CIE302 Structural Analysis I (Fall 2009)

FINAL EXAM

Closed Book, 120 Minutes, 1-Feb-2010

First Name: _____

Family Name: _____

ID Number: _____

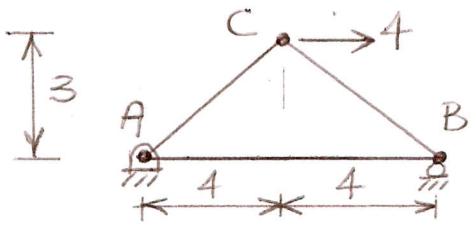
Instructions

- ✓ If the method of solution is not specified, you choose the method .
- ✓ If the virtual structure is not specified, you choose the structure.
- ✓ Ignore shear deformations, unless explicitly stated in the problem.

Problem	1	2	3	4	5	Total
Points						
Max. Points	20	15	15	20	30	100

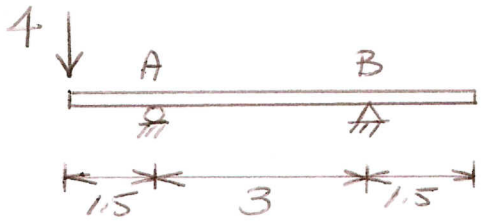
Problem 1 (20 Points)

The truss consists of three members and is supported by a hinge at A and a roller at B. Determine the vertical deflection at C. Use EA = 1.



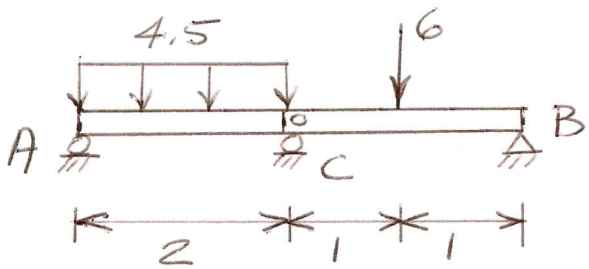
Problem 2 (15 Points)

The beam is supported by a roller at A and a hinge at B. Determine the rotation at B. Use EI = 1.

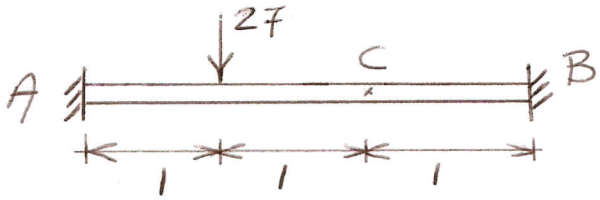


Problem 3 (15 Points)

Two members: AC and BC are connected by a pin at C. Supports are: roller at A, roller at C and hinge at B. Determine the change in rotation at C. Use EI = 1.



For the fixed-fixed beam AB shown in the figure below, determine the vertical reaction at A and the vertical deflection at C. Use EI = 1.



Problem 4 (20 Points)

Use only the Stiffness Method, do not use statics or any other method. The structural model should be as follows:

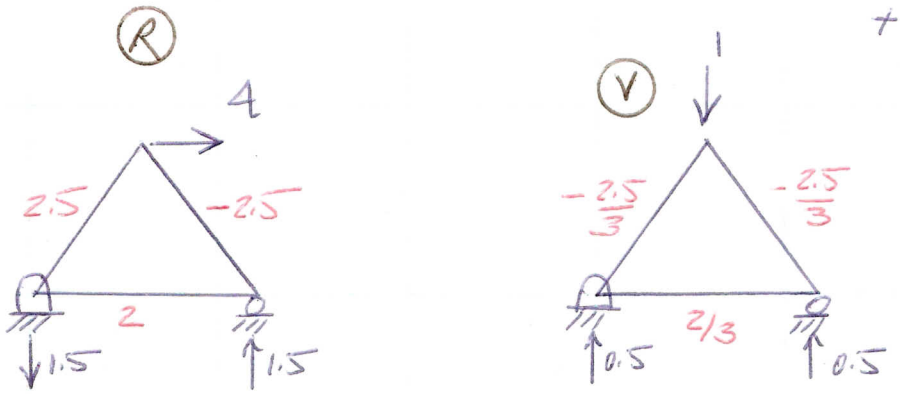
Joints:	Joint 1 is A	Joint 2 is C	Joint 3 is B
Elements:	Element 1 is AC	Element 2 is CB	

Problem 5 (30 Points)

Use Flexibility (if needed), Virtual Work, and Statics. Do not use any other method.

