Lebanese American University
Byblos Campus

School of Engineering and Architecture
Department of Civil Engineering

FLUID MECHANICS

CIE 320

Final Exam

January 31, 2008

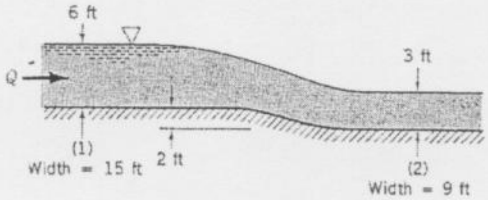
Name: Solution

ID #: ____________________

- *Show all calculations, and indicate the proper units,*
- *Answer briefly and include all necessary explanations,*
- *Answer questions that add up to 100% (i.e., DROP ANY TWO QUESTIONS of 13% each),*
- *Closed book and notes,*
- *Assume any missing information that is necessary,*
- *Questions have weights as indicated.*
- *Exam consists of 21 pages. Do not unstaple the exam.*

Question 1 (10 %)

Water flows down the ramp shown in the channel. The channel width decreases from 15 ft at section (1) to 9 ft at section (2). For the conditions shown, determine the flow rate.



Neglecting the losses due to contraction

Bernoulli eq. ① to ②

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$2 + 6 + \frac{V_1^2}{2g} = 0 + 3 + \frac{V_2^2}{2g} \Rightarrow 5 + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

Continuity eq

$$6 \times 15 V_1 = 3 \times 9 V_2$$

$$\boxed{V_1 = 0.3 V_2}$$

$$5 = \frac{V_2^2}{2g} - (0.3)^2 \frac{V_2^2}{2g} = 0.91 \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{2 \times 32.2 \times 5}{0.91}} = 18.81 \text{ ft/sec}$$

$$Q = A_2 V_2 = 3 \times 9 \times 18.81 = 507.89 \text{ ft}^3/\text{sec}$$

$$\boxed{Q = 507.89 \text{ cfs}}$$

Question 2 (12 %)

Water flows over a spillway at 5000cfs. For dynamic similarity, what should be the model scale if the flow rate over the model is to be 45 cfs? The force exerted on a certain area of the model is measured to be 1.0 lb, what would be the force on the corresponding area of the prototype?

$Q_p = 5000$ cfs $\quad Q_m = 45$ cfs

gravity forces dominate $\Rightarrow$ Froude Number Similarity

$$Fr_p = Fr_m \Rightarrow \frac{V_p}{\sqrt{g_p \cdot L_p}} = \frac{V_m}{\sqrt{g_m L_m}}$$

$$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_r}}\sqrt{\frac{g_p}{g_m}} \Rightarrow V_r = L_r^{1/2} g_r^{1/2}$$

assuming same $g \Rightarrow g_r = 1$

$$\boxed{V_r = L_r^{1/2}}$$

$$Q_r = L_r^2 V_r = L_r^{5/2} \Rightarrow L_r = Q_r^{2/5}$$

$$L_r = \left(\frac{5000}{45}\right)^{2/5} = 6.58$$

$$\boxed{L_r = 6.58 : 1}$$

assume same fluid and same fluid properties $\Rightarrow \rho_r = 1$

$$F_r = \rho_r L_r^2 V_r^2$$

$$F_r = 1 \times L_r^2 \left(L_r^{1/2}\right)^2 = L_r^3$$

$$\frac{F_p}{1} = (6.58)^3 \Rightarrow \boxed{F_p = 284.89 \text{ Lb}}$$

$k = 0.000423$ ft

Question 3 (13 %)

What size of asphalted cast iron pipe (k = 0.005075 in) is needed to carry a water discharge of 12 cfs and with a head loss of 4 ft per 1000 ft of pipe?

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{\left(\frac{\pi}{4}\right)^2\left(D^2\right)^2 2g} = \frac{fLQ^2}{D^5\left(\frac{\pi}{4}\right)^2 2g}$$

$$\frac{h_L}{L} = \frac{fQ^2}{2g\left(\frac{\pi}{4}\right)^2 D^5} \Rightarrow \frac{4}{1000} = \frac{12^2 f}{2\times 32.2\left(\frac{\pi}{4}\right)^2 D^5}$$

$$1.103\times10^{-3} = \frac{f}{D^5}$$

Assume $D = 1$ ft $\Rightarrow \frac{\varepsilon}{D} = 0.000423 \Rightarrow V = 15.28$ ft/sec

$$\Rightarrow R = \frac{1.94\times 15.28\times 1}{2.05\times10^{-5}} = 1.445\times10^{6}$$

