Kinematics & Dynamics of Linkages

Lecture 3: Grashof condition

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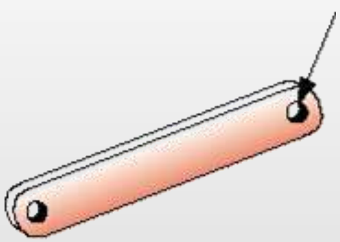
LAU

Review of Kinematic Principles

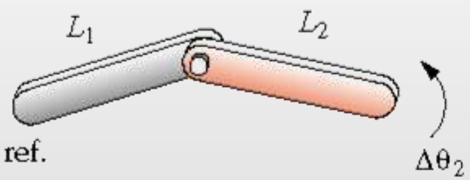
- Determining the motion of mechanisms
- 2 type of motion of any rigid body
  - Translation & Rotation
  - May combine for Complex motion

Linkages

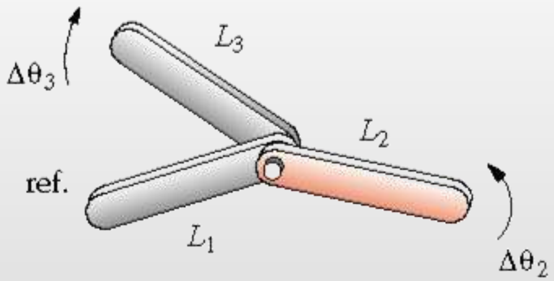
- Link = a rigid body possessing at least 2 nodes
- Nodes = points for attachment to other links
- Joint = connection between 2 or more links



Binary link



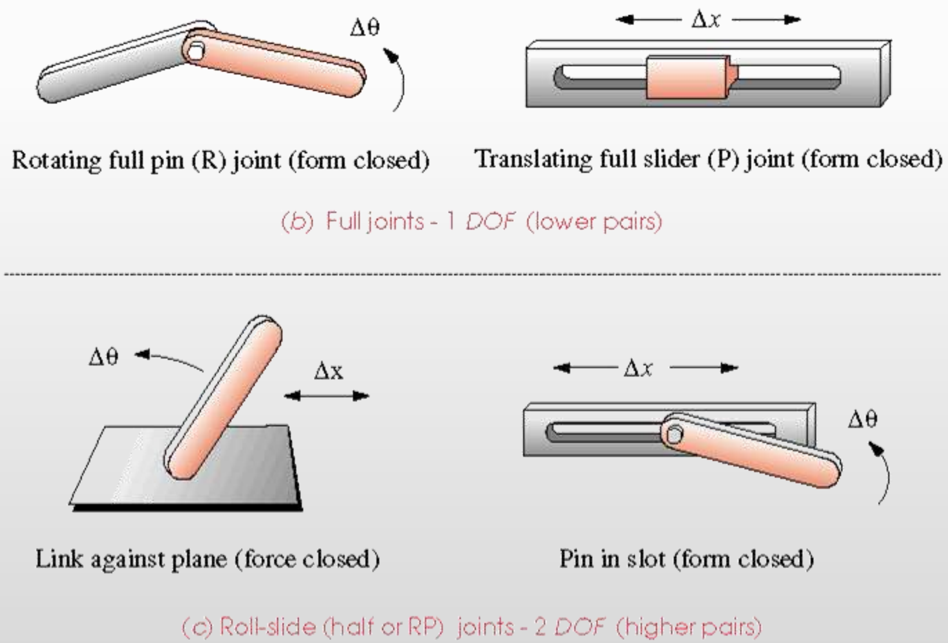
First order pin joint - one *DOF*
(two links joined)



Second order pin joint - two *DOF*
(three links joined)

Classification of Joints by the # of DOF they allow

- Full Joint
  - Rotating pin or translating slider
    - 1 DOF
- Half Joint
  - Roll-slide joint
    - 2 DOF



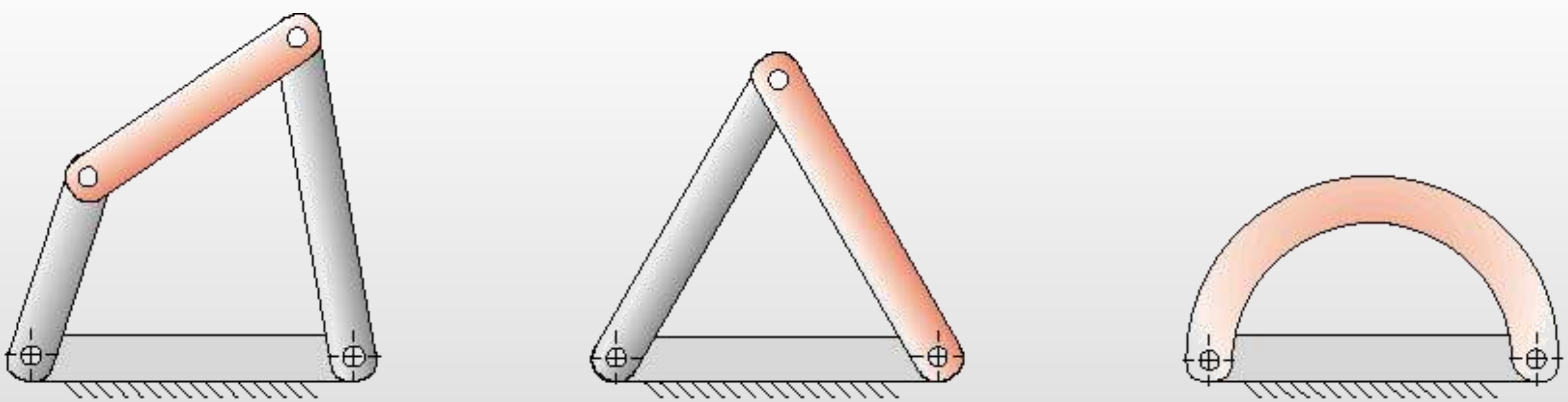
Mobility of a Mechanism

- Gruebler's equation :
  - $M = 3(L - 1) - 2J$
    - Where:
      - L = # of links
      - J = # of joints

- Kutzbach's equation :
  - $M = 3(L - 1) - 2J1 - J2$
    - Where:
      - L = # of links
      - J1 = # of full joints
      - J2 = # of half joints

Mechanisms and Structures

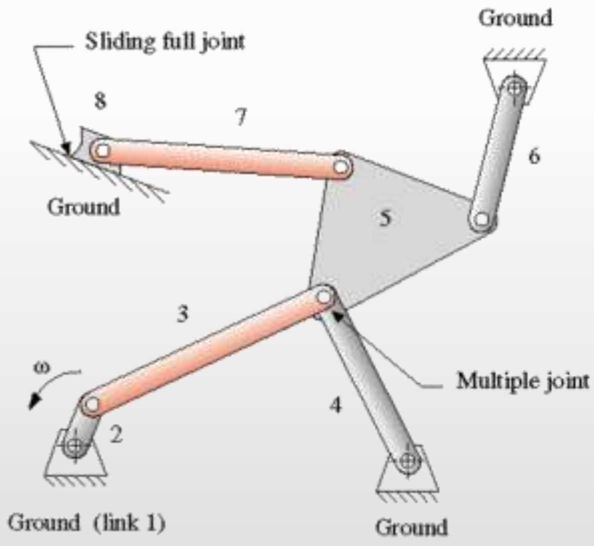
- If DOF > or = 1, then it is a mechanism
- If DOF = 0, then it is a structure
- If DOF = -1, then it is a preloaded structure