1

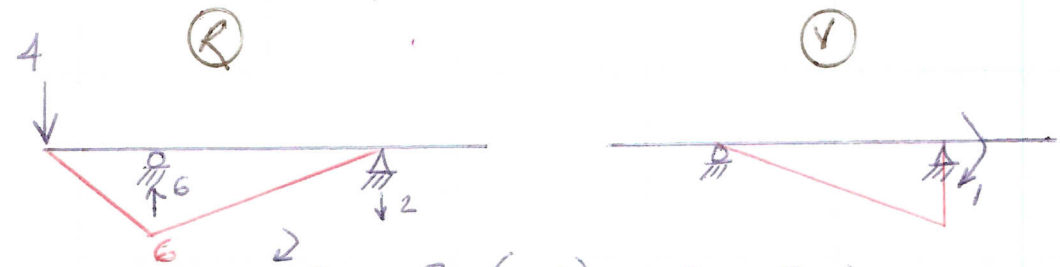
+ TENSION



$$(\downarrow)\ \Delta_C = \left(-\frac{2.5}{3}\right)(2.5)(5) + \left(-\frac{2.5}{3}\right)(-2.5)(5) + \left(\frac{2}{3}\right)(2)(8)$$

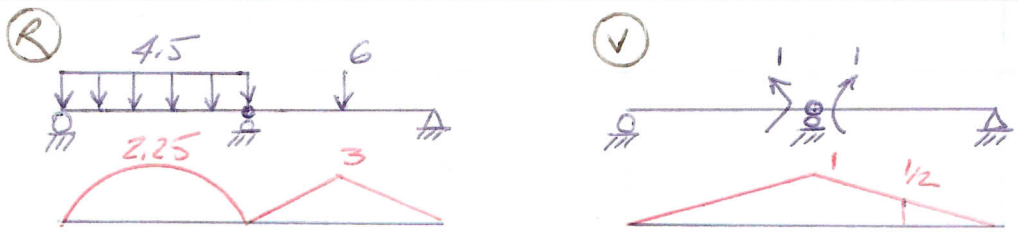
$$\Delta_C = \frac{32}{3}\downarrow$$

2



$$(\curvearrowright)\ \theta_B = \frac{3}{6EI}(1\times 6), \quad \theta_B = \frac{3}{EI}\curvearrowright$$

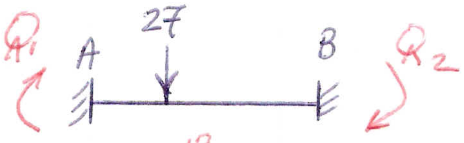
3



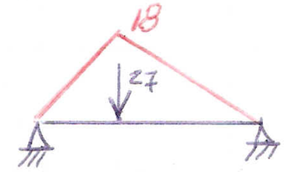
$$\Delta\theta_C = (\curvearrowleft)\theta_{C^-} + (\curvearrowright)\theta_{C^+} = \frac{2}{3}(1\times 2.25) + \frac{1}{6}\left(2\times\frac{1}{2}\times 3 + 1\times 3\right) + \frac{1}{6}\left(2\times\frac{1}{2}\times 3\right)$$

$$\Delta\theta_C = 3\ \curvearrowleft$$

5



①



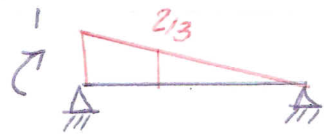
R	V
1	2

$$D_{QL1} = \frac{1}{6}\left(2\times\frac{2}{3}\times 18 + 1\times 18\right) + \frac{2}{6}\left(2\times\frac{2}{3}\times 18\right) = 15$$