Moody Diagram $\Rightarrow f = 0.0165 \Rightarrow$

$$D = \sqrt[5]{\frac{f}{1.103\times10^{-3}}} = 1.72 \text{ ft}$$

Assume $D = 1.72$ ft $\Rightarrow \frac{\varepsilon}{D} = 0.000246$ and $V = 5.16$ ft/sec

$$R = \frac{1.94\times 5.16\times 1.72}{2.05\times10^{-5}} = 0.84\times10^{6}$$

Moody Diagram $\Rightarrow f = 0.0155 \Rightarrow D = 1.696$ ft

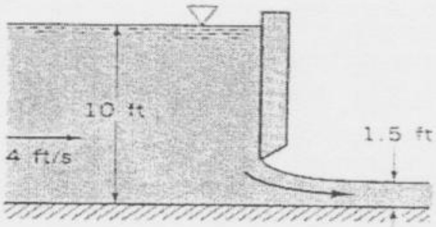
Assume $D = 1.7$ ft $\Rightarrow \frac{\varepsilon}{D} = 0.00025$ and $V = 5.28$

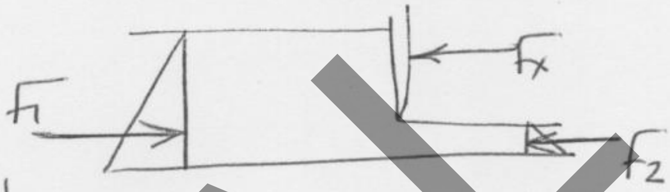
$R = 0.85\times10^{6} \Rightarrow f = 0.0155$

$$\boxed{D = 1.7 \text{ ft}}$$

Question 4 (13 %)

Determine the magnitude of the horizontal component of the anchoring force required to hold in place the sluice gate shown. Compare this result with the size of the horizontal component of the anchoring force required to hold in place the sluice gate when it is closed and the depth of water upstream is 10 ft.





$F_1 = \gamma_1 h_{c1} A_1$

$F_1 = 62.4 \times 5 \times (10 \times 1) = 3120 \text{ Lb}$

$F_2 = \gamma_2 h_{c2} A_2 = 62.4 \times \left(\frac{1.5}{2}\right) 1.5 = 70.2 \text{ Lb}$

Continuity eq

$10 \times 1 \times 4 = 1.5 \times 1 \; V_2 = 26.67 \text{ ft/sec}$

$Q = 40 \text{ cfs}$

Momentum eq

$\Sigma F_x = \rho Q (V_{2x} - V_{1x})$

$F_1 - F_2 - F_x = \rho Q (V_{2x} - V_{1x})$

$3120 - 70.2 - F_x = 1.94 \times 40 \times (26.67 - 4)$

$3120 - 70.2 - F_x = 1758.93$

$F_x = 1290.87 \text{ Lb} \leftarrow$

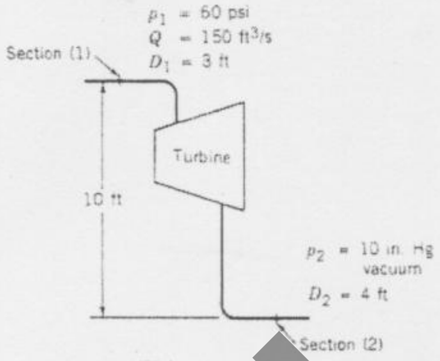
Closed gate

$F = \gamma h_c A = 62.4 \times \left(\frac{10}{2}\right) \times (10 \times 1) = 3120 \text{ Lb}$

$F = 3120 \text{ Lb}$

Question 5 (13 %)

Water is supplied at 150 ft^3/s and 60 psi to a hydraulic turbine through a 3-ft-inside diameter inlet pipe as indicated. The turbine discharge pipe has a 4-ft-inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10 in. Hg vacuum. If the turbine develops 2500 hp, determine the rate of loss of available energy between sections (1) and (2).



