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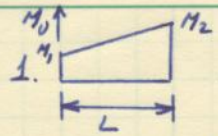
School of Engineering & Architecture

1. 

$M_U = ax + b$ WHERE $a = \frac{M_2 - M_1}{L}$ AND $b = M_1$

$M_L = Ax + B + Cx^2$

A LINEAR M_U IS ENOUGH SINCE THE UNIT LOADS ARE CONCENTRATED LOADS. HENCE, THE MOMENT DIAGRAM IS LINEAR.

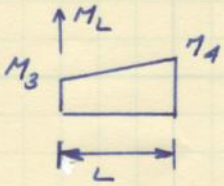
$$\int_0^L M_U M_L \, dx = ?$$

$$M_U M_L = (ax+b)(Ax+B+Cx^2)$$

$$= aAx^2 + aBx + aCx^3 + bAx + bB + bCx^2$$

$$= aCx^3 + (aA + bC)x^2 + (aB + bA)x + bB$$

$$\int_0^L M_U M_L \, dx = aC\frac{L^4}{4} + (aA+bC)\frac{L^3}{3} + (aB+bA)\frac{L^2}{2} + bBL$$

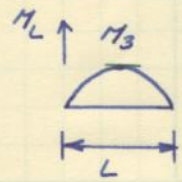


$A = \frac{M_4 - M_3}{L}$, $B = M_3$, $C = 0$

$$\int_0^L M_U M_L \, dx = (M_2 - M_1)(M_4 - M_3)\frac{L}{3} + \left[(M_2 - M_1)M_3 + M_1(M_4 - M_3)\right]\frac{L}{2} + M_1 M_3 L$$

$$= \frac{L}{6}\left(2M_2M_4 - 2M_2M_3 - 2M_1M_4 + 2M_1M_3 + 3M_2M_3 - 3M_1M_3 + 3M_1M_4 - 3M_1M_3 + 6M_1M_3\right)$$

$$= \frac{L}{6}\left(2M_1M_3 + 2M_2M_4 + M_2M_3 + M_1M_4\right) \checkmark$$



$M_L(0) = 0$: $B = 0$

$M_L(L) = 0$: $AL + CL^2 = 0$, $A = -CL$

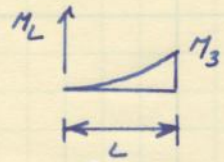
$M_L(\frac{L}{2}) = M_3$: $\underbrace{A\frac{L}{2}}_{-CL} + C\frac{L^2}{4} = M_3$, $C = -\frac{4M_3}{L^2}$

$A = \frac{4M_3}{L}$ $B = 0$ $C = -\frac{4M_3}{L^2}$

$$\int_0^L M_U M_L \, dx = (M_2 - M_1)(-4M_3)\frac{L}{4} + \left[(M_2 - M_1)4M_3 + M_1(-4M_3)\right]\frac{L}{3} + M_1(4M_3)\frac{L}{2}$$

$$= \frac{L}{3}\left(-3M_2M_3 + 3M_1M_3 + 4M_2M_3 - 4M_1M_3 - 4M_1M_3 + 6M_1M_3\right)$$

$$= \frac{L}{3}\left(M_1M_3 + M_2M_3\right) \checkmark$$



$A = 0$, $B = 0$, $C = \frac{M_3}{L^2}$

$$\int_0^L M_U M_L \, dx = (M_2 - M_1)M_3\frac{L}{4} + M_1 M_3 \frac{L}{3}$$

$$= \frac{L}{12}\left(3M_2M_3 - 3M_1M_3 + 4M_1M_3\right)$$

$$= \frac{L}{12}\left(3M_2M_3 + M_1M_3\right) \checkmark$$

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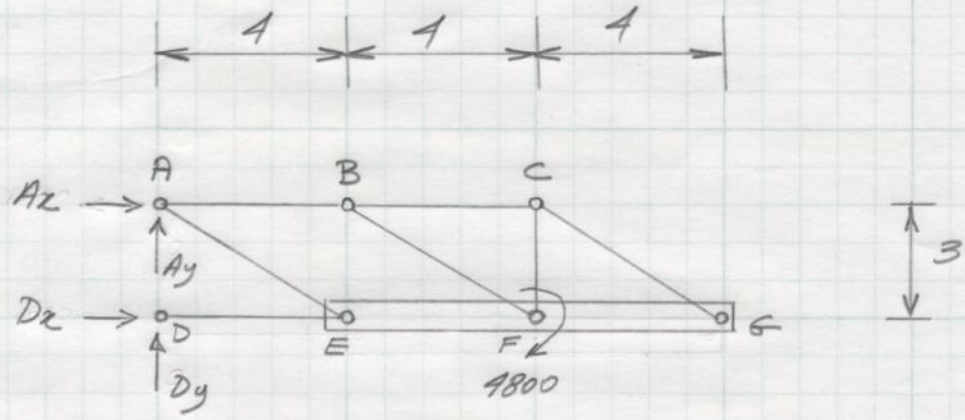
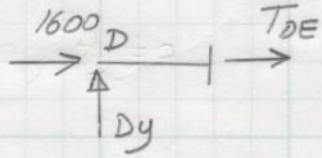


