LEBANESE AMERICAN UNIVERSITY

BYBLOS-LEBANON

MTH 201- Business Mathematics
Instructor: T. Assad
Fall Semester – 2010

Test I
Date: November 4, 2010
Time: 2:00 – 3:30

Key

Student's Name: _____

Student's ID: _____

Exercise	Grade	% of Total
I		10
II		20
III		20
IV		20
V		20
VI		10
Total		100

N.B. Make sure to read all questions. This exam booklet is composed of eight pages. The end of the exam is a question found on page seven. Page eight is a scratch paper you can detach it if you want, provided you write your name on it. Good Luck

1. The demand Q1, for a certain good depends on its own price, P1, and the price of an alternative good P2, according to

$$Q_1 = 30 - 3P_1 + P_2$$

(a) Find Q_1 if $P_1 = 4$ and $P_2 = 5$

(b) Is product two substitutable or complementary? Give a reason for your answer.

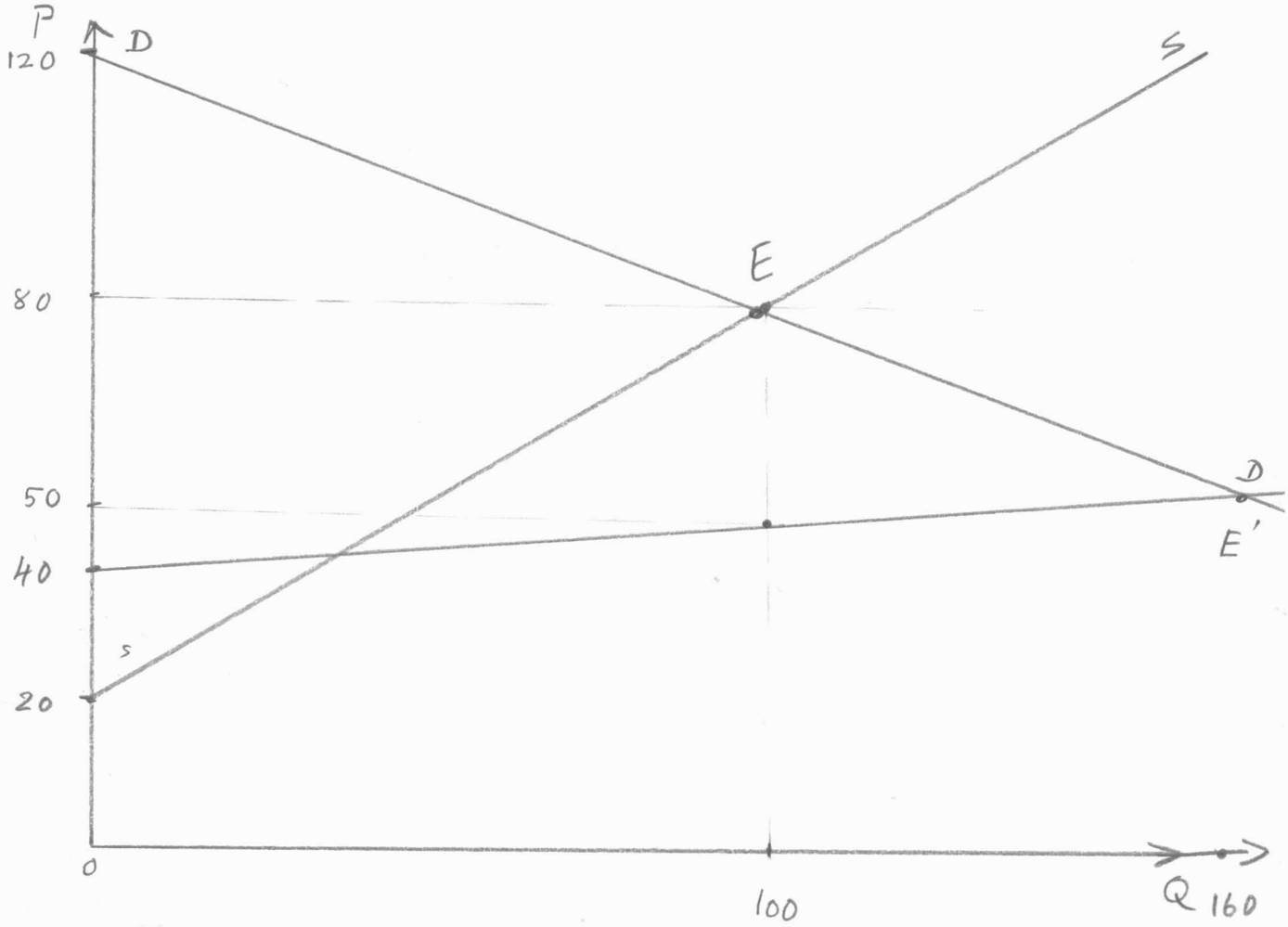
(a) $Q_1 = 30 - (3 \times 4) + 5 = 23$

(b) the second product is substitutable because any increase in its price shall lead to a higher Q_1. Mathematical explanation. the plus infront of P_2 is that indication

2. The market demand curve for a product is given by: P = - 0.4 Q +120

   The supply curve is given by: P = 0.6 Q +20

   a. Sketch the two lines on an "Q" and "P" axis
   b. Find the equilibrium point E
   c. Suppose that a new manufacturing plant is installed, resulting in a new supply curve: P = 0.1 Q + 40; sketch the new supply line on the same axis and determine the new equilibrium point E'
   d. Do you think the market situation is improved after installing the new manufacturing plant. Give a reason for your answer ?



