

## **PROBLEM #1**

The dry unit weight of a particular clayey soil in its natural condition is  $15 \text{ kN/m}^3$ . A sample of this soil is used to determine the water content. The recorded masses before and after 24 hours in the oven at  $105^\circ\text{C}$  are 145.3 and 123.9 g, respectively.

a) If the specific gravity of the soils  $G$  is 2.8, determine the void ratio and the degree of saturation of the soil.

b) 15000 kN of the soil in its natural moist condition are transported to a site for use as an embankment material with trucks having a capacity of  $30\text{m}^3$ . How many truckloads are necessary to complete this operation?

c) Considering that for use in construction the soil has to be dried to a water content of 12%, how much extra water was carried to the construction site? How many truckloads have essentially been “wasted” by carrying water?

**TO RECEIVE CREDIT, SOLVE USING PHASE DIAGRAMS.**

### **SOLUTION:**

a) If the specific gravity of the soils  $G$  is 2.8, determine the void ratio and the degree of saturation of the soil.

Refer to  $1\text{m}^3$  of soil (i.e.  $V=1\text{m}^3$ )

Given  $\gamma_d=15\text{kN/m}^3$

$\Rightarrow$  for  $1\text{m}^3$ ,  $W_s = 15\text{kN}$

**See phase diagram**

From masses measured, water content ( $w_c$ ) =  $(145.3-123.9)/123.9 = 17.3\%$

$\Rightarrow W_w = w_c \times W_s = 0.173 \times 15 = 2.59\text{kN}$

Calculate  $V_s$  and  $V_w$  using the appropriate unit weights:

$V_w = W_w/\gamma_w = 2.59/9.81 = 0.26\text{m}^3$

$V_s = W_s/\gamma_s = 15/(2.8 \times 9.81) = 0.55\text{m}^3$

**See phase diagram**

Now you can calculate  $V_v = V - V_s = 1 - 0.55 = 0.45\text{m}^3$  and  $V_a = V_v - V_w = 0.45 - 0.26 = 0.19\text{m}^3$

•  $e = V_v/V_s = 0.45/0.55 = 0.82$

•  $S = V_w/V_v = 0.26/0.45 = 0.58$

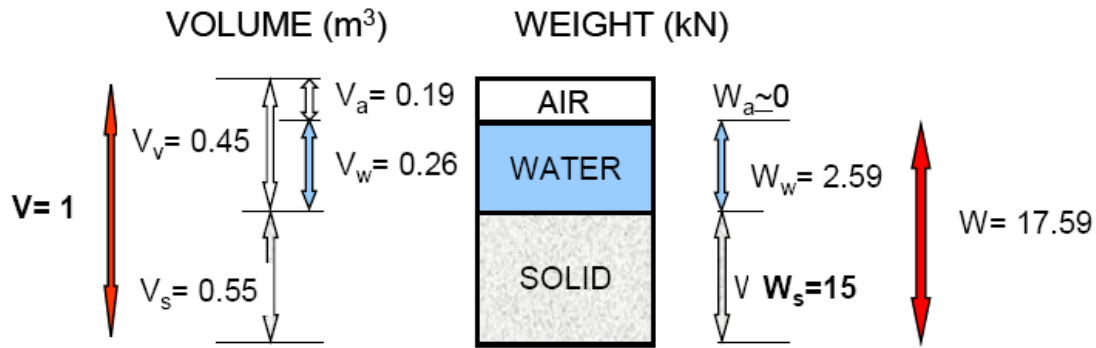
b) 15000 kN of the soil in its natural moist condition are transported to a site for use as an embankment material with trucks having a capacity of  $30\text{m}^3$ . How many truckloads are necessary to complete this operation?