$Q = 150\ ft^3/sec$

$V_1 = \frac{150}{\frac{\pi}{4}(3)^2} = 21.22\ ft/sec$

$V_2 = \frac{150}{\frac{\pi}{4}(4)^2} = 11.94\ ft/sec$

$P_1 = 60\ psi = 60\ \frac{Lb}{in^2} \times \left(\frac{12\ in}{1\ ft}\right)^2 = 8640\ psf$

$\frac{P_2}{\gamma_{Hg}} = \frac{-10\ in}{12} \Rightarrow P_2 = -\frac{10}{12} \times 13.59 \times 62.4 = -706.68\ psf$

$Power = \frac{\gamma Q H_t}{550} \Rightarrow H_t = \frac{P \times 550}{\gamma Q} = \frac{2500 \times 550}{62.4 \times 150}$

$\boxed{H_t = 146.9\ ft}$

Energy eq. ① to ②

$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{g} + H_M = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{g} + h_L$

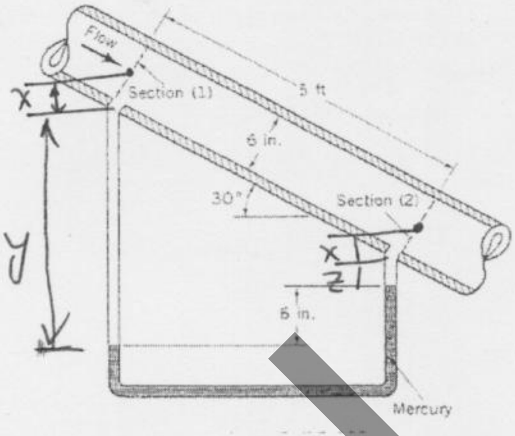
$\frac{8640}{62.4} + 10 + \frac{(21.22)^2}{2 \times 32.2} - 146.9 = \frac{-706.68}{62.4} + 0 + \frac{11.94^2}{2 \times 32.2} + h_L$

$\boxed{h_L = 17.67\ ft}$

Question 6 (13 %)

Water flows steadily down the inclined pipe as indicated. Determine the following:

1. The difference in pressure $P_1 - P_2$
2. The headloss between the two sections.



Manometer eq ① to ②

$$P_1 + \gamma x + \gamma y - \frac{6}{12}\gamma_M - z\gamma - x\gamma = P_2$$

$$P_1 - P_2 = -\gamma y + \gamma z + 0.5\gamma_M = -\gamma(y - z) + 0.5\gamma_M$$

$$y - z = 5\sin 30 + \frac{6}{12} = 2.5 + 0.5 = 3$$

$$P_1 - P_2 = -62.4 \times (3.0) + 0.5 \times (62.4 \times 13.59)$$

$$\boxed{P_1 - P_2 = 236.8 \text{ Psf}}$$

Energy eq ① to ②

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L$$

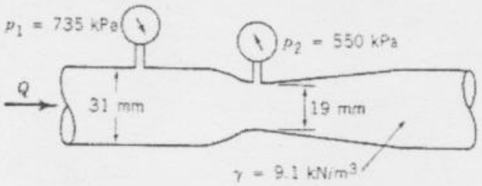
Same diameter $V_1 = V_2$

$$h_L = \frac{236.8}{62.4} + 2.5 = 6.295 \text{ ft}$$

$$\boxed{h_L = 6.295 \text{ ft}}$$

Question 7 (13 %)

1. Determine the flow rate through the Venturi meter shown if ideal conditions exist.
2. For what flow rate through the Venturi will cavitation begin if P_1 = 275 KPa gage, atmospheric pressure is 101 KPa, and the vapor pressure is 5.2 kPa (abs)?



① Ideal flow ⇒ Bernoulli eq.

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{735}{9.1} + \frac{V_1^2}{2g} = \frac{550}{9.1} + \frac{V_2^2}{2g}$$

Continuity $\quad \frac{\pi}{4}\left(\frac{31}{1000}\right)^2 V_1 = \frac{\pi}{4}\left(\frac{19}{1000}\right)^2 V_2$

$$\boxed{V_2 = 2.66\, V_1}$$

$$20.33 = -\frac{V_1^2}{2g} + (2.66)^2 \frac{V_1^2}{2g} = 0.309\, V_1^2$$

$$\boxed{\begin{aligned} V_1 &= 8.10 \text{ m/sec} \\ V_2 &= 21.55 \text{ m/sec} \end{aligned}}$$

$$Q = \frac{\pi}{4}(0.031)^2 \times 8.1$$

$$\boxed{Q = 0.00612 \text{ m}^3/\text{sec}}$$

2) $$\frac{P_{crit}}{\gamma} = -\left(\frac{P_{atm}}{\gamma} - \frac{P_v}{\gamma}\right)$$

$$\frac{P_{crit}}{\gamma} = -\left(\frac{101}{9.1} - \frac{5.2}{9.1}\right) = -10.527 \text{ m}$$

