Chapter 3

Vectors and
Two-Dimensional Motion

Vector vs. Scalar Review

- All physical quantities encountered in this text will be either a scalar or a vector
- A **vector** quantity has both magnitude (size) and direction
- A **scalar** is completely specified by only a magnitude (size)

Vector Notation

- When handwritten, use an arrow: $\vec{A}$
- When printed, will be in bold print with an arrow: $\vec{\mathbf{A}}$
- When dealing with just the magnitude of a vector in print, an italic letter will be used: *A*
  - Italics will also be used to represent scalars

Properties of Vectors

- Equality of Two Vectors
  - Two vectors are **equal** if they have the same magnitude and the same direction
- Movement of vectors in a diagram
  - Any vector can be moved parallel to itself without being affected

More Properties of Vectors

- Negative Vectors
  - Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)
  - $\vec{\mathbf{A}} = -\vec{\mathbf{B}};\ \vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$
- Resultant Vector
  - The **resultant** vector is the sum of a given set of vectors
  - $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Adding Vectors

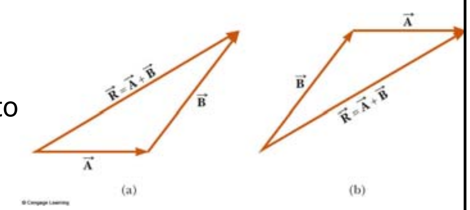
- When adding vectors, their directions must be taken into account
- Units must be the same
- Geometric Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient

Adding Vectors Geometrically (Triangle or Polygon Method)

- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector using the same scale with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{\mathbf{A}}$ and parallel to the coordinate system used for $\vec{\mathbf{A}}$

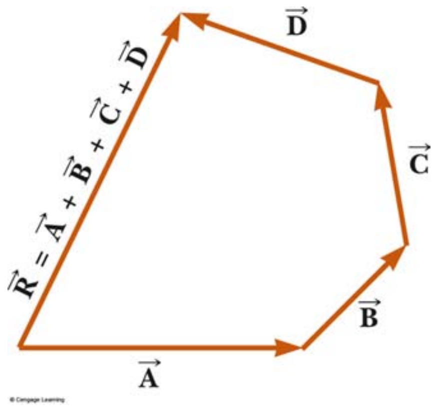
Graphically Adding Vectors, cont.

- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of $\vec{\mathbf{A}}$ to the end of the last vector
- Measure the length of $\vec{\mathbf{R}}$ and its angle
  - Use the scale factor to convert length to actual magnitude



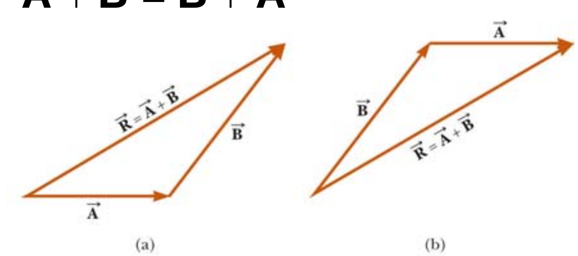
Graphically Adding Vectors, cont.

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



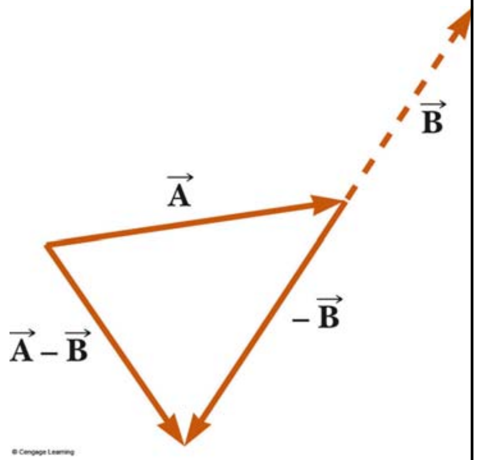
Notes about Vector Addition

- Vectors obey the **Commutative Law of Addition**
  - The order in which the vectors are added doesn't affect the result
  - $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$



