Name: ______________________________

Student ID: ______________________________

Instructions:

1. You must show correct work to receive credit. Correct answers with inconsistent work will not be given credit.
2. Books and notes are not allowed.
3. You may use a simple calculator.
4. Turn off and put away all cell phones.

Page	**Points**	**Points Possible**
2		12
3		14
4		11
5		13
Total		50

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1. Let $B_1 = \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$ be two ordered bases of $\mathbb{R}^2$.

a) Find the transition matrix $[I]_{B_1}^{B_2}$ between B_1 and B_2.

b) If $[v]_{B_1} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, find $[v]_{B_2}$.

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2. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be given by $T(v) = \begin{bmatrix} 1 & 4 & 2 & 0 & 4 \\ 3 & 2 & 0 & 2 & 4 \\ 1 & -6 & -4 & 1 & -5 \\ 6 & 4 & 0 & -1 & 3 \end{bmatrix} v$. Prove or disprove

(a) (8 pts) T is one-to-one.

(b) (4 pts) T is onto.

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3. (6 pts) Let $T : \mathbb{R}^3 \to R^3$ be a linear operator and $B = \{v_1, v_2, v_3\}$ a basis for $\mathbb{R}^3$. Suppose $T(v_1) = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, $T(v_2) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ and $T(v_3) = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$.

(a) Determine whether $w = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$ belongs to $R(T)$.

(b) Find a basis for $R(T)$.

(c) Find a basis for $\mathbb{R}^3$ containing $T(v_1)$.

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4. (8 pts) Let $T : M_{2x2} \to M_{2x2}$ be given by $T(A) = A - A^t$.

(a) Show that T is a linear transformation.

(b) Find $N(T)$.

(c) Find a basis for $N(T)$.

(d) Find $\dim(R(T))$.

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5. (3 pts) Let $W = \{p(x) \in \mathcal{P}_3 | p(1) = p(-1)\}$. Find a basis for W.