THERMODYNAMICS – Exam II

Lebanese American University School of Engineering

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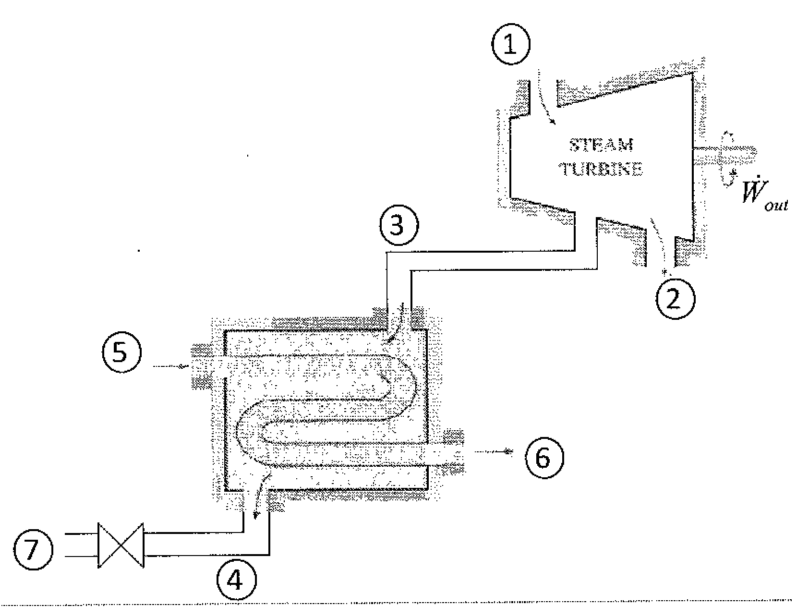
Date: Tuesday, January 5^{th} 2010; 6:00PM
Location: ENG Attic
Instructor: Dr. Wassim HABCHI
Notes: No documents allowed
Value: 20% of Total Grade
Time: 120 Minutes

/100

Problem I (40 points)

1) The adiabatic steam turbine of the Zouk power plant produces 50 MW of electric power. The incoming steam to the turbine is at a pressure P_1= 5MPa and a temperature T_1=1300°C. Water exits the turbine at P_2=150 KPa with a quality x_2=0.95. If the electric generator of the power plant is assumed to have an efficiency of 100%. Determine the mass flow rate of water flowing through the turbine. **(10 points)**

2) The Lebanese government is now considering the use of the Zouk power plant as a co-generation system (simultaneous generation of electricity and heating) for the Mount-Lebanon area (See Figure below). It is proposed that a mass flow rate $\dot{m}_3 = 1\,Kg/s$ of steam be removed from an intermediate stage of the turbine where P_3=2 MPa and T_3=700°C. Determine the new electric power output of the turbine (Assume that the incoming mass flow rate of steam at the turbine inlet is $\dot{m}_1 = 17.7\,Kg/s$ and that the electric generator has an efficiency of 100%). The inlet and outlet conditions of water (points 1 and 2) are the same as previously. **(10 points)**



3) The steam removed from the turbine is used in a well insulated heat exchanger to heat an incoming stream of water with a mass flow rate $\dot{m}_5 = 5\, Kg/s$, a pressure P_5=5MPa and a temperature T_5=25°C. If it is required that the temperature of the cold water be raised to T_6=90°C, determine the exit temperature T_4 of ~~the steam~~ water (Neglect any pressure drops within the heat exchanger). **(15 points)**

4) Finally, for security reasons, it is required that the exit temperature of steam be dropped to 40°C. In order to achieve this task, engineers at the Zouk power plant proposed that the outcoming steam from the heat exchanger be passed through an expansion valve. Determine the required pressure drop ΔP through the valve. **(5 points)**

Solution:

1) Since the generator' sefficiency is 100%, then $\dot{W}_{out,turb}$ = 50 MW

Applying the 1st Low of thermo to the turbine :

$$(\cancel{\dot{Q}_{in}} - \cancel{\dot{Q}_{out}}) + (\cancel{\dot{W}_{in}} - \dot{W}_{out}) + \dot{m}_1 h_1 - \dot{m}_2 h_2 = 0$$