Vector Subtraction

- Special case of vector addition
  - Add the negative of the subtracted vector
- $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$
- Continue with standard vector addition procedure

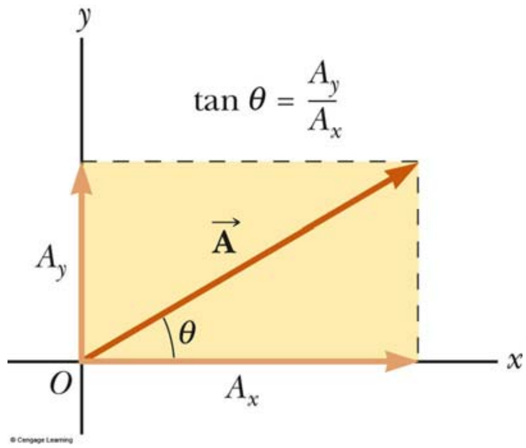


Multiplying or Dividing a Vector by a Scalar

- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes



Components of a Vector, cont.

- The x-component of a vector is the projection along the x-axis
  $A_x = A\cos\theta$
- The y-component of a vector is the projection along the y-axis
  $A_y = A\sin\theta$
- Then, $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$

More About Components of a Vector

- The previous equations are valid ***only if θ is measured with respect to the x-axis***
- The components can be positive or negative and will have the same units as the original vector

More About Components, cont.

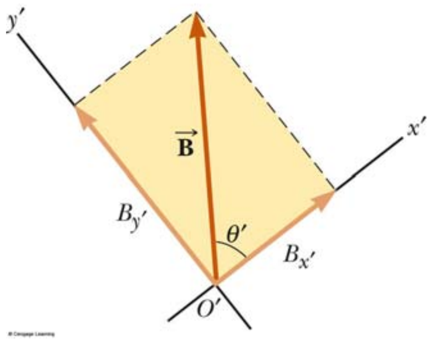
- The components are the legs of the right triangle whose hypotenuse is $\vec{\mathbf{A}}$

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

  - May still have to find θ with respect to the positive x-axis
  - The value will be correct only if the angle lies in the first or fourth quadrant
  - In the second or third quadrant, add 180°

Other Coordinate Systems

- It may be convenient to use a coordinate system other than horizontal and vertical
- Choose axes that are perpendicular to each other
- Adjust the components accordingly



Adding Vectors Algebraically

- Choose a coordinate system and sketch the vectors
- Find the x- and y-components of all the vectors
- Add all the x-components
  - This gives R_x:

$$R_x = \sum v_x$$

Adding Vectors Algebraically, cont.

- Add all the y-components
  - This gives R_y: $R_y = \sum v_y$
- Use the Pythagorean Theorem to find the magnitude of the resultant: $R = \sqrt{R_x^2 + R_y^2}$
- Use the inverse tangent function to find the direction of R:

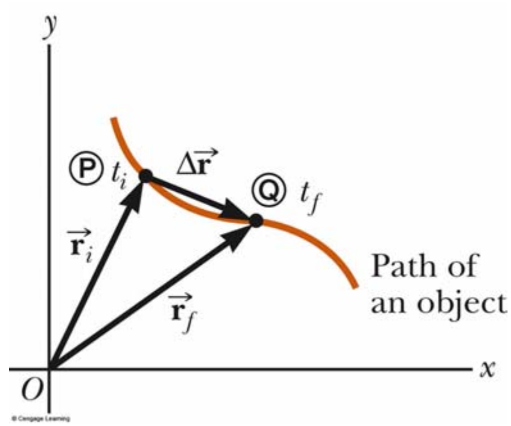
$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

Motion in Two Dimensions

- Using + or – signs is not always sufficient to fully describe motion in more than one dimension
  - Vectors can be used to more fully describe motion
- Still interested in displacement, velocity, and acceleration

Displacement

- The position of an object is described by its position vector, $\vec{\mathbf{r}}$
- The **displacement** of the object is defined as the ***change in its position***
  - $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$



Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero
  - The direction of the instantaneous velocity is along a line that is tangent to the path of the particle and in the direction of motion

