Math 218 **Quiz 2** **Fall 2009**

Name: ______________________________

Student ID:______________________________

Instructions:

1. You must show correct work to receive credit. Correct answers with inconsistent work will not be given credit.
2. Books, notes and calculators are not allowed.
3. Turn off and put away all cell phones.

Page	**Points**	**Points Possible**
2		12
3		12
4		14
5		10
6		12
Total		50

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1. (6 pts) Let $V = \left\{ \begin{bmatrix} 2a+b+3c \\ 2b+2c \\ 4a-3b+c \end{bmatrix} \middle| \, a,b,c \in \mathbb{R} \right\}$. Find $\dim V$.

2. (6 pts) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a mapping with

$$T\left(\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ a \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ b \\ 5 \end{bmatrix}, \quad T\left(\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}\right) = \begin{bmatrix} c \\ 6 \\ 2 \end{bmatrix}.$$

For which values of a, b, c is T a linear operator?

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3. (6 pts) Let $T : V \to W$ be a linear transformation and let U be a subspace of W. Prove that the set $S = \{v \in V | T(v) \in U\}$ is a subspace of V.

4. (6 pts) Let $T : V \to V$ be a linear operator. Determine whether the set $W = \{v \in V | T(v) = v\}$ is a subspace of V.

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5. (4 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} |x| \\ |y| \end{bmatrix}$$

where $|x|$ is the absolute value of x. Determine whether T is a linear transformation.

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ y - z \\ x + 2z \end{bmatrix}.$$

(a) (6 pts) Find a basis for $N(T)$.

(b) (4 pts) Find $\dim(R(T))$.

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7. (4 pts) Let $\{v_1, v_2, \ldots, v_k\}$ be a linearly independent set of vectors in a vector space V. Show that the set $\{v_1, v_2, \ldots, v_{k-1}\}$ cannot be a basis for V.

8. (6 pts) Let

$$V = \left\{ \begin{bmatrix} 1 & 1+t \\ 2+t & 2 \end{bmatrix} \middle| t \in \mathbb{R} \right\}.$$

Define

$$\begin{bmatrix} 1 & 1+t \\ 2+t & 2 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1+s \\ 2+s & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1+t+s \\ 2+t+s & 2 \end{bmatrix}.$$

Find the additive identity and additive inverse.

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9. (4 pts each) Determine whether each of the following statements is true or false. Justify your answer.

(a) If $\{a, b, c\}$ is linearly dependent then a is a linear combination of b and c.

(b) If $T : \mathbb{R}^{12} \to \mathbb{R}^7$ is a linear transformation, then $\dim(N(T)) \geq 5$.

(c) If W is a subspace of a vector space V, then $V - W$, the set of all vectors that are in V but not in W, is also a subspace of V.