Chapter 2 : Motion in One Dimension

- ***Dynamics*** The branch of physics involving the motion of an object and the relationship between that motion and other physics concepts
- ***Kinematics*** is a part of dynamics
  - In kinematics, you are interested in the *description* of motion
  - *Not* concerned with the cause of the motion
- Any motion involves three concepts: displacement, velocity and acceleration

2.1 Position and Displacement

Before describing a motion, you must set up a coordinate system – define an origin and a positive direction.

Position is defined in terms of a **frame of reference**
- A choice of coordinate axes
- Defines a starting point for measuring the motion
  - Or any other quantity
- One dimensional, so generally the x- or y-axis

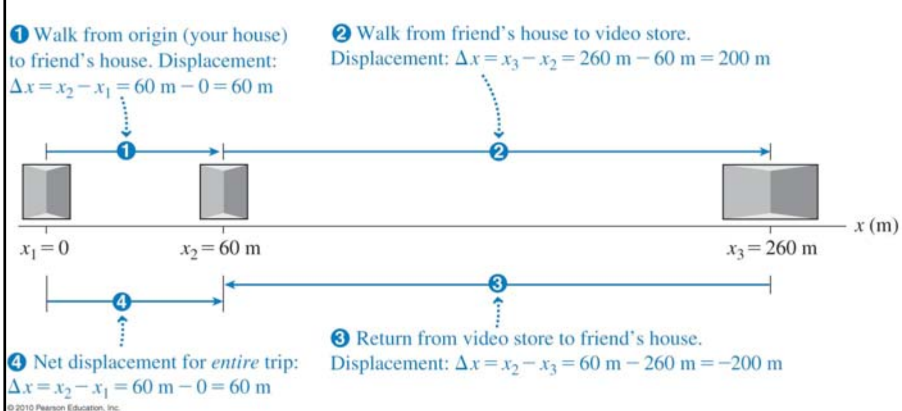
Displacement is defined as the *change in position (in one dimension)*

$$\Delta x \equiv x_f - x_i$$

- f stands for final and i stands for initial
- Units are meters (m) in SI

Distance vs. Displacement

The distance is the total length of travel, in the case below it adds up to 460 m. While the displacement is only the change in the position, and is equal to 60 m in the example below.



2.2 Velocity and Speed

The **average speed** is defined as the total distance traveled divided by the time the trip took.

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$v = \frac{d}{t}$$

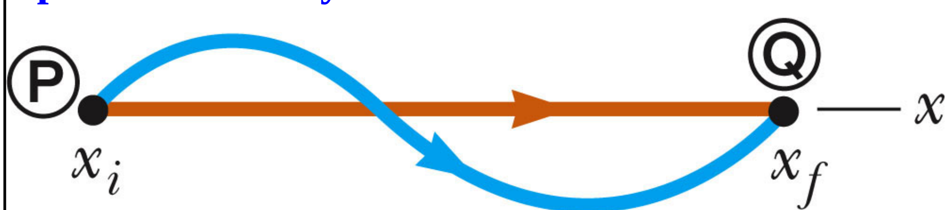
Note that speed is always positive as both distance and time will be positive. SI units (m/s)

The **average velocity** is the rate at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Note that velocity can be positive or negative. Direction will be the same as Δx, + or - is sufficient SI units (m/s)

Speed vs. Velocity

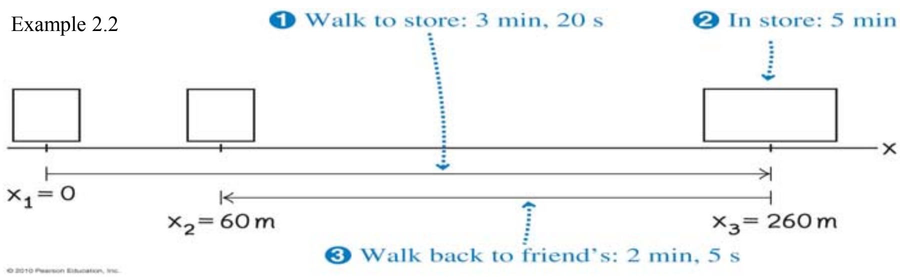


Both cars travel from P to Q in the same amount of time.
Which car has the greatest average speed?
What about the velocity?