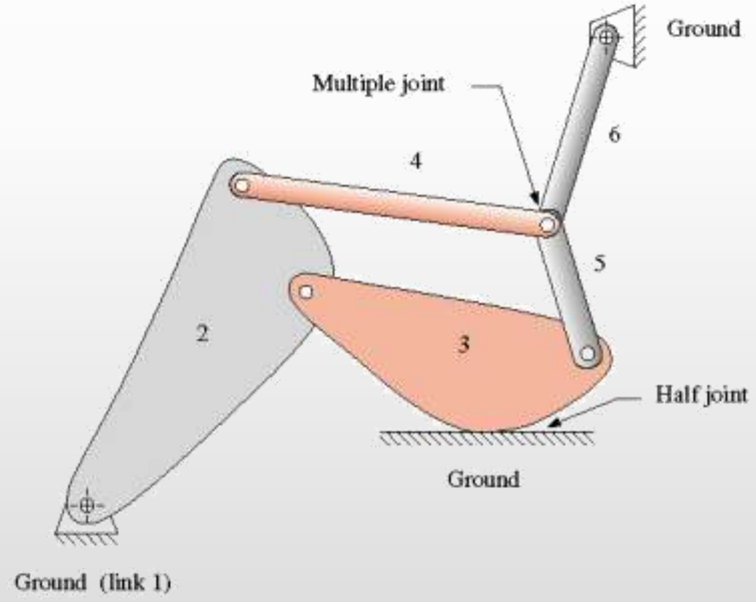


(a) Mechanism—*DOF* = +1 (b) Structure—*DOF* = 0 (c) Preloaded structure—*DOF* = −1