R	V
1	3

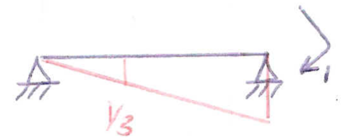
$$D_{QL2} = \frac{1}{6}\left(-2\times\frac{1}{3}\times 18\right) + \frac{2}{6}\left(-2\times\frac{1}{3}\times 18 - 1\times 18\right) = -12$$

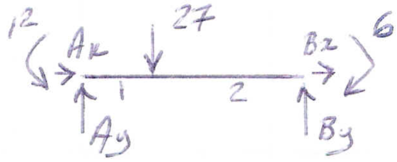
②

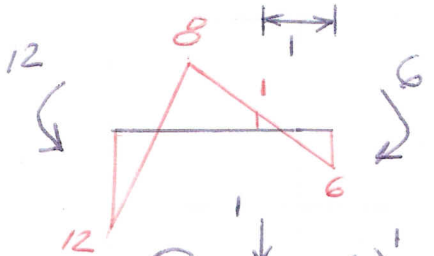


$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = -\frac{2}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 15 \\ -12 \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \end{bmatrix}$$

③

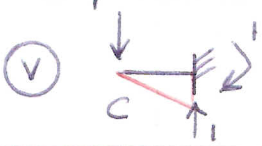






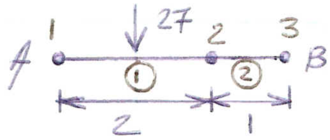
$$\circlearrowleft^{+} \Sigma M_B = 0 : -A_y(3) + 27(2) - 6 + 12 = 0$$

$$A_y = 20 \uparrow$$



$$(1)\Delta_C = \frac{1}{6}\left(2\times 1\times 6 - 1\times 1\right) = \frac{11}{6}, \quad \Delta_C = \frac{11}{6}\downarrow$$

4



$$\underset{\sim}{F} = \underset{\sim}{K}\underset{\sim}{D} + \underset{\sim}{F}^{\circ}$$

① (1, 2): $\underset{\sim}{k} = \frac{1}{8}\begin{bmatrix} & & -12 & 12 \\ & & -12 & 8 \\ & & 12 & -12 \\ & & -12 & 16 \end{bmatrix}$ (columns 1 2 3 4; rows 1 2 3 4)

$$\underset{\sim}{f}^{\circ} = \frac{27}{2}\begin{bmatrix} 1 \\ 1/2 \\ 1 \\ -1/2 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

② (2, 3): $k = \frac{1}{1}\begin{bmatrix} 12 & 6 & & \\ 6 & 4 & & \\ & & & \\ & & & \end{bmatrix}$ (columns 3 4 5 6; rows 3 4 5 6)

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 13.5 & 4.5 \\ 4.5 & 6 \end{bmatrix}\begin{bmatrix} d_3 \\ d_4 \end{bmatrix} + \frac{27}{2}\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$$

$$\begin{bmatrix} d_3 \\ d_4 \end{bmatrix} = \frac{1}{60.75}\begin{bmatrix} 6 & -4.5 \\ -4.5 & 13.5 \end{bmatrix}\frac{27}{2}\begin{bmatrix} -1 \\ 1/2 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -11 \\ 15 \end{bmatrix}$$

d_3 → $\Delta_C = \frac{11}{6}\downarrow$

$$R_1 = K_{13}d_3 + K_{14}d_4 + F_1^{\circ} = 20$$

($K_{13} = -\frac{12}{8}$, $d_3 = -\frac{11}{6}$, $K_{14} = \frac{12}{8}$, $d_4 = \frac{15}{6}$, $F_1^{\circ} = \frac{27}{2}$)

$R_1 = 20 \uparrow$

$\uparrow A_y$