LEBANESE AMERICAN UNIVERSITY

BYBLOS-LEBANON

MTH 201- Business Mathematics
Instructor: T. Assad
Fall Semester – 2009

Test I
Date: November 3, 2009
Time: 2:00 – 3:30

Student's Name: _____

Student's ID: _____

Exercise	Grade	% of Total
I		10
II		10
III		20
IV		20
V		20
VI		20
Total		100

N.B. Make sure to read all questions. This exam booklet is composed of ten pages. The end of the exam is a question found on page eight. Page ten is a scratch paper you can detach it if you want, provided you write your name on it. Good Luck

I. Given Matrix "A":

$$\begin{pmatrix} 8 & \frac{2}{3} \\ 3 & \frac{1}{4} \end{pmatrix}$$

1. Find the determinant of matrix "A". (DET A)

2. Does matrix "A" possess an inverse? If yes find it, if no, state why? In this case, matrix "A" is called (give the name)

3. If this matrix is used to find a solution for two unknowns (x, y), will there be a solution?

1. $\left(8 \times \frac{1}{4}\right) - \left(3 \times \frac{2}{3}\right) = \frac{8}{4} - \frac{6}{3} = 2 - 2 = 0$

2. No; because its det is "0". It is a singular matrix

3. No,

II. Given the following two equations including two unknowns:

$$5x + 2y = 29$$
$$3x - y = 2$$

Use any algebraic technique you know to find the value of x and y

$$5x + 2y = 29$$
$$+\ 6x - 2y = +4$$

$$11x = 33 \Rightarrow x = \frac{33}{11} = 3$$

$$(3 \times 3) - y = 2 \Rightarrow 9 - y = 2 \Rightarrow y = 9 - 2 = 7$$

$$\boxed{x = 3 \ , \ y = 7}$$

III. Given matrix "A", and matrix "B"

$$A = \begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix} \quad B = \begin{bmatrix} 29 \\ 2 \end{bmatrix}$$

1. Find matrix C = A*B

2. Find matrix D = B*A

1. $(5 \times 29) + (2 \times 2) = 145 + 4 = 149$

$(3 \times 29) + (-1 \times 2) = 87 - 2 = 85$

$$C = \begin{bmatrix} 149 \\ 85 \end{bmatrix}$$

2. $\begin{bmatrix} 29 \\ 2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}$ $\Rightarrow$ D doesnot exist, because B. A cannot be multiplied

$2 \times 1 \quad 2 \times 2$

\IV. Given Matrix E and matrix F

$$E = \begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix} \qquad F = \begin{pmatrix} 1/11 & 2/11 \\ 3/11 & -5/11 \end{pmatrix}$$

1. Multiply E*F

2. Based on the results obtained, what relationship exists between the two matrices E and F.

1. $E \cdot F = \frac{5}{11} + \frac{6}{11} = \frac{11}{11} = 1$

$\frac{10}{11} - \frac{10}{11} = 0$

$\frac{3}{11} - \frac{3}{11} = 0$

$\frac{6}{11} + \frac{5}{11} = 1.$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. F is the inverse of E. $F = E^{-1}$.

V. Given the following equations:

$$2P_1 + 3P_2 + 2P_3 = 86$$

$$P_1 - P_2 + P_3 = 13$$

$$3P_1 - 2P_2 + P_3 = 21$$

1. Arrange the above equations in the 3 matrices format: "A", "X", and "B"
2. Find the determinant of matrix "A".
3. Use the Cramer's rule to find the value of P3

1. $\begin{bmatrix} 2 & 3 & 2 \\ 1 & -1 & 1 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 86 \\ 13 \\ 21 \end{bmatrix}$

2. Co-factors: e_{11}, e_{12}, e_{13}:

$e_{11} = \begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix}$, $e_{12} \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}$, $e_{13} \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix}$