Multiple Joints

- Treated as N full joints
  - where N = number of links connected - 1

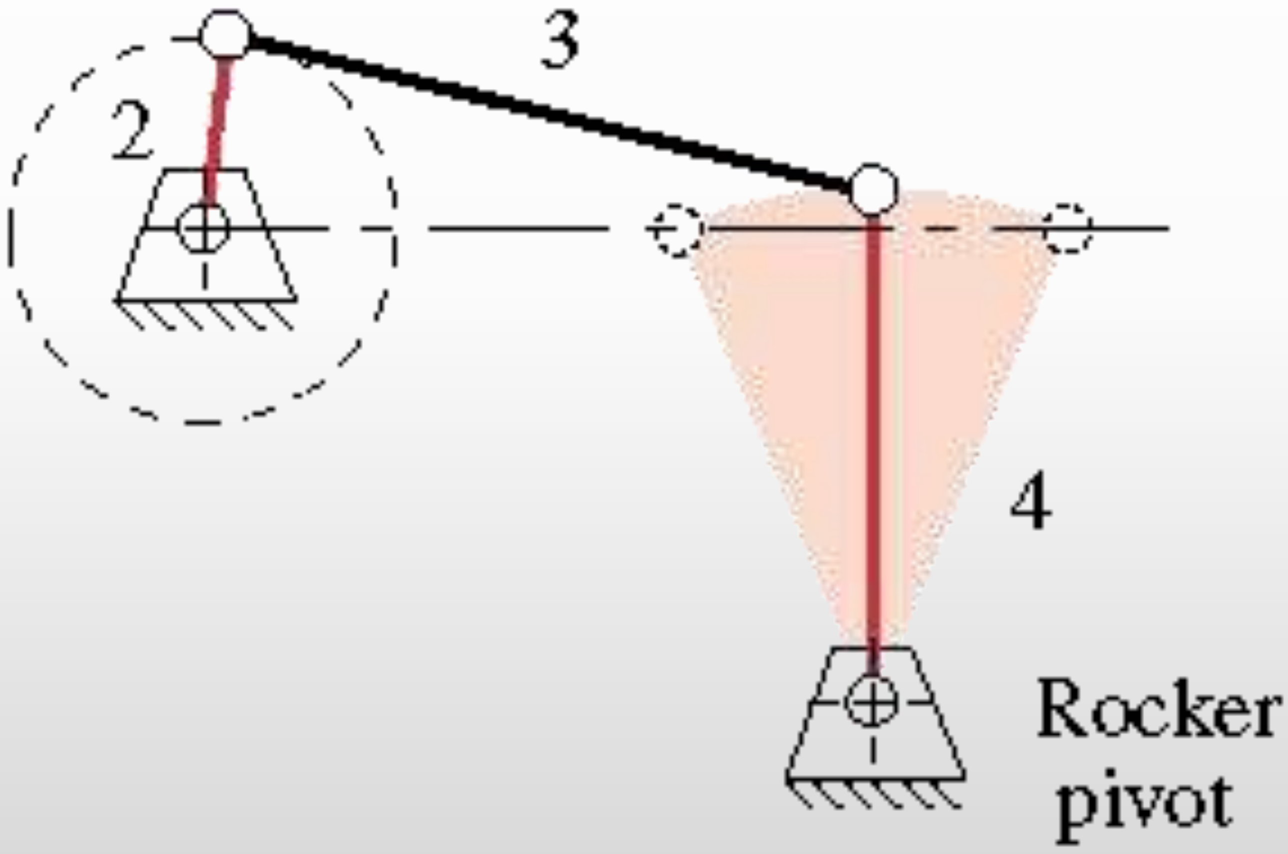




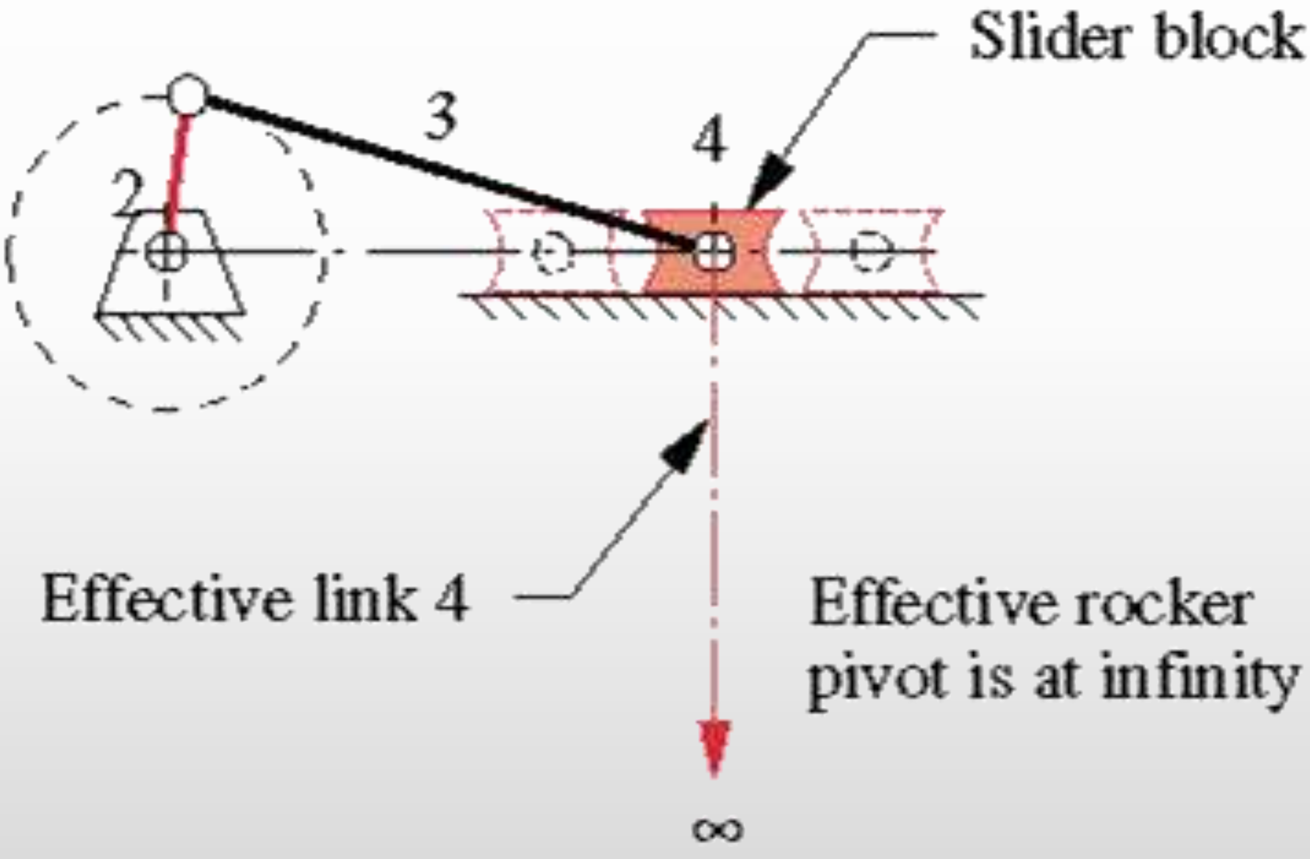
Mechanism Definitions

- **Crank**: A link which makes a complete revolution and is pivoted to ground
- **Coupler**: A connecting rod which has complex motion and is not pivoted to ground
- **Rocker**: A link which has oscillatory (back & forth) rotation & is pivoted to ground
- **Ground**: Any link or links that are fixed (non-moving) with respect to the reference frame

Grashof crank-rocker