- The car on the blue path will have a greater average speed since the distance it traveled is larger
- Cars on both paths have the same average velocity since they had the same displacement in the same time interval

Speed vs. Velocity

Example 2.2



For the entire trip Δt = 8mn, 25s = 625 s.
Average speed = d/t = 460 m/ 625 s = 0.74m/s.
Average velocity = $\Delta x/\Delta t$ = 60 m/ 625 s = 0.096m/s.

Do problem 40 from the book.

Question 2.2 — Cruising Along I

You drive for 30 minutes at 30 mi/hr and then for another 30 minutes at 50 mi/hr. What is your average speed for the whole trip?

a) more than 40 mi/hr

b) equal to 40 mi/hr

c) less than 40 mi/hr

Question 2.2 Cruising Along I

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a) more than 40 mi/hr
b) equal to 40 mi/hr
c) less than 40 mi/hr

It is 40 mi/hr in this case. Because the average speed is distance/time and you spend the same amount of time at each speed, then your average speed would indeed be 40 mi/hr.

Question 2.3 Cruising Along II

You drive 4 miles at 30 mi/hr and then another 4 miles at 50 mi/hr. What is your average speed for the whole 8-mile trip?

a) more than 40 mi/hr
b) equal to 40 mi/hr
c) less than 40 mi/hr

Question 2.3 Cruising Along II

You drive 4 miles at 30 mi/hr and then another 4 miles at 50 mi/hr. What is your average speed for the whole 8-mile trip?

a) more than 40 mi/hr

b) equal to 40 mi/hr

c) less than 40 mi/hr

It is not 40 mi/hr! Remember that the average speed is distance/time. Because it takes longer to cover 4 miles at the slower speed, you are actually moving at 30 mi/hr for a longer period of time! Therefore, your average speed is closer to 30 mi/hr than it is to 50 mi/hr.

Question 2.5 Graphing Velocity I

The graph of position versus time for a car is given below. What can you say about the velocity of the car over time?

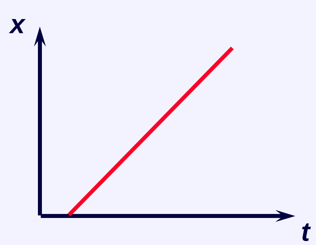
a) it speeds up all the time

b) it slows down all the time

c) it moves at constant velocity

d) sometimes it speeds up and sometimes it slows down

e) not really sure



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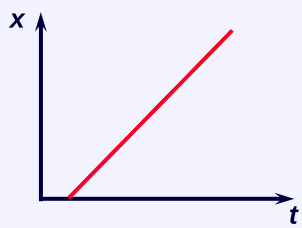
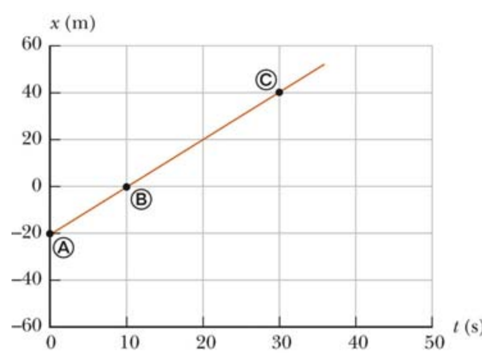


The car moves at a constant velocity because the x vs. t plot shows a straight line. The slope of a straight line is constant. Remember that the slope of x vs. t is the velocity!

