AMERICAN UNIVERSITY OF BEIRUT
Mathematics Department
Math 218 - Quiz I
Fall 2006-2007

Name:.......................................

ID:..

Section: 4 (@ 12:30)   5 (@ 9:30)

Time: 60 min

I- (10 points) Consider the following matrices,

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 7 & 3 \\ 0 & 2 & -3 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 & 8 \\ 1 & 0 & 4 \\ -2 & 0 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 3 & 8 & 0 \end{bmatrix}$$

Calculate $B^T A - C$.

II- Let

$$A = \begin{bmatrix} 3 & 2 & 6 \\ 5 & 1 & 1 \\ 0 & -2 & 3 \end{bmatrix}$$

(a) **(8 points)** Find $det(A)$. (Use any one of the three methods that you know)

(b) **(7 points)** Find $adj(A)$.

(c) **(6 points)** Deduce A^{-1}.

III- (a) **(12 points)** Find the conditions that the b's should satisfy so that the system is consistent. (If there are any)

$$\begin{cases} 3x_1 + 27x_2 + 14x_3 - 3x_4 = b_1 \\ x_2 + 3x_4 = b_2 \\ -2x_1 + 2x_2 - 6x_3 + 12x_4 = b_3 \\ x_1 + 5x_2 + 3x_3 + 2x_4 = b_4 \end{cases}$$

(Continue your answer here)

(b) **(4 points)** Using part (a), explain why the following matrix B is invertible,

$$B = \begin{bmatrix} 3 & 27 & 14 & -3 \\ 0 & 1 & 0 & 3 \\ -2 & 2 & -6 & 12 \\ 1 & 5 & 3 & 2 \end{bmatrix}$$

IV- **(a)** **(5 points)** If A is an invertible matrix, show that the system $Ax = b$ has a unique solution.

(b) (7 points) Assuming that the stated inverses exist, show that,

$$(B + AC)^{-1}A = (C^{-1}A^{-1}B + I)^{-1}C^{-1}$$

(c) (8 points) If E is a symmetric matrix and $AD(E^{-1})^T = (EA^{-1})^{-1}$, show that $D = I$.

V- **(a)** **(5 points)** Prove the following identity without directly computing the determinant.

$$\begin{vmatrix} a_1 & b_1 & c_1 + sa_1 + tb_1 \\ a_2 & b_2 & c_2 + sa_2 + tb_2 \\ a_3 & b_3 & c_3 + sa_3 + tb_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

(b) **(5 points)** Use (a) to show that,

$$\begin{vmatrix} 1 & 2 & 123 \\ 4 & 5 & 456 \\ 7 & 8 & 789 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}.$$

V- It is a known fact that if A is a 3×3 matrix with $|A| = 1$ and some other properties, then multiplication by A is a rotation about some axis of rotation through an angle θ.

For this question consider $u = (a, b, c)$ to be a vector that determines this axis and has length 1, then the standard matrix representing the rotation is,

$$\begin{bmatrix} a^2(1-\cos\theta)+\cos\theta & ab(1-\cos\theta)-c\sin\theta & ac(1-\cos\theta)+b\sin\theta \\ ab(1-\cos\theta)+c\sin\theta & b^2(1-\cos\theta)+\cos\theta & bc(1-\cos\theta)-a\sin\theta \\ ac(1-\cos\theta)-b\sin\theta & bc(1-\cos\theta)+a\sin\theta & c^2(1-\cos\theta)+\cos\theta \end{bmatrix}$$

(10 points) Prove that $\cos\theta = \frac{tr(A)-1}{2}$.

VI- YOU DO NOT NEED TO PROVE ANY OF YOUR ANSWERS IN THIS QUESTION.

a- **(10 points)** Answer **TRUE** or **FALSE**.

i- A system of linear equations that doesn't have infinitely many solutions must have a unique solution.

ii- T is a linear transformation if and only if there is a matrix A such that $T(x) = Ax$ for all x in the domain of T.

iii- If A is an invertible $n \times n$ matrix then for all $n \times 1$ matrices b, there is a nonzero vector x such that $A(3x) = b$.

iv- If A is a square matrix with all of its entries being integers and with $det(A) = -1$, then all the entries of A^{-1} are integers.

b- **(3 points)** A student was multiplying an $m \times k$ matrix A with a $k \times n$ matrix B. He copied one entry of A wrongly. If he wants to correct his mistake, how many entries of the product AB does he have to check?

Your answer:________________________

GOOD LUCK