From the phase diagram, the total unit weight =  $(W_s + W_w)/V = (15 + 2.59)/1 = 17.59 \text{ kN/m}^3$

$\Rightarrow 15000 \text{ kN occupy } 15000/17.59 = 852.8 \text{ m}^3$

$\Rightarrow$  Number of trucks necessary are  $852.8/30 = 28.4$

$\Rightarrow$  **29 truckloads are necessary**



*Phase "boxes" not drawn to scale*

c) Considering that for use in construction the soil has to be dried to a water content of 12%, how much extra water was carried to the construction site? How many truckloads have essentially been "wasted" by carrying water?

- Refer to  $1\text{m}^3$  of soil
- If the  $w_c = 12\%$ , you can use this  $w_c$  to calculate the water present in  $1\text{m}^3$ 
  - $W_w = 0.12 \times 15 = 1.8 \text{ kN}$
- This means that you have to eliminate  $2.59 - 1.8 = 0.79 \text{ kN}$  of water for every  $1\text{m}^3$  of transported soil
- Given that  $852.8 \text{ m}^3$  were transported, the amount of extra water is equal to  $852.8 \times 0.79 = 673.7 \text{ kN}$  of extra water
- This weight corresponds to  $673.7 / 9.81 = 68.7 \text{ m}^3$  of water which means almost **2.5 truckloads** were wasted ( $68.7 / 30 = 2.29$  truckloads)

**See phase diagram**

## **PROBLEM #2**

The following method can be used to determine the density of a specimen of irregular shape, especially of friable samples. The specimen at its natural water content is (1) weighed, (2) painted with a thin coat of wax or paraffin (to prevent water from entering the pores), (3) weighed again to measure  $(M + M_{\text{wax}})$ , and (4) weighed in water (to get the volume of the sample + wax coating – remember Archimedes? If not .... see below). Finally, the natural water content of the specimen is determined. A silt specimen of silty sand is treated in this way.

From the information given below, using a phase diagram and making use only of basic definitions and mass and volume balance, determine:

- (a) total density
- (b) dry density
- (c) void ratio
- (d) degree of saturation of the sample.

### **Given:**

Mass of specimen at natural water content	= 181.8 g
Mass of specimen + wax coating	= 215.9 g
Mass of specimen + wax in water	= 58.9 g
Natural water content	= 2.5%
Soil solid density, $\rho_s$	= 2700 kg/m <sup>3</sup>
Wax solid density, $\rho_{\text{wax}}$	= 940 kg/m <sup>3</sup>
Water density, $\rho_w$	= 1000 kg/m <sup>3</sup>

### **SOLUTION:**

#### **Recognize what you have:**

#### **On Mass Side:**

- Total wet mass is known = 181.8g
- ⇒ ***put on phase diagram***
- Water content is known. It tells you how the mass is shared between water and solids.
- $M_w + M_s = 181.8\text{g}$
- $M_w/M_s = 0.025$
- ⇒  $M_s(1+0.025) = 181.8\text{g}$
- ⇒  $M_s = 177.4\text{g}$
- ⇒  $M_w = 181.8 - 177.4 = 4.4\text{g}$
- ⇒ ***put on phase diagram***

#### **On Volume Side:**

- From  $M_w$  and density of water  $\rho_w = 1000\text{kg/m}^3 = 1\text{g/cm}^3$ ,  $V_w = M_w/\rho_w = 4.4\text{ cm}^3$
- ⇒ ***put on phase diagram***

- From  $M_s$  and density of solids  $\rho_s = 2700\text{kg/m}^3 = 2.7\text{g/cm}^3$ ,  $V_s = M_s/\rho_s = 65.7\text{ cm}^3$   
 $\Rightarrow$  **put on phase diagram**
- We are still missing the Total Volume,  $V$ . To determine  $V$ , we need to make use of Archimedes Law:

$$V_{\text{waxedspecimen}} = (M_{\text{specimen+wax}} - M_{\text{specimen+waxinwater}}) / \rho_w$$

$$\Rightarrow V_{\text{waxedspecimen}} = (215.9 - 58.9) / 1 = 157\text{ cm}^3$$

This value includes the volume occupied by the wax, which needs to be subtracted to obtain the total volume of the soil.