Graphical Interpretation of Velocity (constant)

The straight line indicates constant velocity

The slope of the line is the value of the average velocity

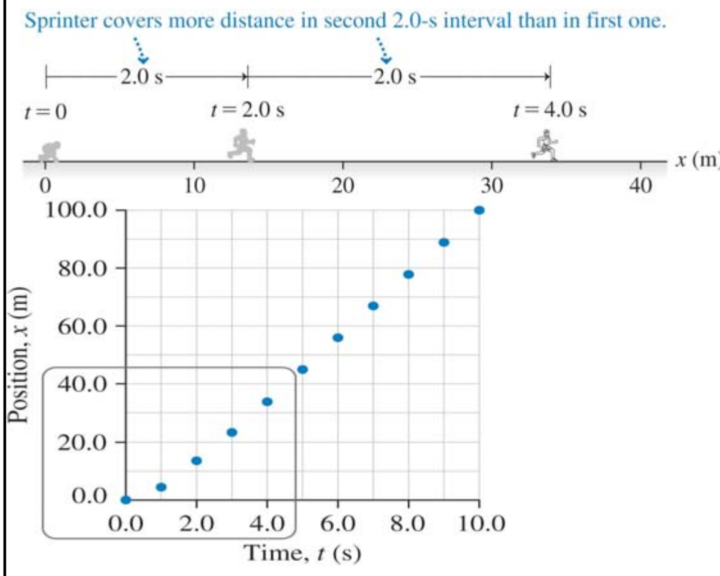


(a)

The general equation for the slope of any line is

$$slope = \frac{change\ in\ vertical\ axis}{change\ in\ horizontal\ axis}$$

Graphical Interpretation of Velocity: example

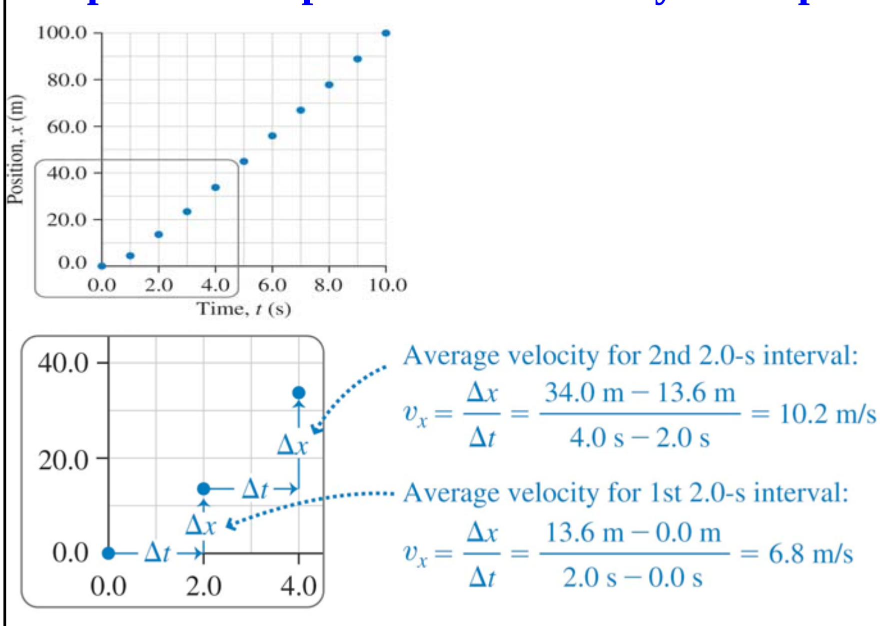


(b) Position-versus-time graph

Time, t (s)	Position, x (m)
0.0	0.0
1.0	4.7
2.0	13.6
3.0	23.4
4.0	34.0
5.0	45.0
6.0	56.0
7.0	67.0
8.0	78.0
9.0	89.0
10.0	100.0

(a) The data

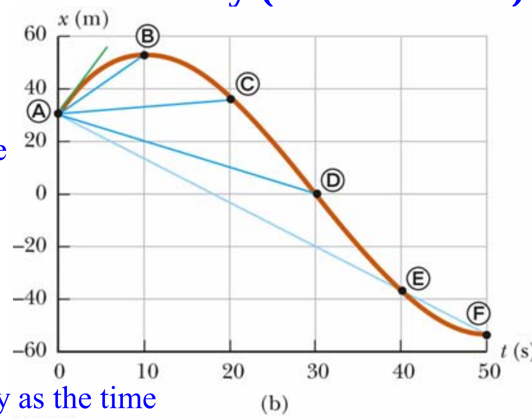
Graphical Interpretation of Velocity: example



(d) How to compute average velocity

Graphical Interpretation of Velocity (non-constant)

The motion is non-constant velocity
The average velocity is the slope of the straight line joining the initial and final points.