a) | Q | 0 | 100 |
   | P | 120 | 80 |

   | Q | 0 | 100 |
   | P | 20 | 80 |

b) $-0.4Q + 120 = 0.6Q + 20 \Rightarrow Q = 120 - 20 = 100 \Rightarrow P = 0.6(100) + 20 = 80$

   $E(100, 80)$

c. | Q | 0 | 100 |
   | P | 40 | 50 |

   $\Rightarrow 0.1Q + 40 = -0.4Q + 120 \Rightarrow 0.5Q = 80 \Rightarrow Q = 160$

   $P = 0.1(160) + 40 = 56 \Rightarrow E'(160, 56)$

D. Yes More Q_D @ a lower Price.

3. Multiply the following two matrices:

$$A = \begin{pmatrix} 5 & 3 \\ 5 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 0.8 & -0.6 \\ -1 & 1 \end{pmatrix}$$

Based on the result obtained, what can be said about matrix "A" and matrix "B".

$$A * B = \begin{bmatrix} 5 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 0.8 & -0.6 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$(5 \times 0.8) + (3 \times -1)$;
$4 - 3 = 1$
$(5 \times -0.6) + (3 \times 1) = 0$

$(5 \times 0.8) + (4 \times -1) = 0$
$(5 \times -0.6) + (4 \times 1) = 1$

obtaining the Identity Matrix is an indicahcin that Matrix B is the inverse of Matrix A.

4. Use the Cramer's rule to determine the value of X

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 2 & 4 \\ 0.5 & 1 & 5 \end{pmatrix} \quad B = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = C = \begin{pmatrix} 35 \\ 140 \\ 120 \end{pmatrix}$$

$$X = \begin{bmatrix} 35 & 1 & 0 \\ 140 & 2 & 4 \\ 120 & 1 & 5 \end{bmatrix}$$

Row 1; Matrix A; cofactors: $\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix}$; $\begin{vmatrix} 3 & 4 \\ 0.5 & 5 \end{vmatrix}$; $\begin{vmatrix} 3 & 2 \\ 0.5 & 1 \end{vmatrix}$

$(2 \times 5) - (4 \times 1) = 6$; $(3 \times 5) - (4 \times 0.5) = 13$; $(3 \times 1) - (2 \times 0.5) = 2$.

$+6$ $\quad -13$ $\quad +2$.

$\text{Det}(A) = (2 \times 6) + (1 \times -13) + (0 \times 2) = -1$

Row 1; Matrix "X"; cofactors: $\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix}$; $\begin{vmatrix} 140 & 4 \\ 120 & 5 \end{vmatrix}$; $\begin{vmatrix} 140 & 2 \\ 120 & 1 \end{vmatrix}$

$10 - 4 = 6$; $(140 \times 5) - (4 \times 120) = 220$; $140 - 240 = -100$

$+6$ $\quad -220$ $\quad -100$

$\text{Det}(X) = (35 \times 6) + (1 \times -220) + (0 \times -100) =$

$210 - 220 = -10$.

$$X = \frac{\text{Det}(X)}{\text{Det}(A)} = \frac{-10}{-1} = 10.$$

5. Find the inverse of the following Matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 4 \\ 0.5 & 1 & 5 \end{bmatrix}$$

cofactors: $\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix}$; $\begin{vmatrix} 3 & 4 \\ 0.5 & 5 \end{vmatrix}$; $\begin{vmatrix} 3 & 2 \\ 0.5 & 1 \end{vmatrix}$

$10 - 4 = 6$ $\quad 15 - 2 = 13$ $\quad 3 - 1 = 2$

$\begin{vmatrix} 1 & 0 \\ 1 & 5 \end{vmatrix}$; $\begin{vmatrix} 2 & 0 \\ 0.5 & 5 \end{vmatrix}$; $\begin{vmatrix} 2 & 1 \\ 0.5 & 1 \end{vmatrix}$

$5 - 0 = 5$ $\quad 10 - 0 = 10$ $\quad 2 - \frac{1}{2} = 1.5$

$\begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix}$; $\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}$; $\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$

$4 - 0 = 4$ $\quad 8 - 0 = 8$ $\quad 4 - 3 = 1.$

$$\begin{bmatrix} 6 & -13 & 2 \\ -5 & 10 & -1.5 \\ 4 & -8 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & -5 & 4 \\ -13 & 10 & -8 \\ 2 & -1.5 & 1 \end{bmatrix} \Rightarrow$$

$$\frac{1}{-1}\begin{bmatrix} 6 & -5 & 4 \\ -13 & 10 & -8 \\ 2 & -1.5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -6 & 5 & -4 \\ 13 & -10 & 8 \\ -2 & 1.5 & -1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 4 \\ 0.5 & 1 & 5 \end{bmatrix} \begin{bmatrix} -6 & 5 & -4 \\ 13 & -10 & 8 \\ -2 & 1.5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

6. For the following matrices

$$2X + 4Y = 4$$
$$3X + 6Y = 6$$

(a) Organize the above equation in a matrix form A*B = C

(b) Find DET(A)

(c) Based on the results obtained, is there a solution. Give a reason for your answer.

(d) What sort of matrices is matrix A. (circle the correct response)

i. Singular matrix

ii. Raw vector matrix

iii. Column matrix

iv. Identity matrix

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Det(A) = 12 − 12 = 0 . No solution
(b) (c)