FBD ①: REAL STRUCTURE

$\overset{+ve}{\circlearrowleft} \Sigma M_D = 0 : -A_x(3) - 4800 = 0 \;,\; A_x = -1600$

$\overset{+ve}{\circlearrowleft} \Sigma M_A = 0 : \; D_x(3) - 4800 = 0 \;,\; D_x = 1600$

FBD AT JOINT D

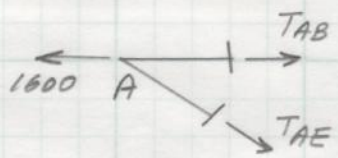


$\overset{+ve}{\rightarrow} \Sigma F_x = 0 : T_{DE} + 1600 = 0 \;,\; T_{DE} = -1600$

$\overset{+ve}{\uparrow} \Sigma F_y = 0 : \; D_y = 0$

FBD ① $\overset{+ve}{\uparrow} \Sigma F_y = 0 : \underset{\nearrow 0}{D_y} + A_y = 0 \;,\; A_y = 0$

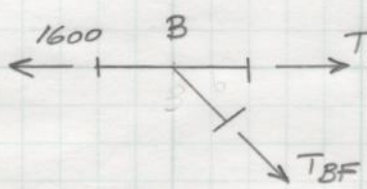
FBD AT JOINT A :



$\overset{+ve}{\uparrow} \Sigma F_y = 0 : -T_{AE}\frac{3}{5} = 0 \;,\; T_{AE} = 0$

$\overset{+ve}{\rightarrow} \Sigma F_x = 0 : T_{AB} - 1600 = 0 \;,\; T_{AB} = 1600$

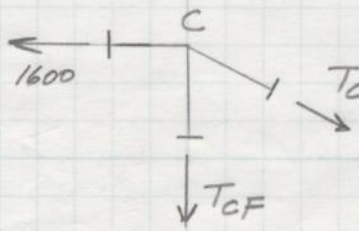
FBD AT JOINT B:



$\overset{+ve}{\uparrow} \Sigma F_y = 0 : -T_{BF}\frac{3}{5} = 0 \;,\; T_{BF} = 0$

$\overset{+ve}{\rightarrow} \Sigma F_x = 0 : \; T_{BC} - 1600 = 0 \;,\; T_{BC} = 1600$

FBD AT JOINT C



$\overset{+ve}{\rightarrow} \Sigma F_x = 0 : \; T_{CG}\frac{4}{5} - 1600 = 0 \;,\; T_{CG} = 2000$

$\overset{+ve}{\uparrow} \Sigma F_y = 0 : -T_{CF} - \underset{\nearrow 2000}{T_{CG}}\frac{3}{5} = 0 \;,\; T_{CF} = -1200$

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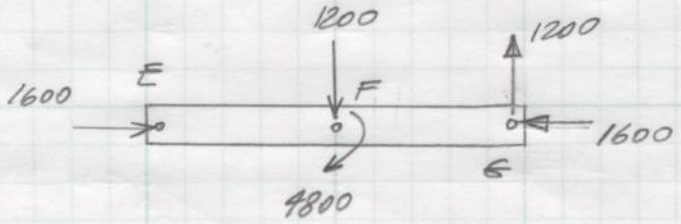
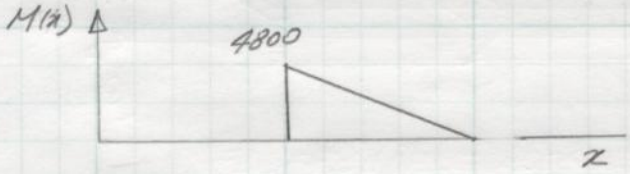
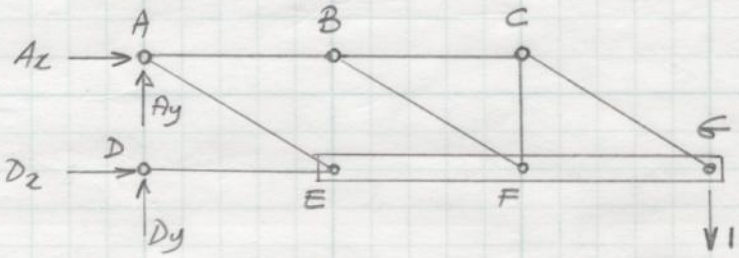