The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

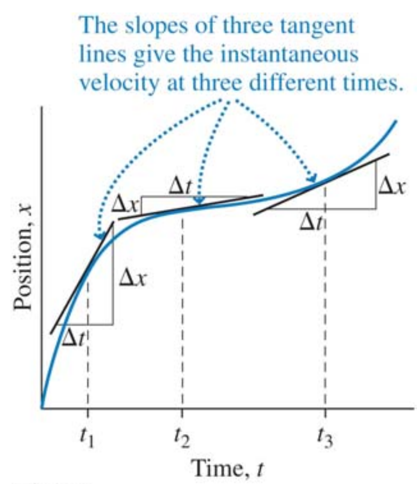
The instantaneous velocity indicates what is happening at every point of time

Graphical Interpretation of Velocity (non-constant)

Note that a positive slope means a positive velocity.

The slope of the line tangent to the position-vs.-time graph is defined to be the instantaneous velocity at that time

The instantaneous speed is defined as the magnitude of the instantaneous velocity



Question 2.6 Graphing Velocity II

The graph of position vs. time for a car is given below. What can you say about the velocity of the car over time?

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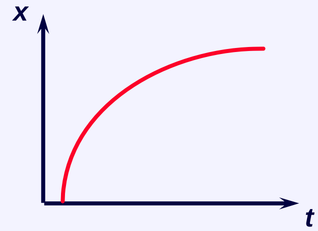
b) it slows down all the time
c) it moves at constant velocity
d) sometimes it speeds up and sometimes it slows down
e) not really sure



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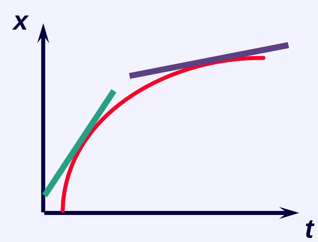
The car slows down all the time because the slope of the *x* vs. *t* graph is diminishing as time goes on. Remember that the slope of *x* vs. *t* is the velocity! At large *t*, the value of the position *x* does not change, indicating that the car must be at rest.



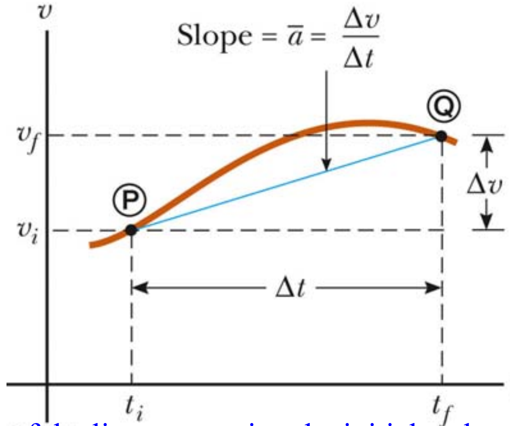
2.3 Acceleration – changing velocity

Changing velocity means an acceleration is present

Acceleration is the rate of change of the velocity

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Units are m/s² (SI), cm/s² (cgs)

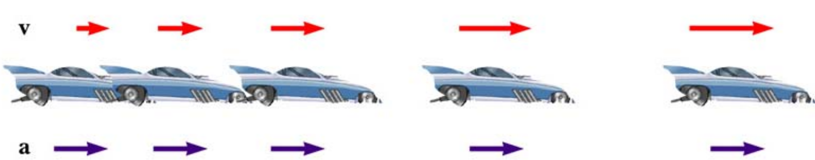


•Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

•Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

•When the instantaneous accelerations are always the same, the acceleration will be uniform or constant.

