

Write your full name, ID number, and circle the section that you are registered in.

Name: _____ ID: _____ Section: **10 AM** **11 AM** **1 PM**

| Problem 1 | Problem 2 | Problem 3 | Problem 4 | Problem 5 | Problem 6 | Score |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| <div></div> | <div></div> | <div></div> | <div></div> | <div></div> | <div></div> | <div></div> |
| / 20 Points | / 20 Points | / 15 Points | / 15 Points | / 15 Points | / 15 Points | / 100 |

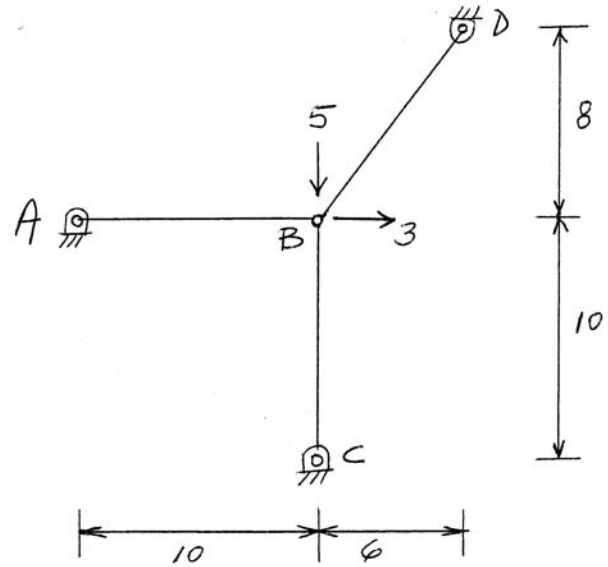
Make sure that you are aware of all the following

- ✚ Show all your calculations, points will be deducted for answers that are not supported by proper calculations.
 - ✚ Use the Virtual Work Method to determine all displacement; using any other method will not count.
 - ✚ Use the specified Virtual structure; using any other virtual structure will not count.
 - ✚ If the Virtual structure is not specified, choose your own Virtual structure.
 - ✚ Ignore shear deformations unless stated otherwise in the problem.
 - ✚ All members have the same E , G , I , J and A , unless stated otherwise in the problem.
 - ✚ Use the specified redundant; using any other redundant will not count.
 - ✚ If a redundant is not specified, choose your own redundant.
 - ✚ Use matrix algebra to solve simultaneous equations. Any other method will not count.
 - ✚ Take advantage of the information provided on the test sheets.
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TRUSS PROBLEM

For the truss shown in the figure, determine the horizontal reaction at D and the horizontal displacement at B.

Use $E = 1$ and $A = 10$



Problem 1 (20 points)

Solve the truss problem given above by using only the Stiffness method. Any other method will not count. Use the structural model given below; points will be deducted for using any other model.

Joints are: Joint 1 is A Joint 2 is B Joint 3 is C Joint 4 is D

Elements are: Element 1 is AB Element 2 is BC Element 3 is BD

$$\begin{array}{c}
 \text{2 } \textcircled{1} \quad \begin{array}{c|c|c} 3 & 4 & 1 & 2 \\ \hline 10 & - & 3 & 2 \\ 00 & - & 1 & 3 \\ \hline - & + & 1 & 2 \end{array} \quad \text{2 } \textcircled{2} \quad \begin{array}{c|c|c} 3 & 4 & 5 & 6 \\ \hline 00 & - & 3 & 2 \\ 01 & - & 1 & 3 \\ \hline - & + & 5 & 6 \end{array} \quad \text{2 } \textcircled{3} \quad \begin{array}{c|c|c} 3 & 4 & 7 & 8 \\ \hline 0.36 & 0.48 & - & 3 \\ 0.48 & 0.64 & - & 4 \\ \hline - & + & 7 & 8 \end{array}
 \end{array}$$