Since it's a steady-flow device. And since it's a single-stream device too $\Longrightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$

$$\Longrightarrow \dot{W}_{out} = \dot{m}(h_1 - h_2) \Longrightarrow \dot{m} = \frac{\dot{W}_{out}}{h_1 - h_2}$$

From $\boxed{\text{Table A-6}} \longrightarrow h_1 = 5405{,}7\, KJ/Kg$

$\boxed{\text{Table A-5}} \longrightarrow h_2 = h_f + x_2 h_{fg} = 467{,}13 + 0{,}95 \times 2226$

$= 2581{,}83\, KJ/Kg$

$$\Longrightarrow \dot{m} = \frac{50 \times 10^3}{5405{,}7 - 2581{,}83} = \boxed{17{,}7\, Kg/s}$$

2) Applying a mass balance to the new turbine system:

$$\dot{m}_1 - \dot{m}_2 - \dot{m}_3 = 0 \Longrightarrow \dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 17{,}7 - 1 = \boxed{16{,}7\, Kg/s}$$

* Energy balance (Neglecting ke and pe):

$$-\dot{W}_{out} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0$$

$$\Rightarrow \dot{W}_{out} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$= 17{,}7 \times 5405{,}7 - 16{,}7 \times 2581{,}83 - 1 \times 3918{,}2$$ from $\boxed{\text{Table A-6}}$

$$= 48646{,}13\ \text{KW} = \boxed{48{,}65\ \text{MW}}$$

And since the generator's efficiency is 100 %, then the new electric power output of the turbine system is 48,65 MW.

3) Applying the 1st Law of thermo to the heat exchanger, we get:

$$(\dot{m}_3 h_3 - \dot{m}_4 h_4) + (\dot{m}_5 h_5 - \dot{m}_6 h_6) = 0$$

However: $\dot{m}_3 = \dot{m}_4 = 1\ \text{Kg/s}$

$\dot{m}_5 = \dot{m}_6 = 5\ \text{Kg/s}$

$$\Rightarrow h_3 - h_4 = -\dot{m}_5 (h_5 - h_6)$$

$$h_4 = h_3 + \dot{m}_5 (h_5 - h_6)$$

with $h_3 = 3918{,}2\ \text{KJ/Kg}$

From $\boxed{\text{Table A-7}}$: $h_5 = h\ @\ 5\text{MPa}, 25°\text{C}$

By interpolation: $h_5 = 109{,}445\ \text{KJ/Kg}$.

And $h_6 = h\ @\ 5\text{MPa}, 90°\text{C} = 380{,}905\ \text{KJ/Kg}$

$$\Rightarrow h_4 = 3918{,}2 + 5\,(109{,}445 - 380{,}905)$$

$$\boxed{h_4 = 2560{,}9\ \text{KJ/Kg}}$$

@4 we have : $P_4 = 2MPa$ & $h_4 = 2560\ KJ/Kg$

but from $\boxed{\text{Table A-5 :}}$ @ 2MPa : $h_f = 908,47\ KJ/Kg$

and $h_g = 2798,3\ KJ/Kg$.

Since $h_f < h_4 < h_g \Rightarrow$ Sat. Liq. vap. mixture

And $T_4 = T_{sat}$ @ 2MPa = $\boxed{212,38\ °C}$

4) The expansion valve being an isenthalpic device $\Rightarrow h_4 = h_7$

$\Rightarrow h_7 = 2560\ KJ/Kg$ & $T_7 = 40°C$

However, from $\boxed{\text{Table A-4}}$, @40°C : $h_f = 167,53\ KJ/Kg$

$h_g = 2573,5\ KJ/Kg$

Since $h_f < h_7 < h_g \rightarrow$ Sat. Liq. vap. mixture

And $P_7 = P_{sat}$ @40°C = 7,3851 KPa

$\Rightarrow \Delta P = P_4 - P_7 = 2000 - 7,3851 = \boxed{1992,61\ KPa}$
$= \boxed{1,99\ MPa}$

