CIE485 Construction Management
HW#4 – Solution

1)

i	j	Act	a	m	b	t_e	TE_i	TL_j	AFS	ATS	σ	σ^2
5	10	A	1	3	5	3	0	3	0	0	0.67	0.44
5	15	B	2	5	6	4.6	0	5.3	0	0.7	0.67	0.44
10	20	C	4	6	7	5.8	3	15.7	0	6.9	0.5	0.25
10	25	E	8	10	12	10	3	13	0	0	0.67	0.44
15	25	d1	0	0	0	0	4.6	13	8.4	8.4	0	0
15	30	F	4	8	9	7.5	4.6	12.8	0	0.7	0.83	0.69
20	35	G	5	7	14	7.8	8.8	23.5	6.9	6.9	1.5	2.25
25	35	L	7	10	16	10.5	13	23.5	0	0	1.5	2.25
30	35	d2	0	0	0	0	12.1	23.5	11.4	11.4	0	0
30	40	M	6	9	12	9	12.1	21.8	0	0.7	1	1
35	45	N	1	3	7	3.3	23.5	26.8	0	0	1	1
40	45	P	3	5	7	5	21.1	26.8	0.7	0.7	0.67	0.44
45	—	—	—	—	—	—	26.8	26.8				

2)

The critical path is : 5 – 10 – 25 – 35 – 45
A , E , L , N

Mean for project duration = Σ means for all critical activities
= 3 + 10 + 10.5 + 3.3
= 26.8 days.

Variance of project duration = Σ Variances of all critical activities
= 0.44 + 0.44 + 2.25 + 1 = 4.13

⇒ Standard deviation of project = $\sqrt{4.13}$ = 2.03

3a)

i	Path i	E[Ti]	Var[Ti]	SD[Ti]	E[Ti] −3SD[Ti]	E[Ti] +3SD[Ti]
1	5-10-25-35-45	26.8	4.13	2.03	20.71	32.89
2	5-15-30-40-45	26.1	2.57	1.6	21.3	30.9
3	5-10-20-35-45	19.9	3.94	1.98	13.96	25.84
4	5-15-25-35-45	18.4	3.69	1.92	12.64	24.16
5	5-15-30-35-45	15.4	2.13	1.46	11.02	19.78
	Max E[Ti]	26.8		Range	21.3	32.89

Now compute correlation coefficients between paths.

ρ_{12} = 0 (no common activities between paths 1 and 2)

$\rho_{13} = \frac{0.44 + 1}{2.03 \times 1.98} = 0.358 < 0.5$

$\rho_{14} = \frac{2.25 + 1}{2.03 \times 1.92} = 0.83 > 0.5$ ⇒ Path 4 can be eliminated because it's represented by path 1.

$\rho_{15} = \frac{1}{2.03 \times 1.46} = 0.337 < 0.5$

$\rho_{23} = 0$, $\rho_{25} = \frac{0.44 + 0.69}{1.6 \times 1.46} = 0.483 < 0.5$

$\rho_{35} = \frac{1}{1.98 \times 1.46} = 0.346 < 0.5$

⇒ So only Path 4 can be eliminated

So $P(\text{Project} \leq 25) = P(T_1 \leq 25) \times P(T_2 \leq 25) \times P(T_3 \leq 25) \times P(T_5 \leq$

$= F_u\left(\frac{25-26.8}{2.03}\right) \times F_u\left(\frac{25-26.1}{1.6}\right) \times F_u\left(\frac{25-19.9}{1.98}\right) \times F_u\left(\frac{25-15}{1.46}\right.$

$= F_u(-0.886) \times F_u(-0.6875) \times F_u(2.576) \times F_u(6.8475$

$= (1 - 0.8133) \times (1 - 0.7549) \times 1 \times 1$

$= 0.1867 \times 0.2451$

$= 0.0457$

3b)

$P(\text{Project} > 27) = 1 - P(\text{Project} \leq 27)$

$= 1 - \left[P(T_1 \leq 27) * P(T_2 \leq 27) \times P(T_3 \leq 27) \times P(T_5 \leq 27)\right]$

$= 1 - \left[F_u\left(\frac{27-26.8}{2.03}\right) \times F_u\left(\frac{27-26.1}{1.6}\right) \times F_u\left(\frac{27-19.9}{1.98}\right) \times F_u\left(\frac{27-15}{1.46}\right)\right]$

$= 1 - (0.5398 \times 0.7123 \times 1 \times 1)$

$= 0.6155$

3c)

$P(TE_{35} \leq 20) = ?$

i	Path i	$E[T_i]$	$Var[T_i]$	$SD[T_i]$	$-3\,SD[T_i]$	$+3\,SD[T_i]$
1	5 – 10 – 25 – 35	23.5	3.13	1.77	18.19	28.81
2	5 – 10 – 20 – 35	16.6	2.94	1.715	11.45	21.745
3	5 – 15 – 25 – 35	15.1	2.69	1.64	10.18	20.02
4	5 – 15 – 30 – 35	12.1	1.13	1.06	8.92	15.28
	Max $E[T_i]$	23.5		Range:	18.19	28.81

$\rho_{12} = 0.14 < 0.5$

$\rho_{13} = \frac{2.25}{1.77 \times 1.64} = 0.775 > 0.5$: So path 3 can be represented by path 1
$\Rightarrow$ Eliminate path 3

$\rho_{14} = 0$, $\rho_{23} = 0$, $\rho_{24} = 0$

$\rho_{34} = \frac{0.44}{1.64 \times 1.06} = 0.253 < 0.5$.

$\Rightarrow$ So only path 3 can be eliminated.

So $P(TE_{35} \leq 20) = P(T_1 \leq 20) \times P(T_2 \leq 20) \times P(T_4 \leq 20)$

$= F_u\left(\frac{20 - 23.5}{1.77}\right) \times F_u\left(\frac{20 - 16.6}{1.7115}\right) \times F_u\left(\frac{20 - 12.11}{1.06}\right)$

$= F_u(-1.97) \times F_u(1.98) \times F_u(7.45)$

$= (1 - 0.97558) \times 0.97615 \times 1$

$= 0.024$