**Lebanese American University** **Fall 2005**

**Byblos**

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**Discrete Structure II** **Date:** 30/01/2006
**Final Exam** **Duration:** 2h

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1. (a) Use propositional resolution to prove the following:

$$\{A \vee B \vee \neg D,\ \neg A \vee C \vee \neg D,\ \neg B,\ D\} \vdash C$$

(b) Find natural deduction proof of the following:

$$[(A \longrightarrow B) \vee (A \longrightarrow C)] \longrightarrow [A \longrightarrow (B \vee C)]$$

2. We consider a digraph $G = (V, E)$ such that $V = \{1, 2, 3\}$ and the adjacency matrix of $G$ is:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(a) Sketch the digraph $G$.

(b) Define the digraph of 2-stage paths $G_2$, and calculate its adjacency matrix.

(c) Sketch, using (b), the digraph $G_2$.

3. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton such that:

- The states set is $Q = \{q_0, q_1, q_2, q_3\}$
- The input alphabet is $\Sigma = \{0, 1\}$
- The transition function $\delta$ is given by the following table:

| $\delta$ | 0 | 1 |
|---|---|---|
| $q_0$ | $q_0$ | $q_1$ |
| $q_1$ | $q_2$ | $q_1$ |
| $q_2$ | $q_0$ | $q_3$ |
| $q_3$ | $q_3$ | $q_3$ |

- The initial state is $q_0$
- The final states set is $F = \{q_3\}$

(a) Sketch the state transition diagram of the finite automaton $M$.

(b) Determine if the two strings 010100 and 00011 are accepted or rejected by $M$.

$\longrightarrow$

(c) Find, using Kleene's algorithm, a representation of the language $L(M)$ by a regular expression.

(d) Explicitly define what the strings of $L(M)$ would be.

4. We consider the following register machine program $P$:

$$\begin{array}{lll} \hat{1} & (1,2,5) & R=2, \quad M=5 \\ \hat{2} & (2,3) & \\ \hat{3} & (2,4) & \\ \hat{4} & (2,1) & \\ \hat{5} & \text{Halt} & \end{array}$$

(a) Find the code $e$ of the program $P$.

(b) Calculate $\{e\}_2(0,2)$, $\{e\}_2(1,1)$, $\{e\}_2(2,1)$ and $\{e\}_2(3,2)$.

(c) Calculate $\{e\}_2(m,n)$ where $m$ and $n$ are in $\mathbb{N}$.

(d) Deduce the function computed by the program $P$.

5. *Bonus question.* Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton such that $Q = \{q_0, q_1\}$, $\Sigma = \{0, 1\}$ and $F = \{q_1\}$. We consider the language $L$ over the alphabet $\Sigma$ defined by

$$L = \{w \in \Sigma^* \ : \ w \text{ is a string containing an odd number of 1's}\}.$$

(a) We assume that $L(M) = L$. Find the transition function $\delta$ (you can give $\delta$ by a table or by a diagram).

(b) Deduce a representation of $L$ by a regular expression.

**MARKS : 1.** [20] **2.** [20] **3.** [30] **4.** [30] **5.** [10]