Lebanese American University **Fall 2005**

Byblos

Discrete Structure II **Date:** 21/12/2005
Test #2 **Duration:** 1h 30

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1. Use propositional resolution to prove the following:
   (a) $\{A \to B \vee C\} \vdash (A \to B) \vee (A \to C)$
   (b) $\{A \to B, B \to A, A \vee B\} \vdash A \wedge B$

2. Find natural deduction proof of the following formula:

$$\exists x\, (\varphi(x) \wedge \psi(x)) \to (\exists x\, \varphi(x) \wedge \exists x\, \psi(x))$$

   Prove, by a counter-example, that the reverse of this formula is not universally valid.

3. Find a resolution proof of $SSSS0 - SS0 = SS0$, assuming the appropriate recursion equations for the difference predicate diff (x, y, z).

4. (a) Give the definition of a tree and a bipartite graph.
   (b) Prove that a graph can be coloured by two colours iff it is a bipartite graph.
   (c) Prove that a tree is a bipartite graph and outline an algorithm which explains how we can find the representation of a tree as a bipartite graph.
   (d) Give an example of a bipartite graph which is not a tree.
   (e) We say that a graph G is a *forest* if it is without circuits.
      i. Verify that a tree is a forest. Give an example of a forest which is not a tree.
      ii. Prove that a forest is a bipartite graph and outline an algorithm which explains how we can find the representation of a forest as a bipartite graph.
      iii. Give an example of a bipartite graph which is not a forest.

5. *Bonus question.*
   We consider the following predicate logic formula:

$$\forall x\, \exists y\, \varphi(x, y) \longrightarrow \forall y\, \exists x\, \varphi(x, y)$$

   This formula is universally valid? Give a deduction proof or a counter-example.

MARKS : 1. [20] **2.** [15] **3.** [15] **4.** [50] **5.** [10]