Relationship between Acceleration and Velocity



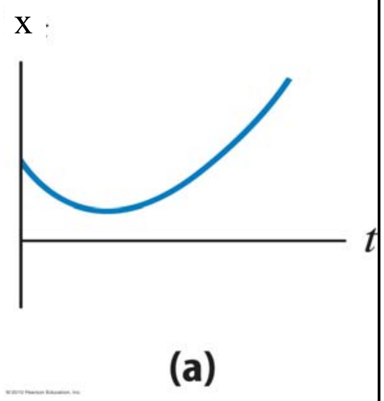
•Positive velocity and positive acceleration

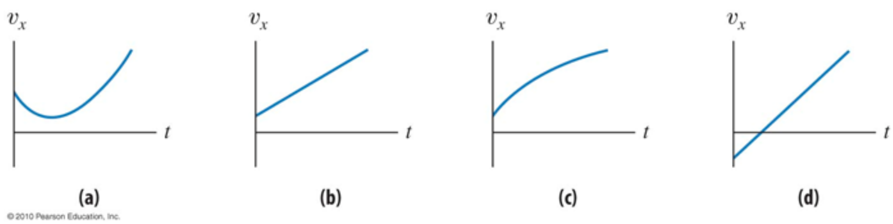
•When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing

•Velocity is positive and acceleration is negative

•When the sign of the velocity and the acceleration are in the opposite directions, the speed is decreasing

Which velocity-versus-time graph goes with the position-versus-time graph shown at right?





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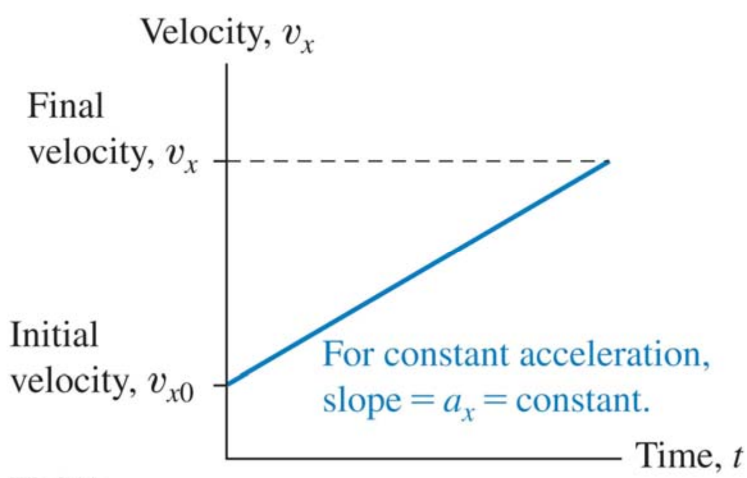
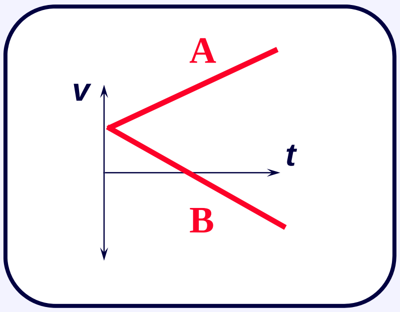
2.4 One-Dimensional Motion with Constant Acceleration



Question 2.8 v versus t graphs II

Consider the line labeled B in the *v* vs. *t* plot. How does the speed change with time for line B?

a) decreases
b) increases
c) stays constant
d) increases, then decreases
e) decreases, then increases



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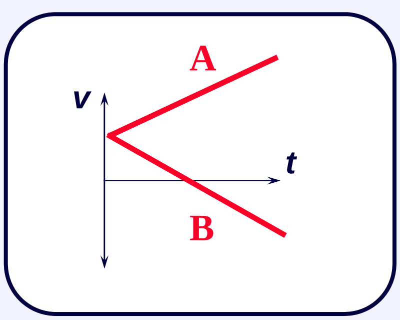

In case B, the initial velocity is positive but the magnitude of the velocity decreases toward zero. After this, the magnitude increases again, but becomes negative, indicating that the object has changed direction.