Problem II (35 points)

A frictionless piston-cylinder device initially contains m_1=0.5Kg of Air at P_1=150KPa and T_1=25°C. The piston is attached to a linear spring it has a diameter of 10cm. The atmospheric pressure is 100KPa. Determine:

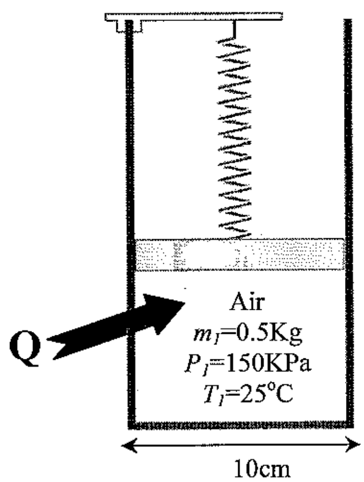
a) The initial volume of Air V_1 in the device **(5 points)**

b) Now, 10KJ of heat are added to the system which leads to an upward motion of the piston. Determine the amount of boundary work associated to this motion if the temperature of Air increases by 25°C. **(10 points)**

c) Determine the final volume of Air V_2 within the cylinder **(15 points)**

d) Determine the force exerted by the spring on the piston in the final state. **(5 points)**

Neglecting the piston's weight



Solution:

a) Air is an ideal gas. Hence, using the ideal-gas relationship, we get:

$$P_1 V_1 = m_1 R T_1$$

From $\boxed{\text{Table A-2a}}$: $R = 0{,}287$ KJ/Kg·K

$$\Rightarrow V_1 = \frac{m_1 R T_1}{P_1} = \frac{0{,}5 \times 0{,}287 \times 298}{150} = \boxed{0{,}285 \ m^3}$$

b) Applying the 1st Law of thermo to the gases inside the cylinder:

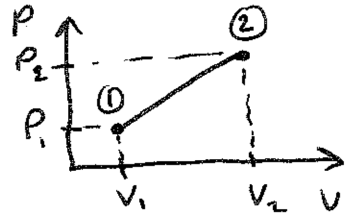
$$Q_i - W_b = m \Delta u = m c_v \Delta T$$

But from $\boxed{\text{Table A2-a}}$: $c_v = 0{,}718$ KJ/Kg.K

$$\Rightarrow W_b = Q_{in} - m c_v \Delta T$$

$\Rightarrow W_b = 10 - 0{,}5 \times 0{,}718 \times 25 = \boxed{1{,}025\ KJ}$

c) the spring force being linear with respect to V, the pressure rise within the cylinder is linear with respect to V, as shown below:



However, W_b is nothing else but the area under the P-V curve:

$$\boxed{W_b = \frac{1}{2}(P_1+P_2)(V_2-V_1)} \quad ①$$

And, from the ideal gas relationship, we know that:

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \Rightarrow P_2V_2 = P_1V_1\frac{T_2}{T_1} = \frac{150 \times 0{,}285 \times 323}{298}$$

$$\boxed{P_2V_2 = 46{,}34\ KJ} \quad ②$$

Eqs. ① and ② form a system of 2 eqs with 2 unknowns P_2 and V_2:

$$\begin{cases} P_2V_2 = 46{,}34 \Rightarrow V_2 = 46{,}34/P_2 \text{ or } P_2 = \dfrac{46{,}34}{V_2} \\ \frac{1}{2}(P_1+P_2)(V_2-V_1) = 1{,}025 \end{cases}$$

$$\Rightarrow \frac{1}{2}(150+P_2)(V_2-0{,}285) = 1{,}025$$

$$\left(150+\frac{46{,}34}{V_2}\right)(V_2-0{,}285) = 2{,}05$$

$$150V_2 - 42{,}75 + 46{,}34 - \frac{13{,}2069}{V_2} = 2{,}05$$

$$150V_2 - \frac{13{,}2069}{V_2} = -1{,}54$$

$$150V_2^2 + 1{,}54V_2 - 13{,}2069 = 0 \quad \begin{cases} V_2 = 0{,}2916\ m^3 \\ V_2 = \cancel{-0{,}301}\ m^3 \end{cases}$$

