx'}{hR^2}\left[1 - \frac{h+d-x'}{\sqrt{(h+d-x')^2+R^2}}\right]$$

$$E(d) = \frac{2k_e Q}{hR^2}\int_{x'=0}^{x'=h}\left[1 - \frac{h+d-x'}{\sqrt{(h+d-x')^2+R^2}}\right]$$

$$= \frac{2k_e Q}{hR^2}\left[h - \int_{x'=0}^{x'=h}\frac{(h+d-x')\,dx'}{\sqrt{(h+d-x')^2+R^2}}\right]$$

to find the value of the integral.

let $h+d-x' = u \Rightarrow du = -dx'$

when $x'=0$ $u = h+d$

when $x'=h$ $u = d$

$$\therefore -\int_{x=0}^{x=h}(\quad)\,dx' = -\int_{u=d+h}^{u=d}\frac{u\,du}{\sqrt{u^2+R^2}}$$

now let

$u^2+R^2 = t$

$\Rightarrow 2u\,du = dt.$

$$\therefore -\int_{u=d+h}^{u=d}(\quad)\,du = -\frac{1}{2}\int_{t=(d+h)^2+R^2}^{t=d^2+R^2}\frac{dt}{t^{\frac{1}{2}}} = +t^{\frac{1}{2}}\Big]_{t=(d+h)^2+R^2}^{t=d^2+R^2}$$

$$\therefore \quad = -\left[\sqrt{(d+h)^2+R^2} - \sqrt{d^2+R^2}\right]$$

$$\therefore \quad E(d) = 2k_e \frac{Q}{h}\left[h - \left[\sqrt{(d+h)^2+R^2} - \sqrt{d^2+R^2}\right]\right]$$

$$E(d) = 2k_e \frac{Q}{h}\left[h - \sqrt{(d+h)^2+R^2} + \sqrt{d^2+R^2}\right]$$

$$\boxed{E(d) = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \sqrt{(h+d)^2+R^2}/h + \sqrt{d^2+R^2}/h\right]}$$

now let's take the extreme cases.

case ※ 1

When R is much less than h $\quad R \ll h$

$$\left[(h+d)^2+R^2\right]^{\frac{1}{2}} = (h+d)\left[1+\frac{R^2}{(h+d)^2}\right]^{\frac{1}{2}}$$

$$= (h+d)\left[1+\frac{1}{2}\frac{R^2}{(h+d)^2}+\dots\right.$$

$$= h+d+\frac{1}{2}\frac{R^2}{(h+d)} \quad \dots \quad 1$$

$$(d^2+R^2)^{\frac{1}{2}} = d\left[1+\frac{R^2}{d^2}\right]^{\frac{1}{2}}$$

$$= d\left[1+\frac{1}{2}\frac{R^2}{d^2}+\dots\right]$$

$$\doteq d+\frac{1}{2}\frac{R^2}{d}$$

$$\therefore \quad E(d) = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1-\sqrt{(h+d)^2+R^2}/h+\sqrt{R^2+d^2}/h\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left[1-\frac{h+d+\frac{1}{2}R^2/(h+d)}{h}+\frac{d+\frac{1}{2}R^2/d}{h}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2 h}\left[h-h-d-\frac{1}{2}\frac{R^2}{h+d}+d+\frac{1}{2}\frac{R^2}{d}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2 h}\left[\frac{1}{2}\frac{R^2}{d}-\frac{1}{2}\frac{R^2}{h+d}\right]$$

$$= \frac{Q\cdot R^2}{4\pi\varepsilon_0 R^2 h}\left[\frac{1}{d}-\frac{1}{(h+d)}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 h}\left[\frac{h+d-d}{d(h+d)}\right]$$

$$E(d) = \frac{k_e Q}{h} \frac{h}{d(h+d)}$$

$$E(d) = \frac{k_e Q}{d(h+d)}$$

and this is the electric field due to a rod on the x-axis at a point also on the x-axis.

Case ** when $d >> h$ it means that h is also very small so

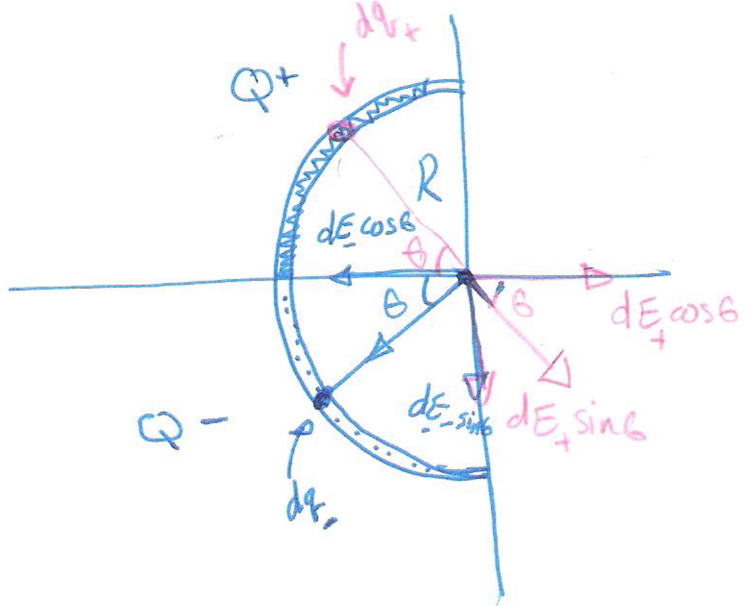
$$E(d) = \frac{k_e Q}{d(h+d)}$$

$$E(d) = \frac{k_e Q}{d^2}$$

which is the electric field due to a point charge.

Suggested problems

a $+Q$ charge is distributed on an arc of 90° and a $-Q$ charge is distributed on an arc of 90° the two arcs are brought together find the Electric field at point "O"



the x-components will cancel.

so the y components. $dE_- \cos\theta$ and $dE_+ \cos\theta$

$$d\vec{E}_x = dE_+ \cos\theta + dE_- \cos\theta = 0$$

$$dE_y = \frac{k_e \lambda_+ R\, d\theta}{R^2} \sin\theta + \frac{k_e \lambda_- R\, d\theta}{R^2} \sin\theta$$

$$\cancel{= \frac{2 k_e \lambda R}{R^2} \sin}$$

$$E_y = \frac{k_e \lambda}{R}\left[\int_{\theta=\frac{\pi}{2}}^{0} \sin\theta \, d\theta - \int_{\theta=0}^{-\frac{\pi}{2}} \sin\theta \, d\theta\right]$$

$$= \frac{-k_e \lambda}{R}\left[\cos\theta \Big]_{\frac{\pi}{2}}^{0} - \cos\theta \Big]_{0}^{-\frac{\pi}{2}}\right]$$

$$= \frac{-k_e \lambda}{R}\left[1 + 1\right]$$

$$E_y = \frac{-2k_e \lambda}{R}$$

$$\boxed{\vec{E}_y = \frac{-2k_e \lambda}{R}\hat{j}}$$