Question 2.10 Acceleration II

When throwing a ball straight up, which of the following is true about its velocity *v* and its acceleration *a* at the highest point in its path?

a) both $v = 0$ and $a = 0$
b) $v \neq 0$, but $a = 0$
c) $v = 0$, but $a \neq 0$
d) both $v \neq 0$ and $a \neq 0$
e) not really sure

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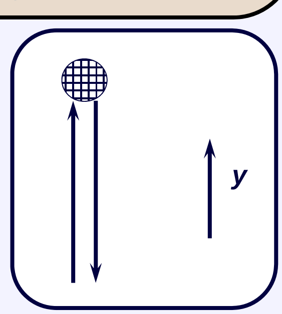
e) not really sure

At the top, clearly $v = 0$ because the ball has momentarily stopped. But the velocity of the ball is changing, so its acceleration is definitely not zero! Otherwise it would remain at rest!!



Follow-up: …and the value of *a* is…?

Kinematic equations

$$\Delta x = v_{average}\, t = \left(\frac{v_o + v_f}{2}\right) t$$

$x_f = x_0 + v_{average}\, t$

- Gives displacement as a function of velocity and time
- Use when you don't know and aren't asked for the acceleration

$$v = v_o + at$$

- Shows velocity as a function of acceleration and time
- Use when you don't know and aren't asked to find the displacement

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

$x_f = x_0 + v_0 t + ½\, a\, t^2$

- Gives displacement as a function of time, velocity and acceleration
- Use when you don't know and aren't asked to find the final velocity

$$v^2 = v_o^2 + 2a\Delta x$$

- Gives velocity as a function of acceleration and displacement
- Use when you don't know and aren't asked for the time

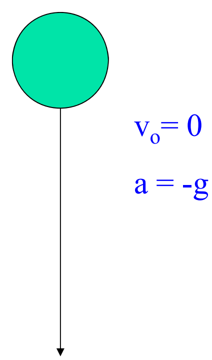
- Solve example 2.9 & 2.10 from the book.

2.5 Free Fall
Gravity: a case of constant acceleration

- All objects moving under the influence of gravity only are said to be in free fall
  - Free fall does not depend on the object's original motion
- All objects falling near the earth's surface fall with a constant acceleration
- The acceleration is called the acceleration due to gravity, and indicated by *g*
- *g* is always directed downward, toward the center of the earth ($g = 9.80\ \mathrm{m/s^2}$)
- An object falling in air is subject to air resistance (and therefore is not freely falling).

Free Fall – an object dropped

- Initial velocity is zero
- Up is positive
- In the kinematic equations use y instead of x since vertical
- Acceleration is $g = 9.80\ \mathrm{m/s^2}$



Kinematic equations for free fall:

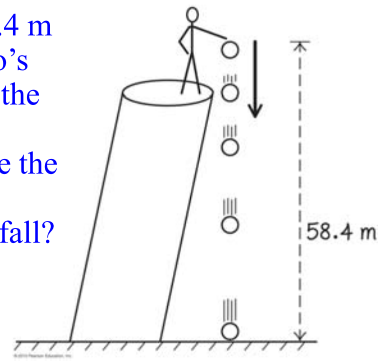
$v_y = v_{y0} - gt$	(Predicts velocity; SI unit: m/s)	(2.11)
$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$	(Predicts position; SI unit: m)	(2.12)
$v_y^2 = v_{y0}^2 - 2g\Delta y$	(Relates final and initial velocity, acceleration, and displacement; SI unit: $(\mathrm{m/s})^2$ or $\mathrm{m^2/s^2}$)	(2.13)

Example 2.11 Tower of Pisa

The top floor of the Tower of Pisa is 58.4 m above the ground. You duplicate Galileo's experiment by dropping two balls from the Tower.

a) What's their velocity when they strike the ground?

b) How much time does it take them to fall?



