

## Two-Factor ANOVA

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BUS 301: Int. Bus. Stats  
Lebanese American University

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Overview

## Today's Lecture

- Case II, reminder, Due by 5pm, 28 November.
- Quiz 2: In-class, 9 December 2013
- Assignment 5
- Overview of Week
- Review
- Two-Factor ANOVA (14.5)

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Case Study Reminder

## Case Study Part II, due 28 November 2013

- Done as partners – each pair of two people hands in one letter.
- Submission is a letter...do NOT hand in reams of excel sheets.
- Clearly write your findings; use tables and charts as necessary.

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## Assignment 5, Due Start of Class, 4 December

Assignment requires you to fill out a worksheet and solve the following four book problems:

- ➊ 13.93
- ➋ 14.9
- ➌ 14.37
- ➍ 14.75

NOTE: We will work on Problems 14.9 and 14.37 together in the lab on Friday.

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## Overview of Week

Monday: Two-Factor ANOVA (on Quiz 2)

Wednesday (Today): Two-Factor ANOVA, reviewed (on Quiz 2)

Friday: In Computer Lab (Practice with ANOVA.)

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## Example

A survey was conducted wherein Americans aged between 37 and 45 were asked how many jobs they have held in their lifetimes. Also recorded were gender and educational attainment. Can we infer that differences exist between genders and educational levels?

Male E1	Male E2	Male E3	Male E4	Female E1	Female E2	Female E3	Female E4
10	12	15	8	7	7	5	7
9	11	8	9	13	12	13	9
12	9	7	5	14	6	12	3
16	14	7	11	6	15	3	7
14	12	7	13	11	10	13	9
17	16	9	8	14	13	11	6
13	10	14	7	13	9	15	10
9	10	15	11	11	15	5	15
11	5	11	10	14	12	9	4
15	11	13	8	12	13	8	11

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### Four Types of Hypotheses

#### One-Way ANOVA

$H_0$ : The means of all the groupings are equal.  
 $H_1$ : At least one mean differs.

#### Differences between Levels of Factor A

$H_0$ : The means of the  $a$  levels of factor A are equal.  
 $H_1$ : At least one mean differs.

#### Differences between Levels of Factor B

$H_0$ : The means of the  $b$  levels of factor B are equal.  
 $H_1$ : At least one mean differs.

#### Test of Interaction between Factor A and Factor B

$H_0$ : Factors A and B do not interact to affect the mean responses.  
 $H_1$ : Factors A and B do interact to affect the mean responses.

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### One-Way ANOVA Results

	A	B	C	D	E	F	G
1	Anova: Single Factor						
2							
3	SUMMARY						
4	Groups	Count	Sum	Average	Variance		
5	Male E1	10	126	12.60	8.27		
6	Male E2	10	110	11.00	8.67		
7	Male E3	10	106	10.60	11.60		
8	Male E4	10	90	9.00	5.33		
9	Female E1	10	115	11.50	8.28		
10	Female E2	10	112	11.20	9.73		
11	Female E3	10	94	9.40	16.49		
12	Female E4	10	81	8.10	12.32		
13							
14							
15	ANOVA						
16	Source of Variation	SS	df	MS	F	P-value	F crit
17	Between Groups	153.35	7	21.91	2.17	0.0467	2.1397
18	Within Groups	726.20	72	10.09			
19							
20	Total	879.55	79				

What can we conclude? Reject  $H_0$ ; There are differences.

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### Overview of Two Factor Data Structure

Factor B	Factor A			
	1	2	...	$\alpha$
1	$\begin{bmatrix} x_{111} \\ x_{112} \\ \vdots \\ x_{11r} \end{bmatrix}$ $\bar{x}[AB]_{11}$	$\begin{bmatrix} x_{211} \\ x_{212} \\ \vdots \\ x_{21r} \end{bmatrix}$ $\bar{x}[AB]_{21}$	...	$\begin{bmatrix} x_{\alpha 11} \\ x_{\alpha 12} \\ \vdots \\ x_{\alpha 1r} \end{bmatrix}$ $\bar{x}[AB]_{\alpha 1}$ $\bar{x}[B]_1$
2	$\begin{bmatrix} x_{121} \\ x_{122} \\ \vdots \\ x_{12r} \end{bmatrix}$ $\bar{x}[AB]_{12}$	$\begin{bmatrix} x_{221} \\ x_{222} \\ \vdots \\ x_{22r} \end{bmatrix}$ $\bar{x}[AB]_{22}$	...	$\begin{bmatrix} x_{\alpha 21} \\ x_{\alpha 22} \\ \vdots \\ x_{\alpha 2r} \end{bmatrix}$ $\bar{x}[AB]_{\alpha 2}$ $\bar{x}[B]_2$
...				
...				
$b$	$\begin{bmatrix} x_{1b1} \\ x_{1b2} \\ \vdots \\ x_{1br} \end{bmatrix}$ $\bar{x}[AB]_{1b}$	$\begin{bmatrix} x_{2b1} \\ x_{2b2} \\ \vdots \\ x_{2br} \end{bmatrix}$ $\bar{x}[AB]_{2b}$	...	$\begin{bmatrix} x_{\alpha b1} \\ x_{\alpha b2} \\ \vdots \\ x_{\alpha br} \end{bmatrix}$ $\bar{x}[AB]_{\alpha b}$ $\bar{x}[B]_b$
	$\bar{x}[A]_1$	$\bar{x}[A]_2$	...	$\bar{x}[A]_\alpha$
				$\bar{\bar{x}}$

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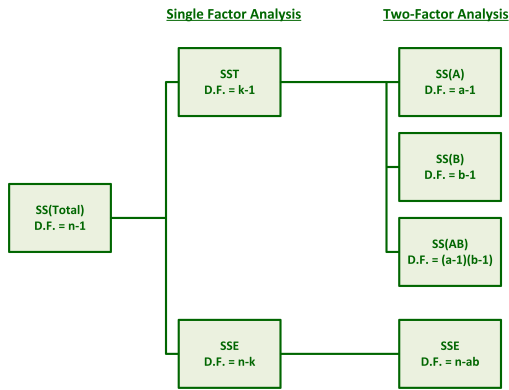
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### Test for Differences in A and B and Interactions



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### Four Test Statistics

One-Way ANOVA

$$F = \frac{MST}{MSE}$$

Differences between Levels of Factor A

$$F = \frac{MS(A)}{MSE}$$

Differences between Levels of Factor B

$$F = \frac{MS(B)}{MSE}$$

Test of Interaction between Factor A and Factor B

$$F = \frac{MS(AB)}{MSE}$$

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### Example in Excel

What can we conclude from the Two-Factor With Replication ANOVA in Excel?

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## Next Time

- In Lab

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