Grashof slider-crank

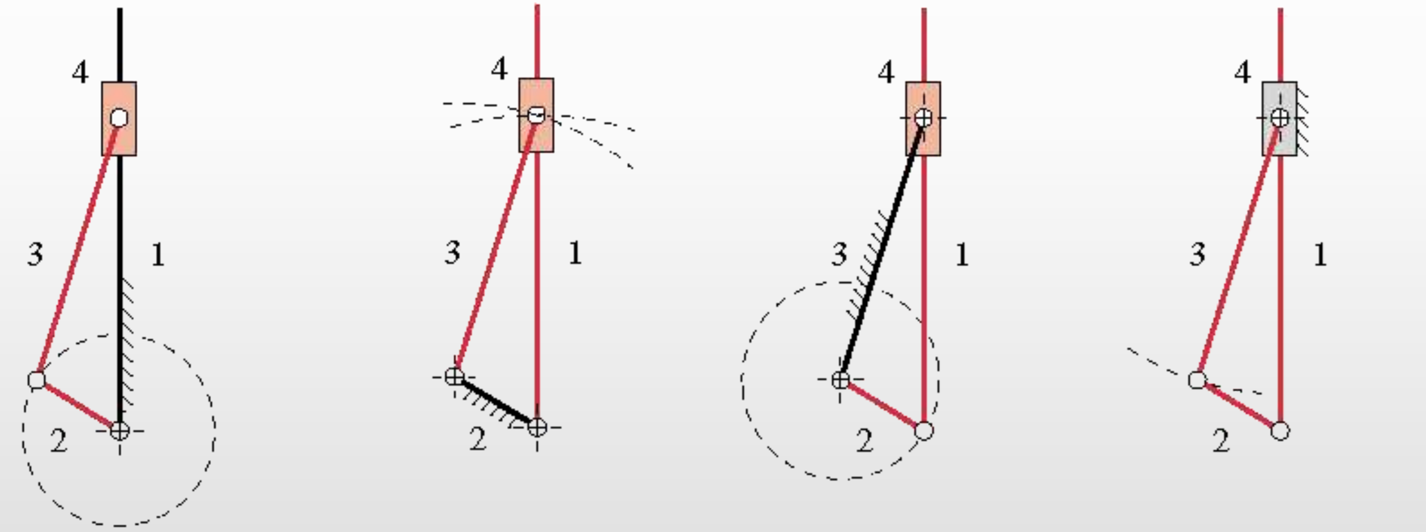


Linkage Transformation

- Rules that can be applied to planar kinematic chains
- Any full joint can be replaced by a half joint, but this will increase the DOF by one.
- Removal of a link will reduce the DOF by one.
- The combination of the rules above will keep the original DOF unchanged.

Inversion

- It is created by grounding a different link in the kinematic chain.
- We have as many inversions of a given linkage as it has links.



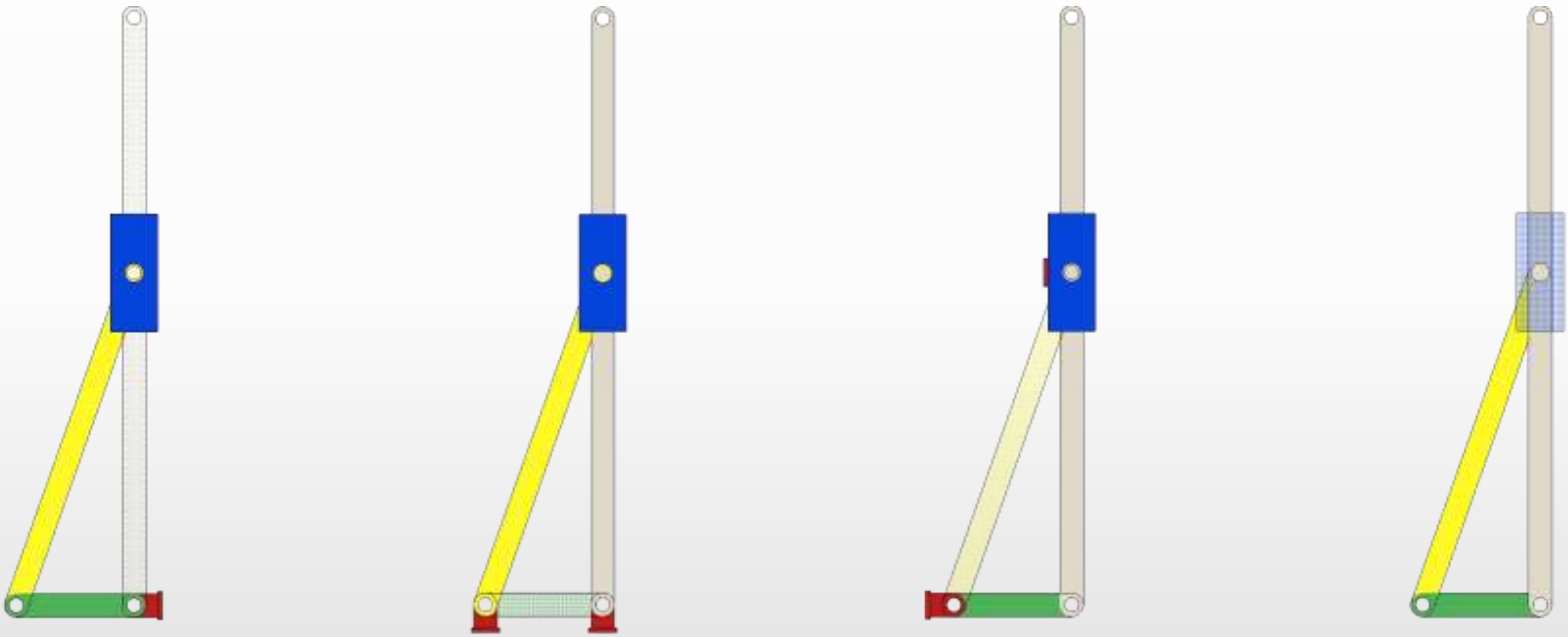
(a) Inversion # 1 slider block translates