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{275}{9.1} + \frac{V_1^2}{2(9.81)} = -10.527 + (2.66)^2 \frac{V_1^2}{2 \times 9.81}$$

$$40.746 = 0.3096\, V_1^2$$

$$V_1 = 11.47 \text{ m/sec}$$

$$\boxed{Q = 0.00865 \text{ m}^3/\text{sec}}$$

Question 8 (13 %)

What horsepower must be supplied to the water to pump 2.5 cfs at 68°F from the lower to the upper reservoir? Assume that the head loss in the pipes is given by $h_L = 0.015\,(L/D)(V^2/2g)$, where L is the length of the pipe in feet, D is the pipe diameter in ft. ***SKETCH E.L. and H.G.L.***

E. E. ① to ②

$$0 + 90 + 0 + h_p = 0 + 140 + 0 + h_{L_1} + h_{L_2}$$

$$V_1 = \frac{2.5\ ft^3/sec}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2 ft^2} = 7.16\ ft/sec \qquad \frac{V_1^2}{2g} = \frac{V_2^2}{2g} = 0.796\ ft$$

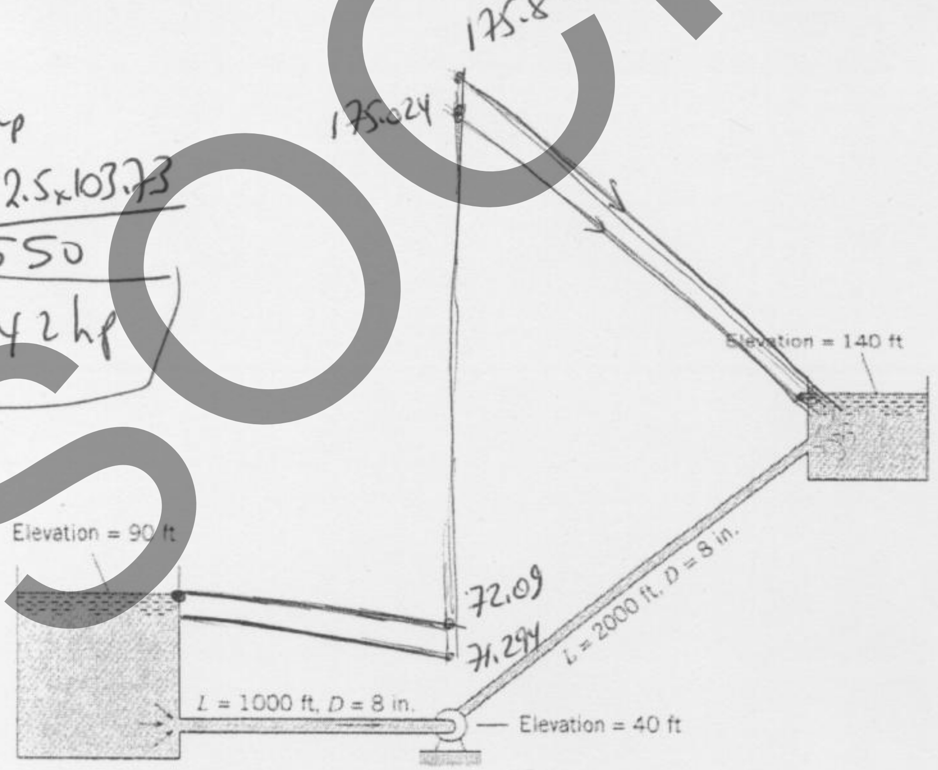
$$V_2 = \frac{2.5}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2} = 7.16\ ft/sec$$

$$h_p = 140 - 90 + 0.015 \times \frac{1000}{8/12}\frac{(7.16)^2}{2 \times 32.2} + \frac{0.015 \times 2000 \times (7.16)^2}{(8/12)\ 2 \times 32.2}$$

