Student ID:\_\_\_\_\_

## Instructions:

- 1. You must show correct work to receive credit. Correct answers with inconsistent work will not be given credit.
- 2. Books and notes are not allowed.
- 3. You may use a simple calculator.
- 4. Turn off and put away all cell phones.

Page	Points	Points Possible
2		12
3		14
4		11
5		13
Total		50

**1.** The augmented matrix of a linear system is  $A = \begin{bmatrix} 2 & 2 & -2 & | & a \\ 3 & 2 & -2 & | & b \\ 2 & 1 & -1 & | & c \end{bmatrix}$ .

(a) (8 pts) Determine the values of a, b, c for which the system is consistent.

(b) (4 pts) If a = 4, b = 1 and c = -1, how many solutions does the system have?

Math 21	18
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**2.** (6 pts) If A is a  $4 \times 4$  matrix with det(A) = 3, find det $(4A^3A^t(2A)^{-1})$ .

**3.** (8 pts) Let 
$$A = \begin{bmatrix} 2 & 6 & 2 \\ 3 & 5 & 1 \\ -1 & 3 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$
. Prove or disprove: The linear system  $A\mathbf{x} = \mathbf{b}$  has a unique solution for every vector  $\mathbf{b}$  in  $\mathbb{R}^4$ .

**4.** (3 pts) Prove or disprove: If **v** is any vector in a vector space V then  $\{\mathbf{v}\}$  is linearly independent.

5. (8 pts) For which values of a is the set  $S = \{ax^3, x^3 + (2a-1)x^2, a^2x^2 + (a-4)x\}$  linearly independent?

**6.** (5 pts) Let A and B be two  $n \times n$  matrices with  $det(A) = det(B) \neq 0$ . Find a matrix C such that det(C) = 1 and A = CB.

7. (8 pts) Let V and W be subspaces of the Euclidean space  $\mathbb{R}^5$ . Determine whether the set  $S = \{\mathbf{v} - \mathbf{w} \mid \mathbf{v} \in V, \mathbf{w} \in W\}$  is a subspace of  $\mathbb{R}^5$ .