$$\begin{bmatrix} 1.36 & 0.48 \\ 0.48 & 1.64 \end{bmatrix} \begin{bmatrix} d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix} \quad \begin{bmatrix} d_3 \\ d_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1.64 & -0.48 \\ -0.48 & 1.36 \end{bmatrix} \begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 3.66 \\ -4.12 \end{bmatrix}$$

$$\Delta_B = d_3, \Delta_B = 3.66 \rightarrow$$

$$R_7 = K_{73} d_3 + K_{74} d_4 = 0.66$$

$\uparrow -0.36$ $\uparrow -0.48$

$$D_x = R_7, D_x = 0.66 \rightarrow$$

Problem 2 (20 points)

Solve the truss problem given on Page 2 by using the Flexibility method with Q_1 = axial force in BD.

①

②

$$D_{Q1} = \frac{(0.6)(3)}{10}(10) + \frac{(0.8)(-5)}{10}(10) = -2.2$$

$$F_{11} = \frac{(0.6)(0.6)}{10}(10) + \frac{(0.8)(0.8)}{10}(10) + \frac{(1)(1)}{10}(10) = 2$$

$$D_{Q1} = D_{Q1} + F_{11} Q_1$$

$$0 = -2.2 + 2 Q_1, \quad Q_1 = 1.1$$

$$D_y \uparrow \rightarrow \sum F_x = 0: D_x = 0.66$$

③

④

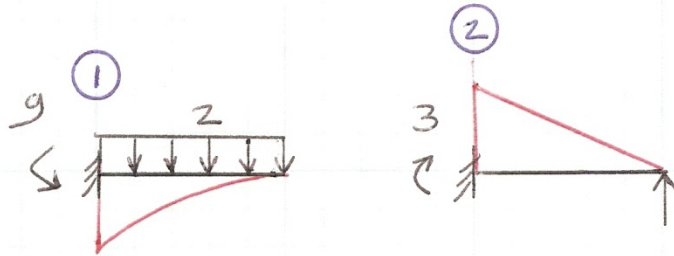
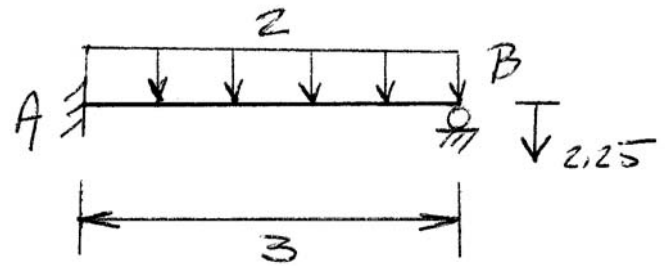
⑤

$$\Delta_B = \frac{(1)(3.66)}{10}(10) = 3.66 \rightarrow$$

Problem 3 (15 points)

For the beam shown in the figure, determine the vertical reaction at A. In addition to the applied distributed load = 2, there is a support movement at B = 2.25 down.

Use $E = 1$ and $I = 1$



$$D_{QL1} = \frac{3}{12} (-3 \times 3 \times 9) = -20.25 \quad F_{11} = \frac{3}{6} (2 \times 3 \times 3) = 9$$