Acceleration

- The average acceleration is defined as the rate at which the velocity changes

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

Unit Summary (SI)

- Displacement
  - m
- Average velocity and instantaneous velocity
  - m/s
- Average acceleration and instantaneous acceleration
  - m/s^2

Ways an Object Might Accelerate

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change
  - Even though the magnitude is constant
- Both the magnitude and the direction can change

Projectile Motion

- An object may move in both the x and y directions simultaneously
  - It moves in two dimensions
- The form of two dimensional motion we will deal with is an important special case called **projectile motion**

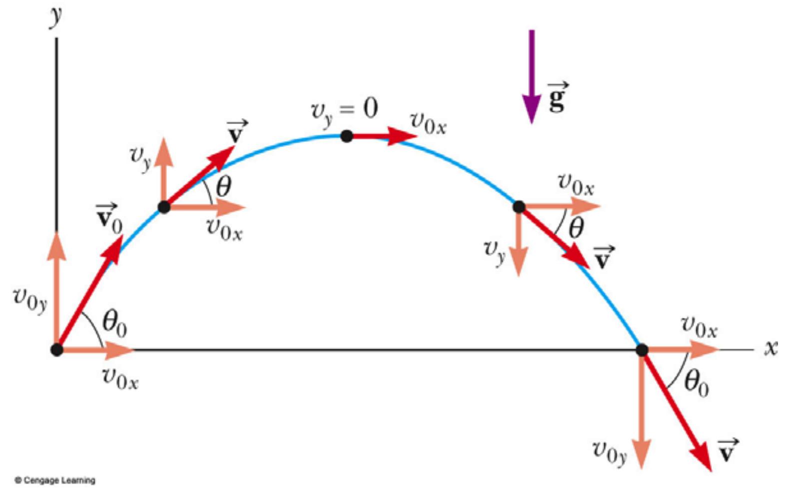
Assumptions of Projectile Motion

- We may ignore air friction
- We may ignore the rotation of the earth
- With these assumptions, an object in projectile motion will follow a parabolic path

Rules of Projectile Motion

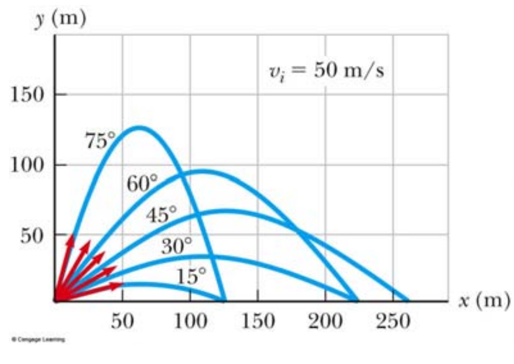
- The x- and y-directions of motion are completely independent of each other
- The x-direction is uniform motion
  - $a_x = 0$
- The y-direction is free fall
  - $a_y = -g$
- The initial velocity can be broken down into its x- and y-components
  - $v_{Ox} = v_O \cos\theta_O \quad v_{Oy} = v_O \sin\theta_O$

Projectile Motion



Projectile Motion at Various Initial Angles

- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°



Some Details About the Rules

- x-direction
  - $a_x = 0$
  - $v_x = v_{o_x} = v_o \cos\theta_o = constant$
  - $x = v_{ox}t$
    - This is the only operative equation in the x-direction since there is uniform velocity in that direction

More Details About the Rules

- y-direction
  - $v_{o_y} = v_o \sin\theta_o$
  - Free fall problem
    - a = -g
  - Take the positive direction as upward
  - Uniformly accelerated motion, so the motion equations all hold

Velocity of the Projectile

- The velocity of the projectile at any point of its motion is the vector sum of its x and y components at that point

$$v = \sqrt{v_x^2 + v_y^2} \quad and \quad \theta = \tan^{-1}\frac{v_y}{v_x}$$

  - Remember to be careful about the angle's quadrant

Projectile Motion Summary

- Provided air resistance is negligible, the horizontal component of the velocity remains constant
  - Since $a_x = 0$
- The vertical component of the velocity v_y is equal to the free fall acceleration -*g*