M-DIAGRAM



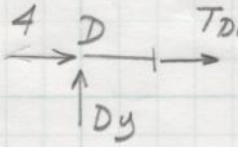


FBD ②: <u>VIRTUAL STRUCTURE</u>

$\overset{+ve}{\circlearrowleft} \Sigma M_D = 0: \quad -A_x(3) - 1(12) = 0 \quad , A_x = -4$

$\overset{+ve}{\rightarrow} \Sigma F_x = 0: \quad \underset{-4}{A_x} + D_x = 0 \quad , \quad D_x = 4$

FBD AT JOINT D

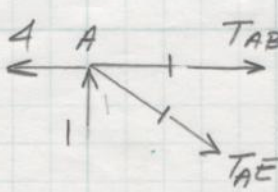


$\overset{+ve}{\rightarrow} \Sigma F_x = 0: \quad T_{DE} + 4 = 0 \quad , T_{DE} = -4$

$\overset{+ve}{\uparrow} \Sigma F_y = 0: \quad D_y = 0$

FBD ① $\quad \overset{+ve}{\uparrow} \Sigma F_y = 0: \quad \underset{0}{D_y} + A_y - 1 = 0 \quad , \quad A_y = 1$

FBD AT JOINT A



$\overset{+ve}{\uparrow} \Sigma F_y = 0: \quad -T_{AE}\frac{3}{5} + 1 = 0, \; T_{AE} = \frac{5}{3}$

$\overset{+ve}{\rightarrow} \Sigma F_x = 0: \quad T_{AB} + \underset{5/3}{T_{AE}}\frac{4}{5} - 4 = 0, \; T_{AB} = \frac{8}{3}$

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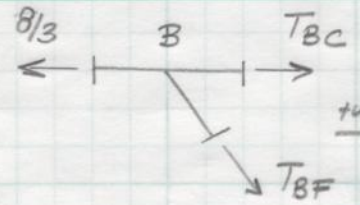
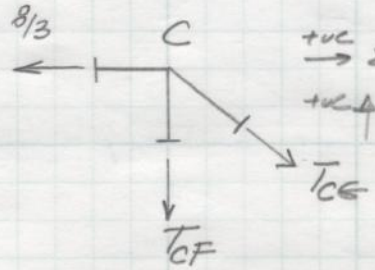
FBD AT JOINT B



$\uparrow^{+ve}$ $\Sigma F_y = 0: \; -T_{BF}\frac{3}{5} = 0, \; T_{BF} = 0$

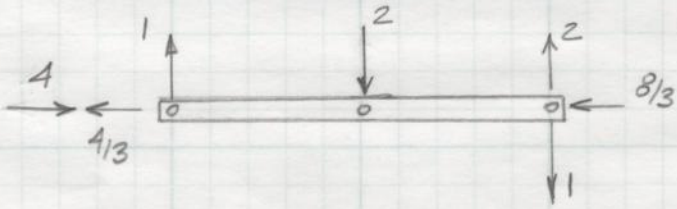
$\xrightarrow{+ve}$ $\Sigma F_x = 0: \; T_{BC} - \frac{8}{3} = 0, \; T_{BC} = \frac{8}{3}$

FBD AT JOINT C



$\xrightarrow{+ve}$ $\Sigma F_x = 0: \; T_{CE}\frac{4}{5} - \frac{8}{3} = 0, \; T_{CE} = \frac{10}{3}$

$\uparrow^{+ve}$ $\Sigma F_y = 0: \; -T_{CF} - \underbrace{T_{CE}}_{\frac{10}{3}}\frac{3}{5} = 0, \; T_{CF} = -2$



MEMBER	L	P_U	P_L	$P_U P_L L$
DE	4	-4	-1600	25600
AE	5	5/3	0	0
AB	4	8/3	1600	17066.7
BF	5	0	0	0
BC	4	8/3	1600	17066.7
CF	3	-2	-1200	7200
CE	5	10/3	2000	33333.3
				100,266.7

$$\Sigma \frac{P_U P_L}{EA} L = \frac{100,266.7}{EA}$$

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MEMBER EFG

$E = 30 \times 10^6$ psi

$$\frac{P_U P_L L}{EA} + \int \frac{M_U M_L}{EI} dx$$

$$= \frac{(8/3)(1600)(8)}{EA} + \frac{4}{6EI}(2 \times 4 \times 4800)$$

$$= \frac{34,133.3}{EA} + \frac{25,600}{EI}$$

TRUSS SYSTEM

BEAM

$$\Delta = \frac{1}{E}\left(\frac{100,266.7}{A} + \frac{34,133.3}{A} + \frac{25,600}{I}\right)$$

$$\Delta = \frac{1}{E}\left(\frac{134,400}{A} + \frac{25,600}{I}\right)$$