$$h_p = 50 + 17.91 + 35.82 \Longrightarrow \boxed{h_p = 103.73\ ft}$$

$$Power = \gamma Q h_p = \frac{62.4 \times 2.5 \times 103.73}{550}$$

$$\boxed{P = 29.42\ hp}$$



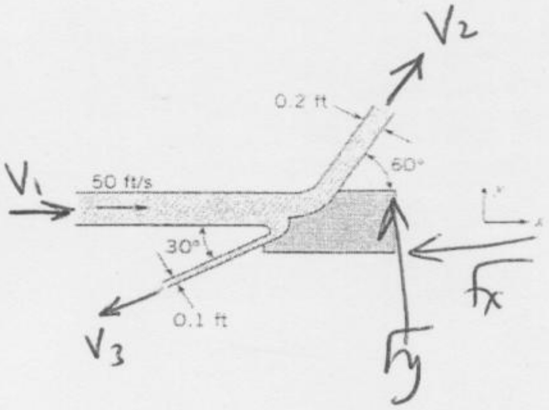
$$h_p = 140 - 90 + 0.015 \times \frac{1000}{8/12} \times \frac{(7.16)^2}{2 \times 32.2} + \frac{0.015 \times 2000 \times (7.16)^2}{(8/12)\ 2 \times 32.2}$$

$$h_p = 50 + 17.91 + 35.82 = 103.73 \text{ ft}$$

$$\boxed{h_p = 103.73 \text{ ft}}$$

Question 9 (13 %)

The horizontal two-dimensional water jet is deflected by the fixed vane. What force per foot of width is required to hold the vane in place?



Bernoulli ① to ②

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{g}$$

$$0 + 0 + \frac{V_1^2}{g} = 0 + 0 + \frac{V_2^2}{g} \Rightarrow \boxed{V_1 = V_2 = 50 \text{ ft/sec}}$$

Bernoulli ① to ③

$$0 + 0 + \frac{V_1^2}{g} = 0 + 0 + \frac{V_3^2}{g} \Rightarrow \boxed{V_1 = V_3 = 50 \text{ ft/sec}}$$

$F_1 = F_2 = F_3 = 0$ free jets

$Q_3 = 0.1 \times 1 \times 50 = 5$

$Q_2 = 0.2 \times 1 \times 50 = 10$

Momentum eq in the x direction

$$Q_1 = Q_2 + Q_3 \Rightarrow Q_1 = 15 \text{ cfs}.$$

$$\sum F = \rho Q V_{out} - \rho Q V_{in}$$

$$-F_x = 1.94 \times (Q_2 V_2 \cos 60 - Q_3 V_3 \cos 30) - 1.94\, Q_1 V_1$$

$$-F_x = 1.94 \times (10 \times 50 \cos 60 - 5 \times 50 \cos 30) - 1.94 \times 15 \times 50$$

$$\boxed{F_x = 1390 \text{ Lb}}$$

Momentum in the y direction

$$F_y = 1.94 \times 10 \times 50 \times \sin 60 - 1.94 \times 5 \times 50 \times \sin 30 - 0$$

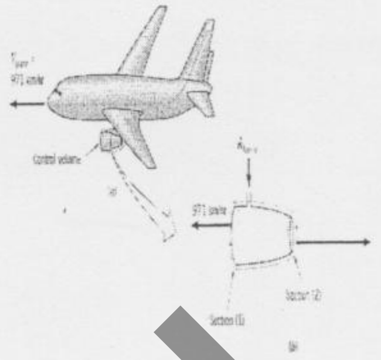
$$\boxed{F_y = 597.54 \text{ Lb} \uparrow}$$

$$F = \sqrt{F_x^2 + F_y^2} = \boxed{1513 \text{ Lb}} \quad \theta = \tan^{-1}\frac{F_y}{F_x} = 23.26°$$

23.26 F

Question 10 (13 %)

An airplane moves forward at a speed of 971 km/hr as shown in the figure. The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2, and the exhaust gas density is 0.515 kg/m^3. Estimate the mass flow rate of fuel into the engine in kg/hr.



$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV + \int_{CS} \rho (W \cdot n) \, dA = 0$$

Steady $\frac{\partial}{\partial t}\int_{CV} \rho \, dV = 0$ flow relative to moving C.V. is considered steady on a time-averaged basis

Since a fixed observer noted that the exhaust gases were moving away from the engine at a speed 1050 $\frac{km}{hr}$ the speed of the exhaust gases relative to the moving C.V. W2 is determined:

$$V_2 = W_2 + V_{plane} \Rightarrow W_2 = V_2 - V_{plane}$$

$$W_2 = 1050 - (-971) = 2021 \ km/hr$$

$$-\dot{m}_{fuel\ in} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

$$\dot{m}_{fuel\ in} = \rho_2 A_2 W_2 - \rho_1 A_1 W_1$$

$$\dot{m}_{fuel\ in} = (0.515 \times 0.558 \times 2021 \times 1000) - 0.736 \times 0.8 \times 971 \times 1000$$

$$= 580800 - 571700 = 9100 \ kg/hr$$

due to rounding 9050 kg/hr