$e_{11} = -1+2 = 1$ $e_{12} = 1-3 = -2$, $e_{13} = -2+3 = +1$ $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$

$Det(A) = (2\times1) + (3\times2) + (2\times1) = 10$

$P_3 = \begin{bmatrix} 2 & 3 & 86 \\ 1 & -1 & 13 \\ 3 & -2 & 21 \end{bmatrix}$

$e_{11}, e_{12}, e_{13} \Rightarrow e_{11} \begin{vmatrix} -1 & 13 \\ -2 & 21 \end{vmatrix}$, $e_{12} \begin{vmatrix} 1 & 13 \\ 3 & 21 \end{vmatrix}$, $e_{13} \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix}$

$\Rightarrow -21 + 26$ $\quad 21 - 39$ $\quad -2 + 3$

$\Rightarrow 5$ $\quad -18$ $\quad 1$

$+5$ $\quad 18$ $\quad 1$

$Det(P_3) = (5\times2) + (3\times18) + (86\times1) =$

$P_3 = \frac{Det(P_3)}{Det(A)} = \frac{10 + 54 + 86}{10} = \frac{150}{10} = 15$ $= 150$ $\boxed{P_3 = 15}$

VI. Given the following demand and supply function:

$P = -Q_D + 13$

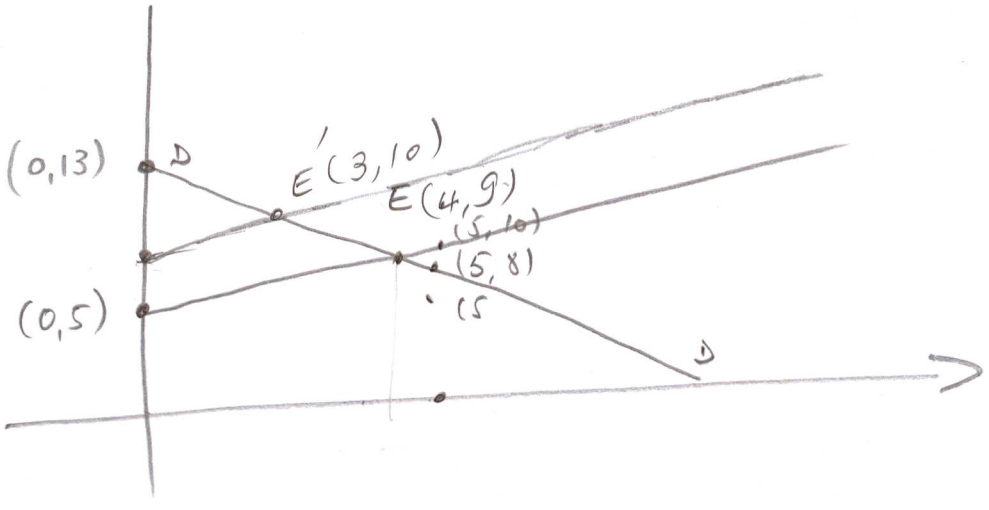
$P = Q_S + 5$

1. Sketch both lines on the same "X" and "Y" axis
2. Find the equilibrium point "E"
3. Find the new equilibrium E' if the government imposes $2 per unit. Who pays the tax and how much?
4. Sketch the new line after the imposition of the $2 tax on the already existing "x" and "y" axis.

1.

Q_D	0	5
P	13	8

Q_S	0	5
P	5	10



$E(4, 9)$

2. $-Q + 13 = Q + 5 \Rightarrow 2Q = 8 \Rightarrow Q = 4. \Rightarrow P = 9$

3. $P - 2 = Q + 5 \Rightarrow P = Q + 7.$

$Q + 7 = -Q + 13 \Rightarrow 2Q = 6 \Rightarrow Q = 3 \Rightarrow P = 10$

$E'(3, 8)$

Difference: $10 - 9 = 1 \Rightarrow$ is paid by the Consumer.
$1 \Rightarrow$ is paid by the supplier.