The 2nd answer is meaningless and $\boxed{V_2 = 0{,}2916\, m^3}$

d) We know that $P_2 V_2 = 46{,}34$ KJ.

$$\Rightarrow P_2 = \frac{46{,}34}{0{,}2916} = 158{,}916 \, KPa$$

Applying an equilibrium of forces on the piston in the final state, we have:

$$P_2 \times A = P_{atm} \times A + F_s$$

$$\Rightarrow F_s = (P_2 - P_{atm}) \times A$$

$$= (158{,}916 - 100) \times \frac{\pi \times 0{,}1^2}{4}$$

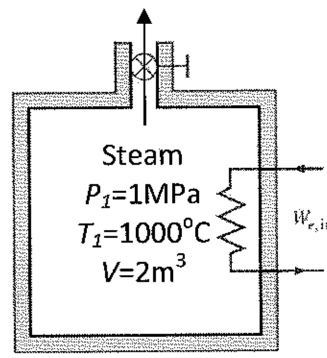
$$\Rightarrow F_s = 0{,}4627 \, KN \Rightarrow \boxed{F_s = 462{,}7 \, N}$$

Problem III (25 points)

An insulated 2m³ rigid tank contains superheated steam at a pressure P_1=1MPa and a temperature T_1=1000°C. A valve connected to the tank is now opened, and steam is allowed to escape until the pressure inside the tank drops to 0.1MPa. The steam's temperature inside the tank is maintained constant during the process by an electric resistance heater placed inside the tank. The exit pressure and temperature of steam through the valve are regulated to 0.1MPa and 800°C respectively. Determine:

a) The final mass of steam m_2 inside the tank. **(10 points)**

b) The electric work done during this process. **(15 points)**



Solution:

a) This is an unsteady-flow process studied over a finite period of time between the opening and closing events of the valve.

At the final state, we know that $P_2 = 0{,}1\,MPa$ and $T_2 = 1000°C$

→ From $\boxed{\text{Table A-6}}$: $v_2 = 5{,}8755\ m^3/Kg$.

But $m_2 = \frac{V}{v_2} = \frac{2}{5{,}8755} = \boxed{0{,}34\ Kg}$

b) Mass balance: $-m_{out} = m_2 - m_1$

But $m_1 = \frac{V}{v_1}$

From $\boxed{\text{Table A-6}}$, @ 1MPa & 1000°C → $v_1 = 0{,}58721\ m^3/Kg$

$\Rightarrow m_1 = \frac{2}{0{,}58721} = \boxed{3{,}4059\ Kg}$

And $m_{out} = m_1 - m_2 = 3,4059 - 0,34 = \boxed{3,0659\ Kg}$

* Energy balance:

$$(\cancel{Q_{in}}^{0} - \cancel{Q_{out}}^{0}) + (W_{in} - \cancel{W_{out}}^{0}) + (\cancel{E_{mass,in}} - E_{mass,out}) = m\,\Delta e_{sys}$$

Neglecting ke, pe, Δke and Δpe, we get:

$$W_{elec,in} - m_{out}\,h_{out} = m_2 u_2 - m_1 u_1$$

$$\boxed{W_{elec,in} = m_{out}\,h_{out} + m_2 u_2 - m_1 u_1}$$

From Table A-6: $h_{out} = 4160,2\ KJ/Kg$ (@ 0,1 MPa, 800°C)

$u_2 = 4055\ KJ/Kg$ (@ 0,1 MPa, 1000°C)

$u_1 = 4052,7\ KJ/Kg$ (@ 1 MPa, 1000°C)

$$\Rightarrow W_{elec,in} = 3,0659 \times 4160,2 + 0,34 \times 4055 - 3,4059 \times 4052,7$$

$$\boxed{W_{elec,in} = 330,37\ KJ}$$