$$\Rightarrow M_{\text{wax}} = M_{\text{specimen+wax}} - M_{\text{specimen}} = 215.9 - 181.8 = 34.1\text{ g}$$

$$\Rightarrow V_{\text{wax}} = M_{\text{wax}} / \rho_{\text{wax}} = 34.1 / 0.94 = 36.3\text{ cm}^3$$

$$\Rightarrow \text{Total Volume, } V = V_{\text{waxedspecimen}} - V_{\text{wax}} = 157 - 36.3 = 120.7\text{ cm}^3$$

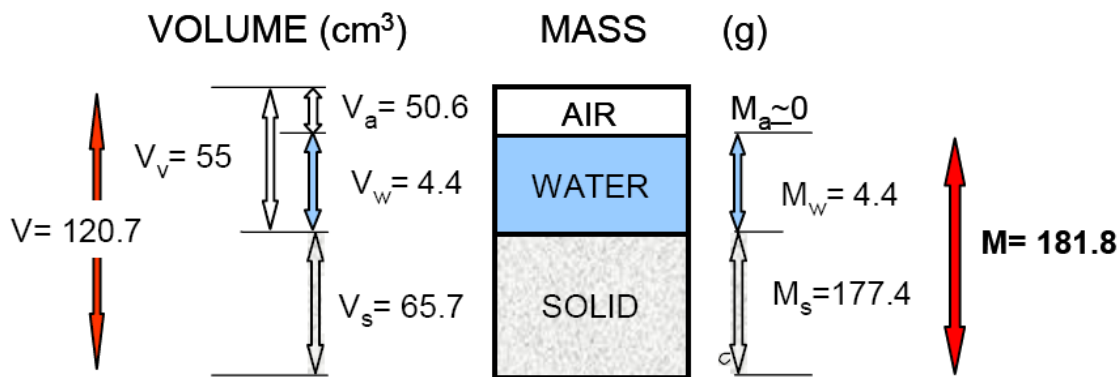
$\Rightarrow$  **put on phase diagram**

From  $V$ ,  $V_s$ , and  $V_w$ , we can now determine the volume occupied by air

$$\Rightarrow V_a = V - V_s - V_w = 120.7 - 65.7 - 4.4 = 50.6\text{ cm}^3$$

$\Rightarrow$  **put on phase diagram**

$$\Rightarrow V_v = V_a + V_w = 50.6 + 4.4 = 55\text{ cm}^3$$



Phase "boxes" not drawn to scale

Now you can calculate the quantities needed:

- Total density  $\rho = M/V = 181.8/120.7 = 1.506\text{ g/cm}^3$
- Dry density  $\rho_d = M_s/V = 177.4/120.7 = 1.469\text{ g/cm}^3$
- Void ratio  $e = V_v/V_s = 55/65.7 = 0.84$
- Degree of saturation  $S = V_w/V_v = 4.4/55 = 0.08 \sim 8\%$

### PROBLEM #3

A sample of dry sand having a unit weight of  $105 \text{ lbs/ft}^3$  and a specific gravity of 2.70 is placed in the rain. During the rain the volume of the sample remains constant, but the degree of saturation increases to 40%. Determine the unit weight and the water content of the soil after being in the rain. Solve using phase diagrams.

### SOLUTION:

In this problem you need to construct two phase diagrams:

#### 1- BEFORE THE RAIN

Assume that you are working with  $V = 1 \text{ ft}^3$ . Because the soil is **DRY**