$$D_{q1} = D_{QL1} + F_{11} \phi_1$$

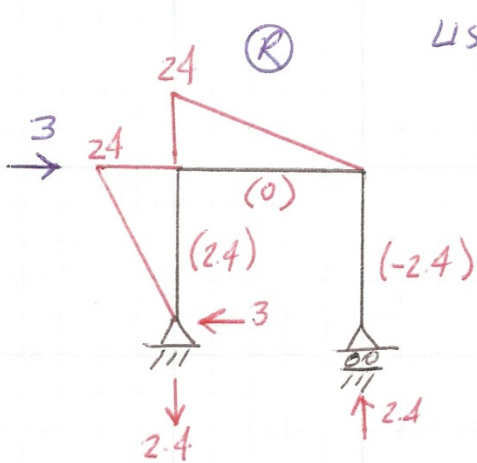
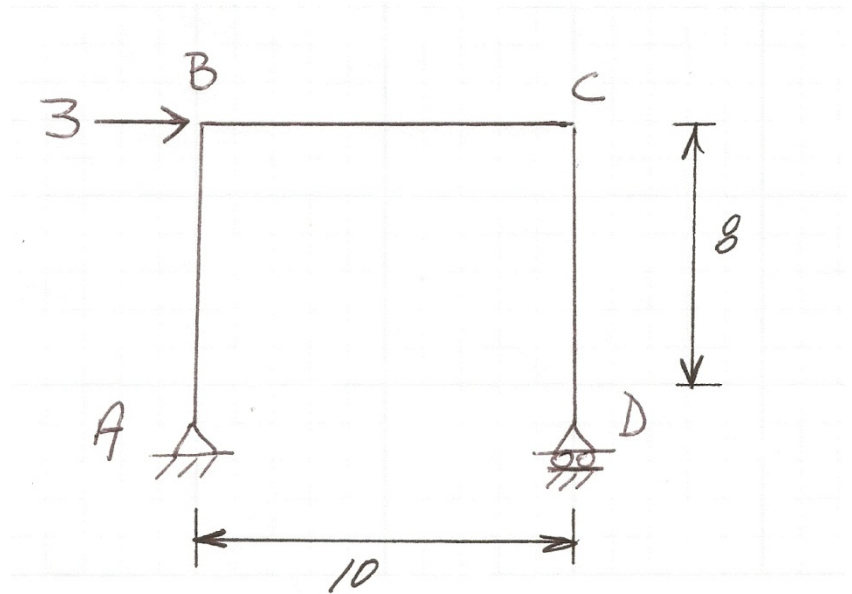
$$MA \quad -2.25 = -20.25 + 9\phi_1, \quad \phi_1 = 2$$

$$\uparrow^+ \sum F_y = 0 : A_y = 4$$

Problem 4 (15 points)

For the frame shown in the figure the supports are a hinge at A and roller at D. Determine the horizontal displacement at B.

Use $E = 1$, $A = 3$, and $I = 288$



USE $\Delta_B = \Delta$

$$(1) \Delta_B = \frac{8}{6EI} (2 \times 8 \times 24) + \frac{10}{6EI} (2 \times 8 \times 24) + \frac{(0.8)(24)(8)}{EA} + \frac{(-0.8)(-24)(8)}{EA}$$

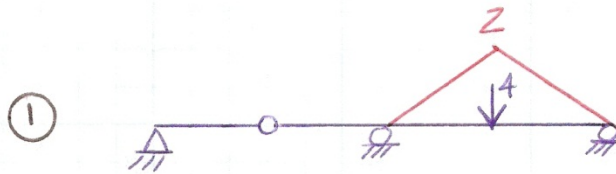
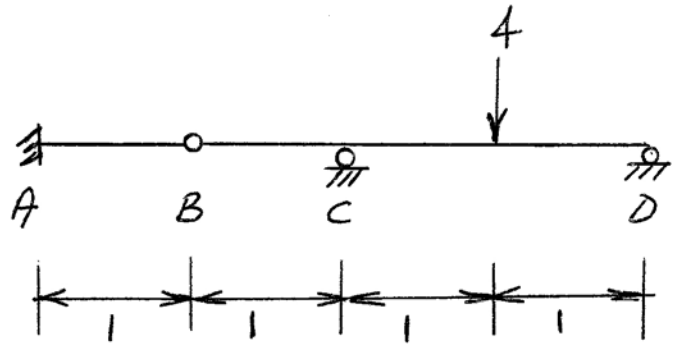
$$E = 1, A = 3, I = 288$$

$$\Delta_B = 14.24 \rightarrow$$

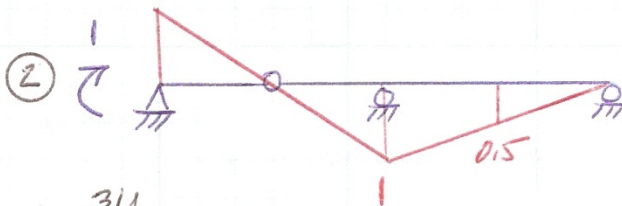
Problem 5 (15 points)

Two members AB and BD are connected by a pin at B. The supports are a fixity at A, rollers at C and D. Determine the vertical reaction at A. Use the Flexibility method with Q_1 = moment at A.

