

Name: _____

Student ID: _____

Instructions:

1. You must show correct work to receive credit. Correct answers with inconsistent work will not be given credit.
2. Books, notes and calculators are not allowed.
3. Turn off and put away all cell phones.

Question	Points	Points Possible
1		5
2		5
3		10
4		10
5		5
6		5
7		5
8		5
Total		50

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1. (5 pts) Determine whether \mathbf{v} is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ where

$$\mathbf{v} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}.$$

2. (5 pts) Determine the values of $a, b,$ and c for which the linear system is consistent.

$$\begin{cases} x + 2y - 3z = a \\ 2x + 3y + 3z = b \\ 5x + 9y - 6z = c \end{cases}$$

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3. (5 pts each) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

(a) Find the inverse of A.

(b) Use (a) to solve the system $A\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$.

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4. (5 pts each) If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and $\det(A) = 5$, find

(a) $\det(2A^2A^tA^{-1})$

(b) $\det \left(\begin{bmatrix} a_1 & a_1 + 2b_1 & 3a_1 + c_1 \\ a_2 & a_2 + 2b_2 & 3a_2 + c_2 \\ a_3 & a_3 + 2b_3 & 3a_3 + c_3 \end{bmatrix} \right)$

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5. (5 pts) Find all values of a for which the inverse of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & a & 0 \\ 1 & 2 & a \end{bmatrix}$ exists.

6. (5 pts) Let A and B be two $n \times n$ matrices. If AB is invertible, must both A and B be invertible? Justify your answer.

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7. (5 pts) Let A and B be symmetric matrices. Show that AB is symmetric **if and only if** $AB = BA$.

8. (5 pts) Let A be a 4×4 invertible matrix. Find $\det(A)$ if $A^t = 2A^{-1}$.