(b) Inversion # 2 slider block has complex motion

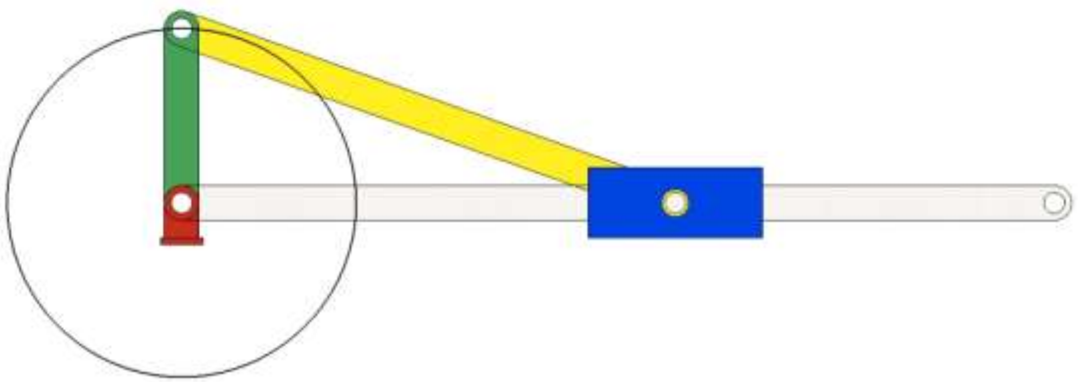
(c) Inversion # 3 slider block rotates

(d) Inversion # 4 slider block is stationary

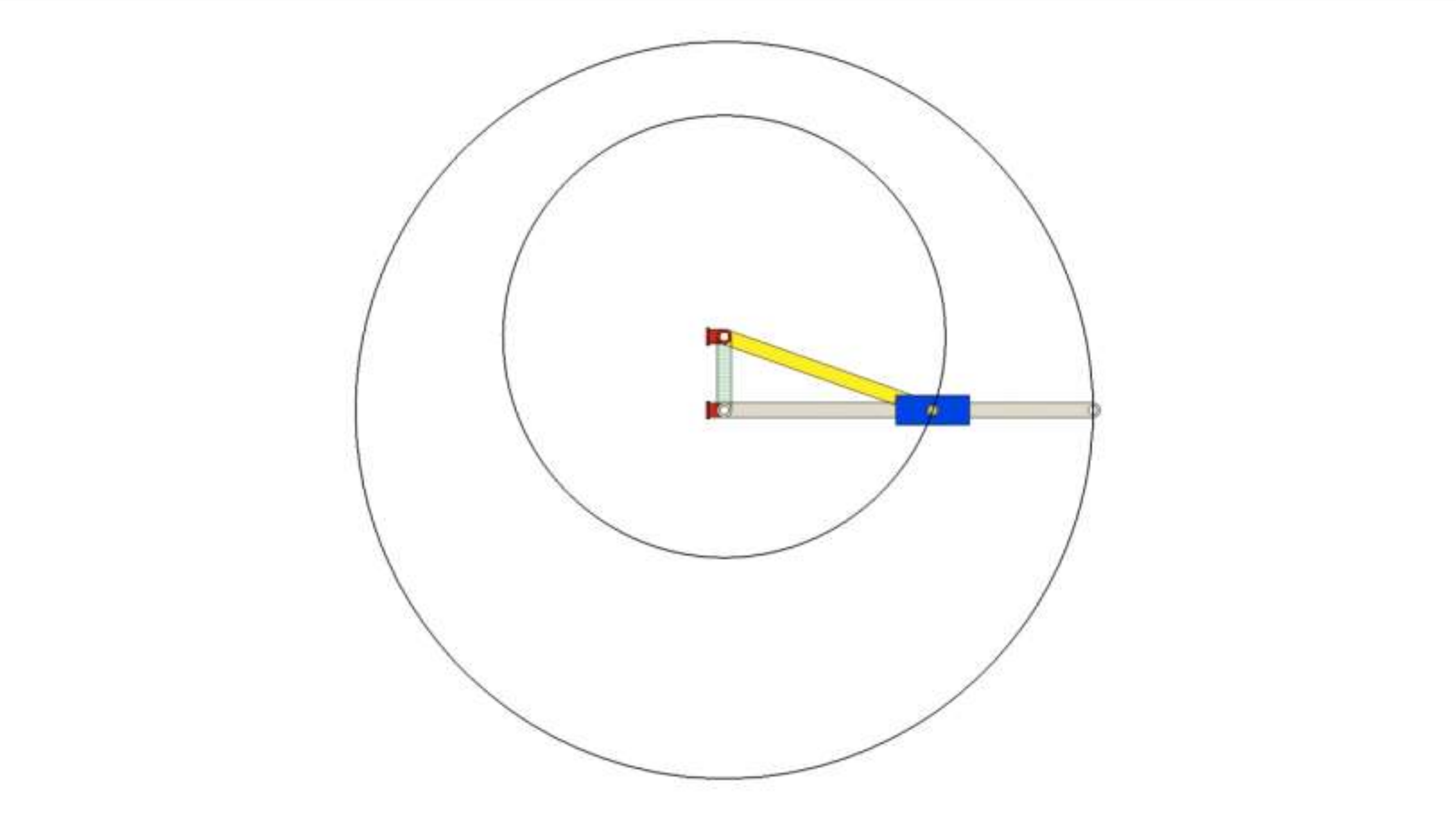
Slider Crank Inversions on SolidWorks



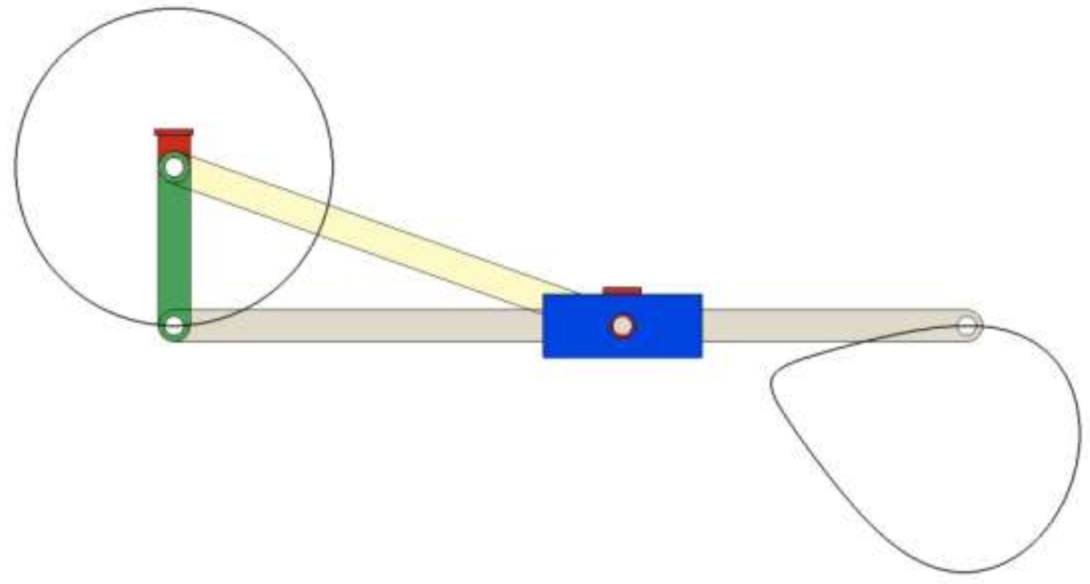
Inversion 1



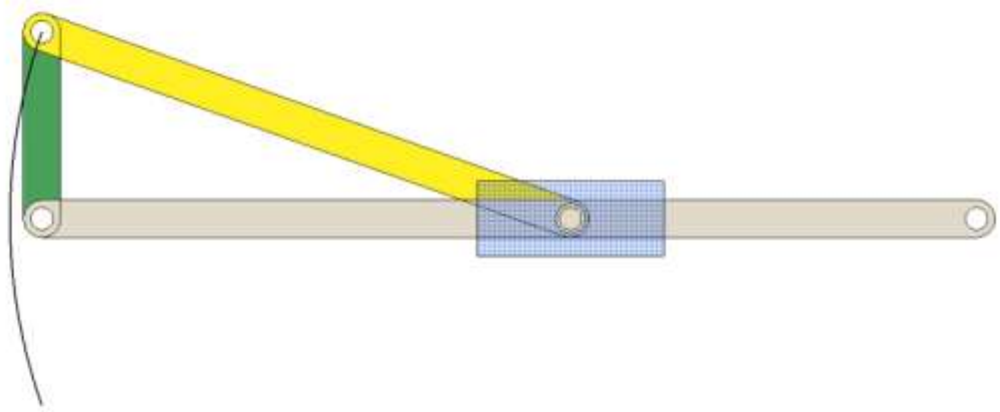
Inversion 2



Inversion 3

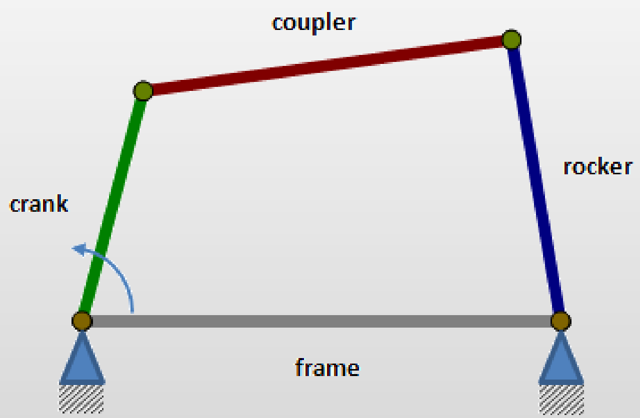


Inversion 4

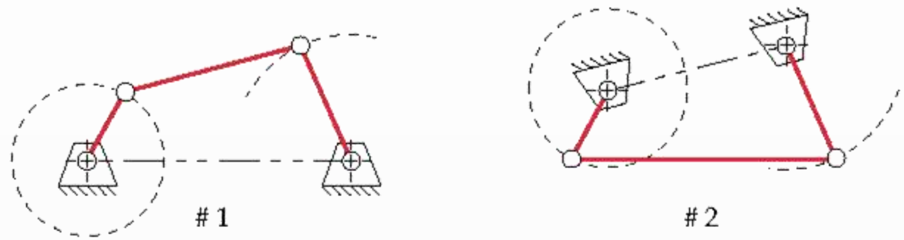


4-Bar Linkages

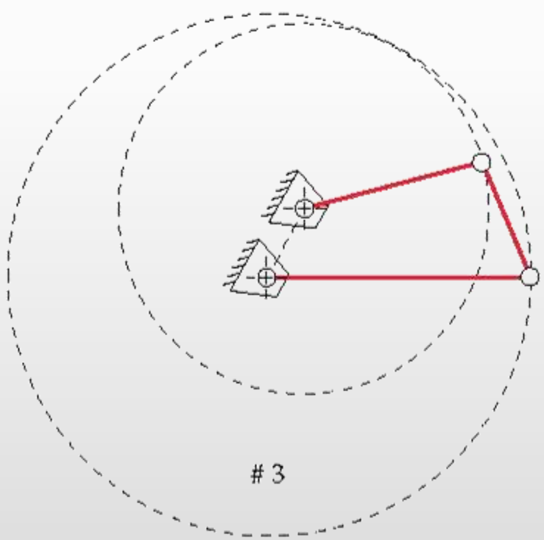
- 3 connected links connected to a ground link
  - The most common device used in machinery
  - Have a single degree of freedom mobility
  - Simplest possible pin-jointed mechanism for 1 DOF
  - Simplicity is one mark of good design



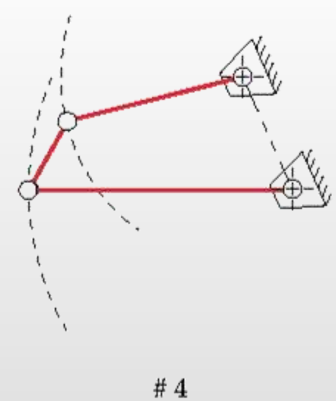
Inversion of a 4-bar linkage



(a) Two non-distinct crank-rocker inversions (GCRR)