Projectile Motion Summary, cont

- The vertical component of the velocity v_y and the displacement in the y-direction are identical to those of a freely falling body
- Projectile motion can be described as a superposition of two independent motions in the *x*- and *y*-directions

Problem-Solving Strategy

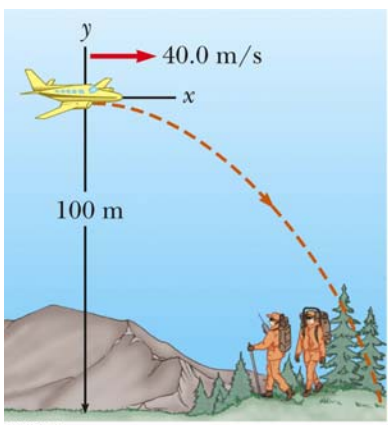
- **Select** a coordinate system and sketch the path of the projectile
  - Include initial and final positions, velocities, and accelerations
- **Resolve** the initial velocity into x- and y-components
- **Treat** the horizontal and vertical motions independently

Problem-Solving Strategy, cont

- **Follow** the techniques for solving problems with constant velocity to analyze the horizontal motion of the projectile
- **Follow** the techniques for solving problems with constant acceleration to analyze the vertical motion of the projectile

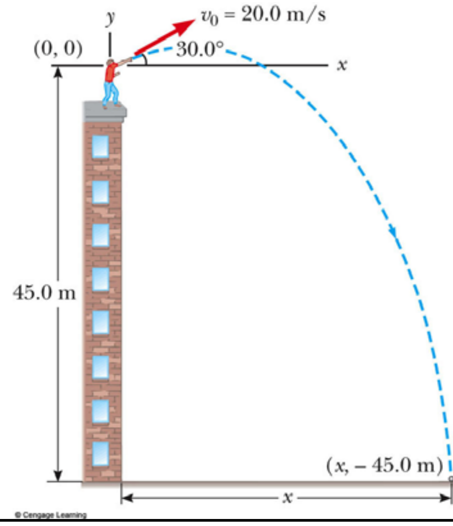
Some Variations of Projectile Motion

- An object may be fired horizontally
- The initial velocity is all in the x-direction
  - $v_o = v_x$ and $v_y = 0$
- All the general rules of projectile motion apply



Non-Symmetrical Projectile Motion

- Follow the general rules for projectile motion
- Break the y-direction into parts
  - up and down
  - symmetrical back to initial height and then the rest of the height



Special Equations

- The motion equations can be combined algebraically and solved for the range and maximum height

$$\Delta x = \frac{v_o^2 \sin 2\theta_o}{g}$$

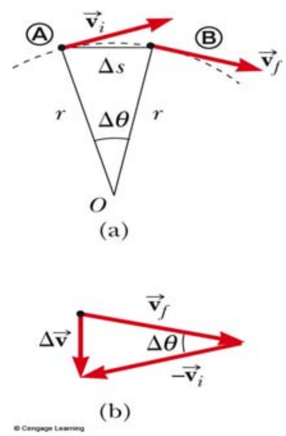
$$\Delta y_{max} = \frac{v_o^2 \sin^2 \theta_o}{g}$$

Centripetal Acceleration

- An object traveling in a circle, even though it moves with a constant speed, will have an acceleration
- The centripetal acceleration is due to the change in the *direction* of the velocity

Centripetal Acceleration, cont.

- Centripetal refers to "center-seeking"
- The direction of the velocity changes
- The acceleration is directed toward the center of the circle of motion



Centripetal Acceleration, final

- The magnitude of the centripetal acceleration is given by
- $$a_c = \frac{v^2}{r}$$
  - This direction is toward the center of the circle

centripetal Acceleration and Angular Velocity

- The angular velocity and The centripetal acceleration can also be related to the angular velocity and the linear velocity are related
- (v = ωr)
- The centripetal acceleration can also be related to the angular velocity

$$a_C = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$