Question 2.11 Free Fall I

You throw a ball straight up into the air. After it leaves your hand, at what point in its flight does it have the maximum value of acceleration?

a) its acceleration is constant everywhere

b) at the top of its trajectory

c) halfway to the top of its trajectory

d) just after it leaves your hand

e) just before it returns to your hand on the way down

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a) its acceleration is constant everywhere
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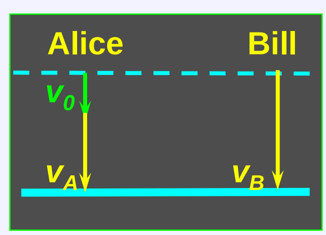
d) just after it leaves your hand
e) just before it returns to your hand on the way down

The ball is in free fall once it is released. Therefore, it is entirely under the influence of gravity, and the only acceleration it experiences is *g*, which is constant at all points.

Question 2.12 Free Fall II

Alice and Bill are at the top of a building. Alice throws her ball downward. Bill simply drops his ball. Which ball has the greater acceleration just after release?

a) Alice's ball
b) it depends on how hard the ball was thrown
c) neither—they both have the same acceleration
d) Bill's ball



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a) Alice's ball
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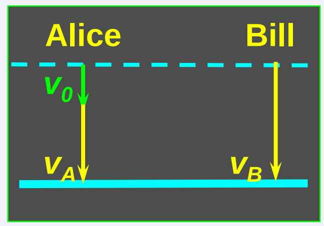
Both balls are in free fall once they are released, therefore they both feel the acceleration due to gravity (*g*). This acceleration is independent of the initial velocity of the ball.

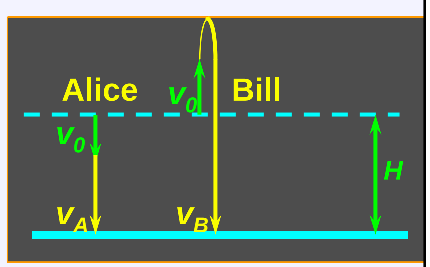


Follow-up: which one has the greater velocity when they hit the ground?

Question 2.14 Up in the Air II

Alice and Bill are at the top of a cliff of height H. Both throw a ball with initial speed v_0, Alice straight down and Bill straight up. The speeds of the balls when they hit the ground are v_A and v_B. If there is no air resistance, which is true?

a) $v_A < v_B$
b) $v_A = v_B$
c) $v_A > v_B$
d) impossible to tell



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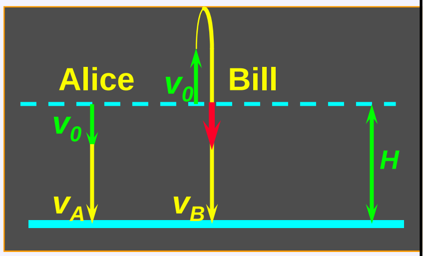
a) $v_A < v_B$

b) $v_A = v_B$

c) $v_A > v_B$

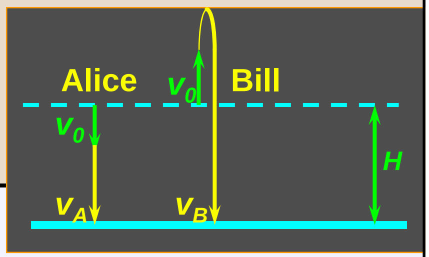
d) impossible to tell

Bill's ball goes up and comes back down to Bill's level. At that point, it is moving downward with v_0, the same as Alice's ball. Thus, it will hit the ground with the same speed as Alice's ball.



Assessment Question

Alice and Bill are at the top of a cliff of height H. Both throw a ball with initial speed v_0, Alice straight down and Bill straight up. There is no air resistance, which is true?



1) Compare the speeds v_A and v_B of the balls when they hit the ground.

2) Which ball arrives first? Alice's or Bill's ball?

3) Compare the accelerations of both balls.

The result: Out of 20 attendees, 14 had all three questions right.
6 had a combination of wrong 1 or 2 answers. None had them all 3 wrong.

Summary of Chapter 2

- Distance: total length of travel
- Displacement: change in position
- Average speed: distance/time
- Average velocity: displacement/time
- Instantaneous velocity: average velocity measured over an infinitesimally small time
- Average acceleration: change in velocity divided by change in time
- Constant acceleration: equation of motion relate position, velocity, acceleration and time
- Freely falling objects: constant acceleration, $g = 9.80 m/s^2$