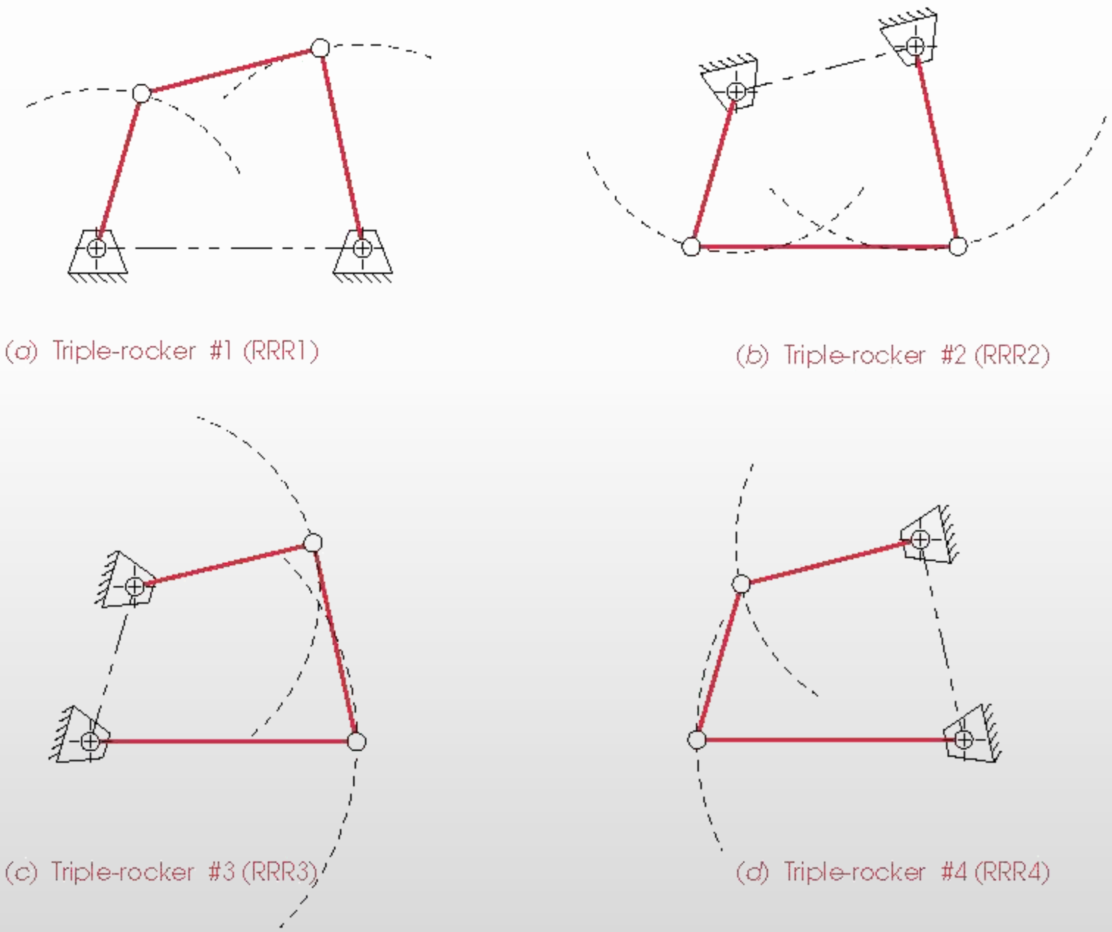


(b) Double-crank inversion (GCCC)
(drag link mechanism)



(c) Double-rocker inversion (GRCR)
(coupler rotates)

Inversion of a 4-bar linkage



Grashof condition

- Mobility of a 4–bar linkage is equal to : M=1
- **Grashof** derived a very simple relationship that predicts the rotation behavior based only on the link lengths

Let:

$\boldsymbol{S}$ = length of the shortest link
$\boldsymbol{L}$ = length of the longest link
$\boldsymbol{P}$ = length of one remaining link
$\boldsymbol{Q}$ = length of the other remaining link

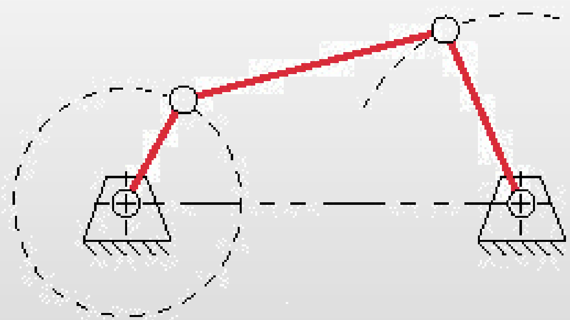
Grashof condition

- The linkage is Grashof : the **shortest link** will be capable of making a full revolution with respect to the ground plave
- The linkage is Non-Grashof : no link will be capable of a complete revolution relative to any other link.

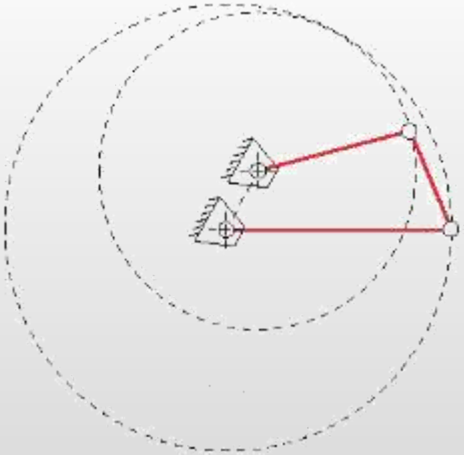
  - Type I : $S + L < P + Q$ (Grashof)
  - Type II : $S + L > P + Q$ (Non−Grashof)
  - Type III : $S + L = P + Q$ (Grashof)

Type I - Grashof

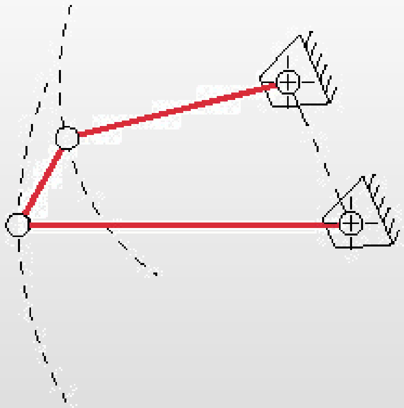
- S + L < P + Q
  - and ground either link adjacent to S - Crank-rocker
  - and ground S - Double-crank
  - and ground link opposite S – Grashof Double-rocker