$$\Rightarrow W = W_s = 105 \text{ lbs}$$

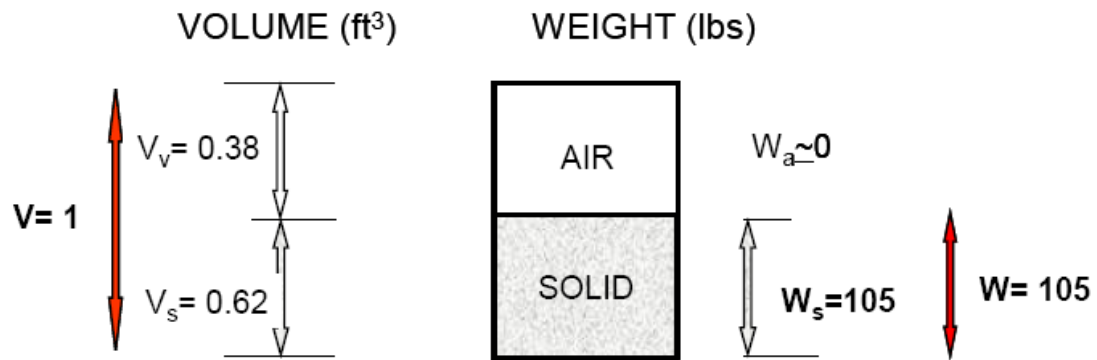
$$\Rightarrow W_w = 0$$

$V_s$  can be calculated from  $W_s$  and  $\gamma_s = G_s \gamma_w = 168.5 \text{ lbs/ft}^3$

$$\Rightarrow V_s = W_s / \gamma_s = 105 / 168.5 = 0.62 \text{ ft}^3$$

From  $V$  and  $V_s$ , we can find  $V_v = 1 - 0.62 = 0.38 \text{ ft}^3$

**NOTE: All voids are occupied by air**



*Phase "boxes" not drawn to scale*

#### 2- AFTER THE RAIN

- $V$  remains constant but degree of saturation  $S$  goes from 0% to 40%, which means that water now fills part of the voids that were previously only occupied by air.
- $V_v$  and  $V_s$  do not change.
- This means that after the rain,  $V_w = 0.40V_v$   
 $\Rightarrow V_w = 0.40 \times 0.38 = 0.15 \text{ ft}^3$   
 $\Rightarrow$  A cubic foot of the soil now holds a weight of water

$$W_w = \gamma_w V_w = 62.4 \times 0.15 = 9.4 \text{ lbs}$$

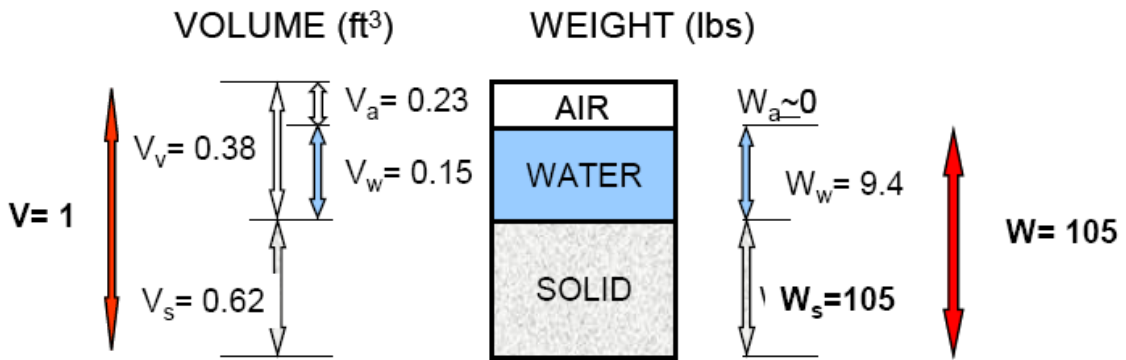
⇒ The total weight of the cubic foot of soil has become

$$W = W_s + W_w = 105 + 9.4 = 114.4 \text{ lbs}$$

You now have all the weight and volume information for the soil after the rain. You can calculate any quantity you are asked for:

$$\gamma = \frac{W}{V} = \frac{114.4}{1} = 114.4 \text{ lbs/ft}^3$$

$$w_c = \frac{W_w}{W_s} = \frac{9.4}{105} = 0.09 \sim 9\%$$



*Phase "boxes" not drawn to scale*

**PROBLEM #4**

For the data given below ( $G_s=2.64$ ):

- a) Plot the compaction curves
- b) Establish maximum dry unit weight and optimum water content for each test
- c) Compute the degree of saturation (S) at the optimum point for the modified data using a phase diagram.

