

Name: _____

Student ID: _____

Instructions:

1. This exam has 5 pages. Please make sure you have all pages.
2. The point value of each problem occurs to the left of the problem.
3. **You must show correct work to receive credit.** Correct answers with inconsistent work or with no justification will not be given credit.
4. Only non-graphing and non-programmable calculators are allowed.
5. **Turn off and put away all cell phones.**

Page	Points	Points Possible
2		11
3		12
4		9
5		8
Total		40

Name: _____

1. (4 pts) For which values of k do the vectors $\begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} k \\ 2 \\ 2 \end{bmatrix}$ form a basis for \mathbb{R}^3 ?

2. Let $t : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by $T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 6x + 3y \\ 4x + 2y + z \\ 4z \end{bmatrix}$.

(a) (4 pts) Find a basis for $R(T)$.

(b) (3 pts) Is T an isomorphism?

Name: _____

3. Let $T : \mathcal{P}_2 \rightarrow \mathcal{P}_1$ be given by $T(p(x)) = xp(1) + p(0)$.

(a) (4 pts) Show that T is a linear transformation.

(b) (4 pts) Find $\ker(T)$.

(c) (4 pts) Find $R(T)$.

Name: _____

4. (4 pts) Let $T : V \rightarrow W$ be a linear transformation. Determine whether $\{\mathbf{v} \in V \mid T(\mathbf{v}) \neq \mathbf{0}\}$ is a subspace of V .
5. (5 pts) Suppose S and T are subspaces of a vector space V . Define $W = \{2\mathbf{u} + 3\mathbf{v} \mid \mathbf{u} \in S, \mathbf{v} \in T\}$. Determine whether W is a subspace of V .

Name: _____

6. (3 pts) Find a linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ and a linearly independent subset $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ of \mathbb{R}^3 such that $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.
7. (5 pts) Let $T : V \rightarrow W$ be a linear transformation and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a subset of V . Show that if T is one-to-one and $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly dependent.