Use $E = 1$, and $I = 1$.



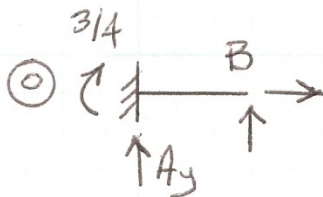
$$D_{Q1} = \frac{1}{6} (-2 \times 0.5 \times 2 - 1 \times 2) + \frac{1}{6} (-2 \times 0.5 \times 2) = -1$$



$$F_{11} = \frac{2}{6} (2 \times 1 \times 1 + 2 \times 1 \times 1 - 1 \times 1 - 1 \times 1) = \frac{4}{3}$$

$$D_{Q1} = P_{Q1} + F_{11} Q_1$$

$$0 = -1 + \frac{4}{3} Q_1 \Rightarrow Q_1 = \frac{3}{4}$$

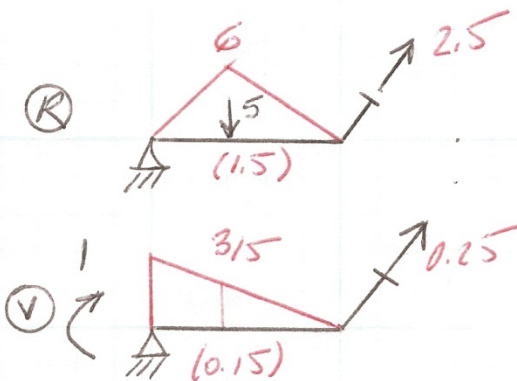
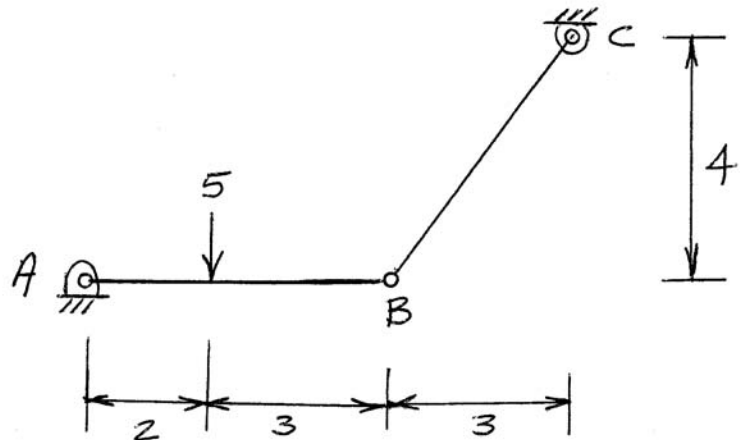


$$\sum M_B = 0 : A_y = -3/4 \quad A_y = 0.75 \downarrow$$

Problem 6 (15 points)

Two members AB and BC are connected by a pin at B. The supports are hinges at A and C. Determine the rotation at A. Use Virtual structure same as the Real structure.

Use $E = 1$, $A = 1$, and $I = 1$



$$\begin{aligned}
 (1) \theta_A &= \frac{2}{6} \left(2 \times \frac{3}{5} \times 6 + 1 \times 6 \right) \\
 &\quad + \frac{3}{6} \left(2 \times \frac{3}{5} \times 6 \right) \\
 &\quad + \frac{(0.15)(1.5)}{1} (5) + \frac{(0.25)(2.5)}{1} (5) \\
 \theta_A &= 12.25 \curvearrowright
 \end{aligned}$$