Modified		Standard		Low Energy	
$\gamma$ (kN/m <sup>3</sup> )	w.c. (%)	$\gamma$ (kN/m <sup>3</sup> )	w.c. (%)	$\gamma$ (kN/m <sup>3</sup> )	w.c. (%)
20.08	9.3	18.13	9.3	17.7	10.9
21.13	12.8	18.8	11.8	18.06	12.3
20.43	15.5	19.68	14.3	19.85	16.3
19.79	18.7	20.16	17.6	20.12	20.1
19.5	21.1	19.97	20.8	19.78	22.4
		19.53	23.0		

**SOLUTION:**

- a) Plot the compaction curves

Before plotting the compaction curves, note that you need to calculate the dry unit weights ( $\gamma_d$ ) from the values of  $\gamma$  and  $w_c$ .

$$\text{Use } \gamma_d = \frac{\gamma}{(1 + w_c)}$$

See data in table below and plots on figure in next page.

Modified			Standard			Low Energy		
$\gamma$ (kN/m <sup>3</sup> )	w.c. (%)	$\gamma_d$ (kN/m <sup>3</sup> )	$\gamma$ (kN/m <sup>3</sup> )	w.c. (%)	$\gamma_d$ (kN/m <sup>3</sup> )	$\gamma$ (kN/m <sup>3</sup> )	w.c. (%)	$\gamma_d$ (kN/m <sup>3</sup> )
20.08	9.3	<b>18.37</b>	18.13	9.3	<b>16.59</b>	17.7	10.9	<b>15.96</b>
21.13	12.8	<b>18.73</b>	18.8	11.8	<b>16.82</b>	18.06	12.3	<b>16.08</b>
20.43	15.5	<b>17.69</b>	19.68	14.3	<b>17.22</b>	19.85	16.3	<b>17.07</b>
19.79	18.7	<b>16.67</b>	20.16	17.6	<b>17.14</b>	20.12	20.1	<b>16.75</b>
19.5	21.1	<b>16.10</b>	19.97	20.8	<b>16.53</b>	19.78	22.4	<b>16.16</b>
			19.53	23	<b>15.88</b>			

b) Establish maximum dry unit weight and optimum water content for each test

See derivation of optimum moisture content and maximum dry unit weight on the plot below. Note that the plot also shows curves for S = 70, 90, and 100% calculated using:

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w V_s}{V_s + V_v} = \frac{G_s \gamma_w}{1 + e} = \frac{G_s \gamma_w}{1 + \frac{G_s w c}{S}}$$

c) Compute the degree of saturation (S) at the optimum point for the modified data using a phase diagram.

For the Modified data,  $\gamma_d = 18.8 \text{ kN/m}^3$  for  $w_c = 11.7\%$

Using the phase relations, calculate the degree of saturation:

Set  $V = 1 \text{ m}^3$ , then  $W_s = 18.80 \text{ kN}$

From  $w_c = 11.7\%$ ,  $W_w = 0.117 \times 18.8 = 2.20 \text{ kN}$

From the weights above and the unit weights of water and of the soil ( $\gamma_w = 9.81 \text{ kN/m}^3$  and  $\gamma_s = G_s \times \gamma_w = 2.64 \times 9.81 = 25.90 \text{ kN/m}^3$ ), you can calculate  $V_w$  and  $V_s$ :

$$\Rightarrow V_w = W_w / \gamma_w = 2.20 / 9.81 = 0.22 \text{ m}^3$$

$$\Rightarrow V_s = W_s / \gamma_s = 18.80 / 25.90 = 0.73 \text{ m}^3$$

$$\Rightarrow V_v = 1 - 0.73 = 0.27 \text{ m}^3$$

$$\Rightarrow S = V_w / V_v = 0.22 / 0.27 = 0.81 \sim 81\%$$