Crank-rocker

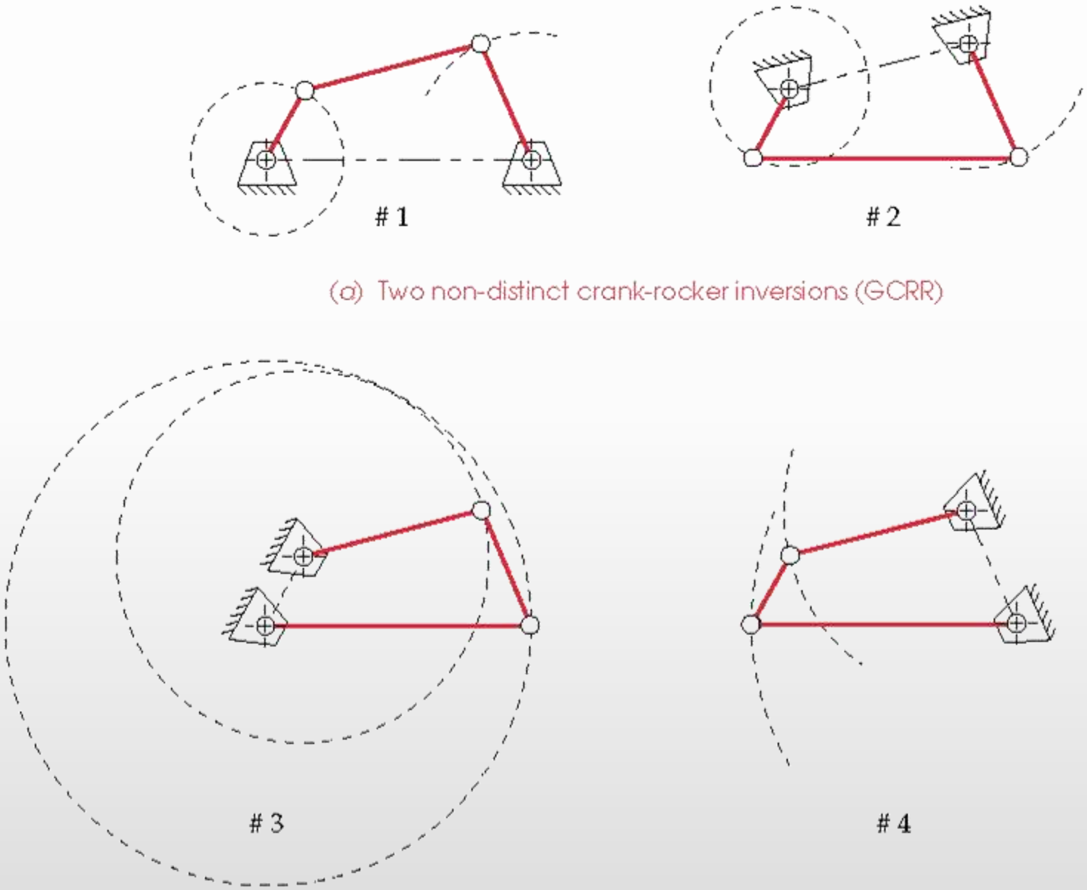


Double-crank



Double-rocker

Grashof linkages

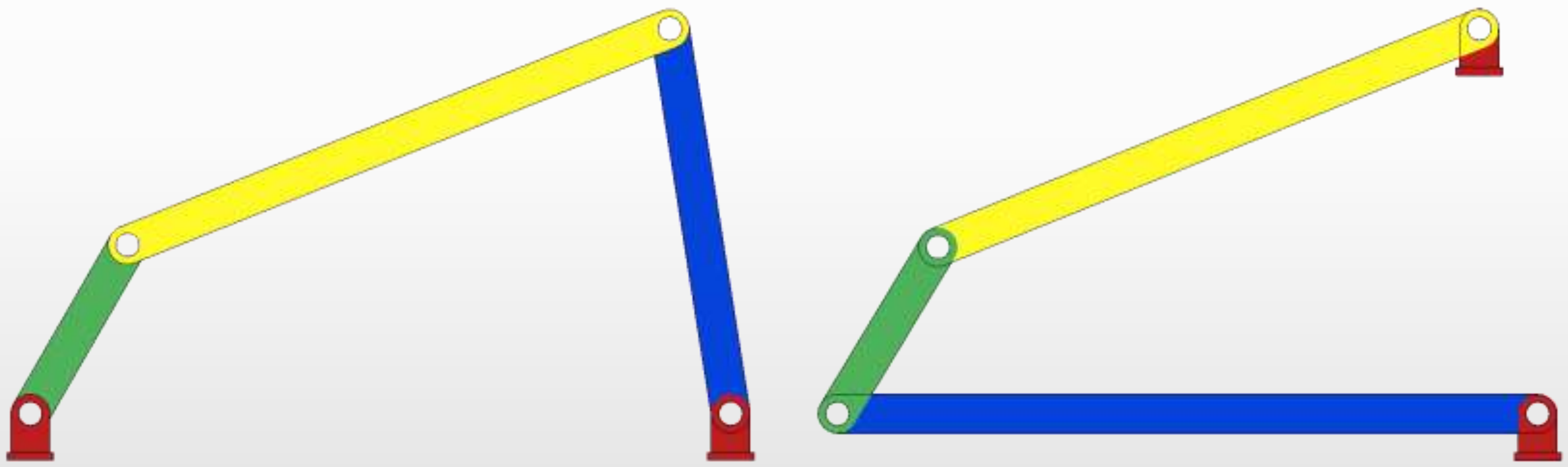


(a) Two non-distinct crank-rocker inversions (GCRR)

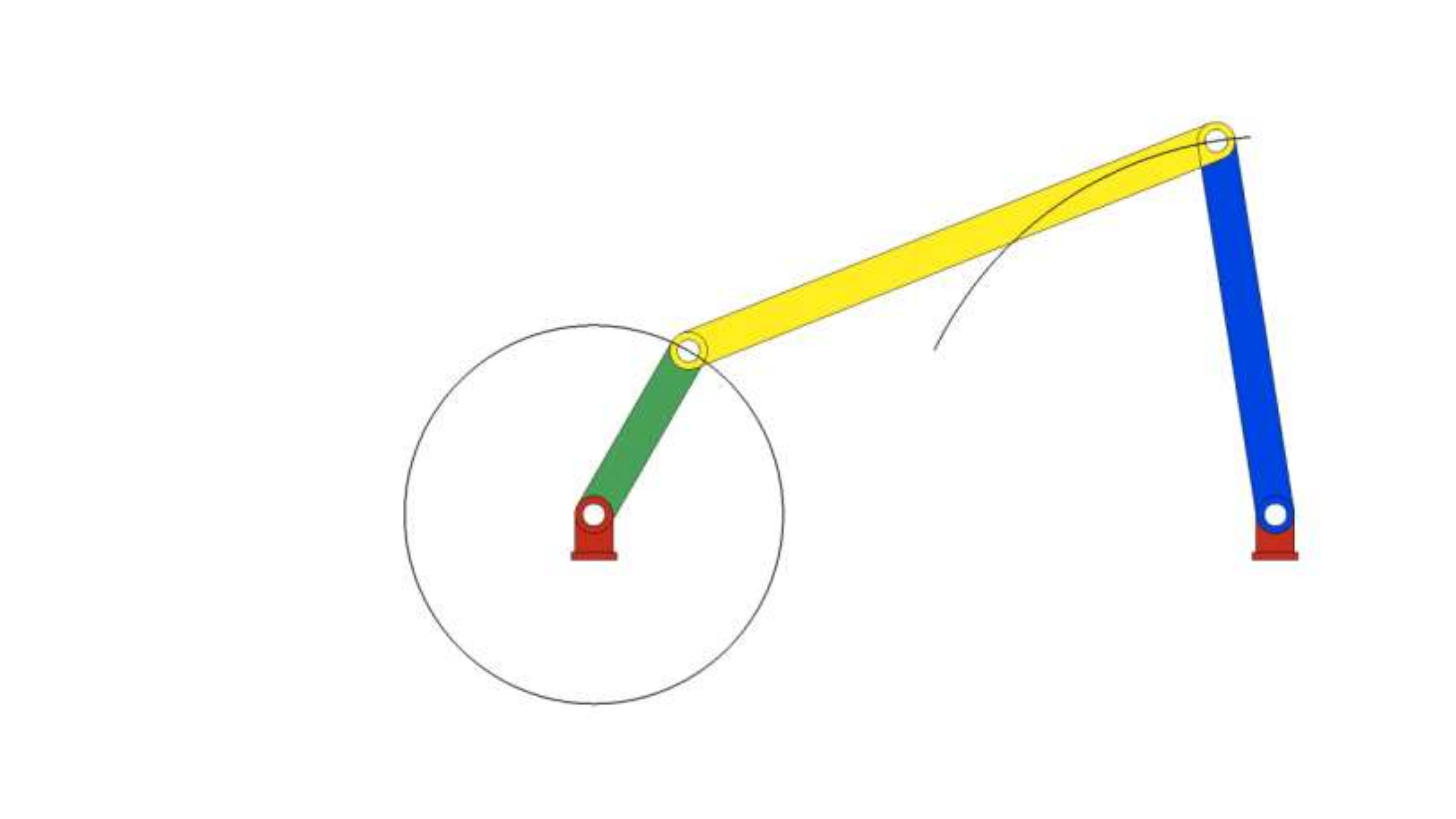
(b) Double-crank inversion (GCCC)
(drag link mechanism)

(c) Double-rocker inversion (GRCR)
(coupler rotates)

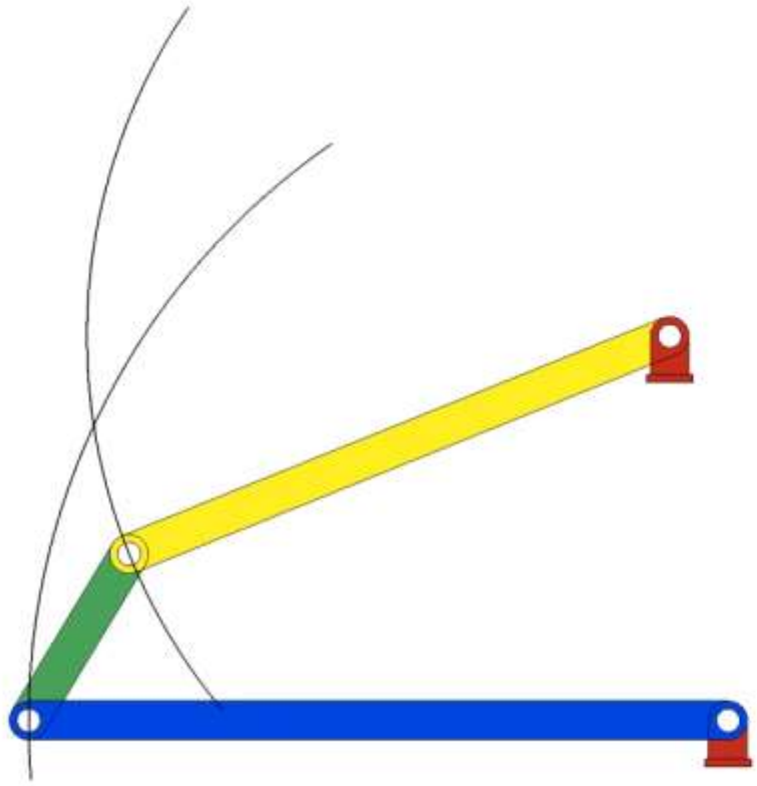
4Bar Inversions on SolidWorks



Inversion 1 animation

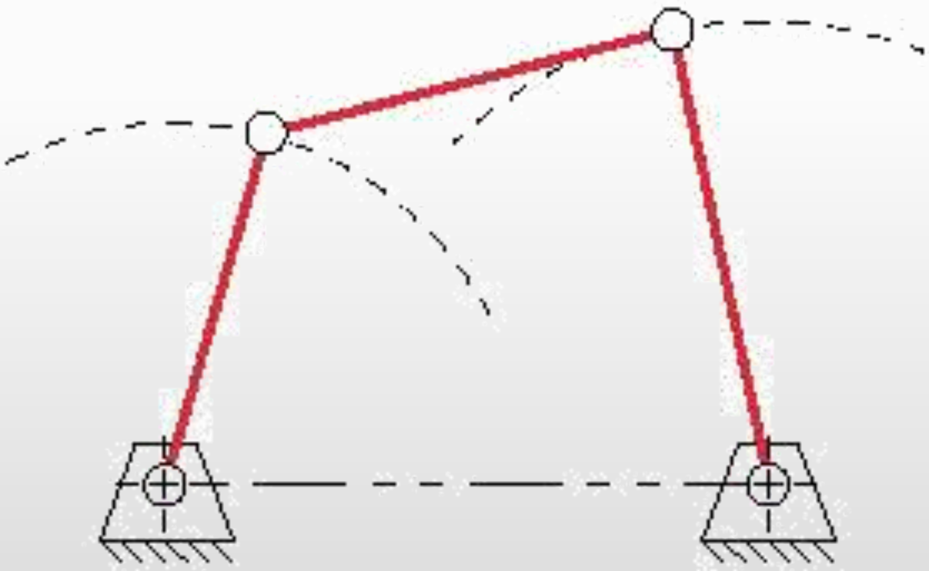


Inversion 2 animation

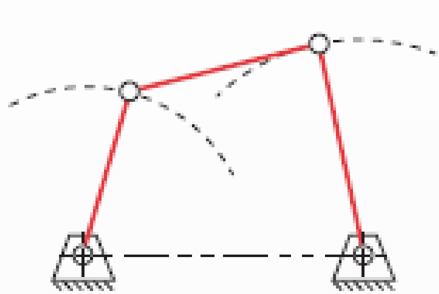


Type II – Non Grashof

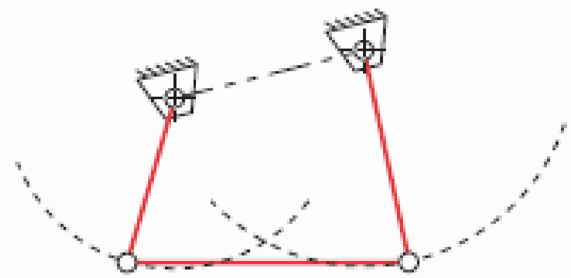
- $S + L > P + Q$
  - all inversions will be Triple-rockers



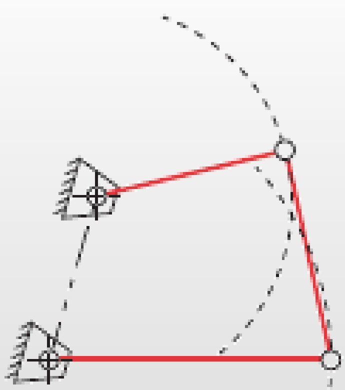
Non-Grashof linkages



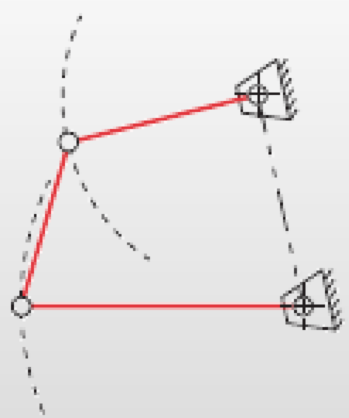
(a) Triple-rocker #1 (RRR1)



(b) Triple-rocker #2 (RRR2)



(c) Triple-rocker #3 (RRR3)

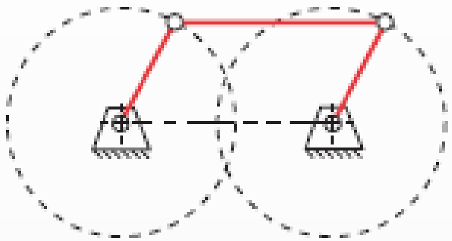


(d) Triple-rocker #4 (RRR4)

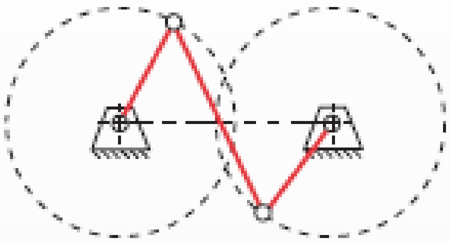
Type III – Grashof

- $S + L = P + Q$ (special case)
  - All inversions will be either Double-cranks or crank-rockers
  - But, they will have "change points" twice per revolution of the input crank when links become co–linear (toggle positions).
  - At these collinear positions, the linkage behavior is unpredictable as it may assume either of two configurations.
  - Motion must be limited to avoid reaching the change points

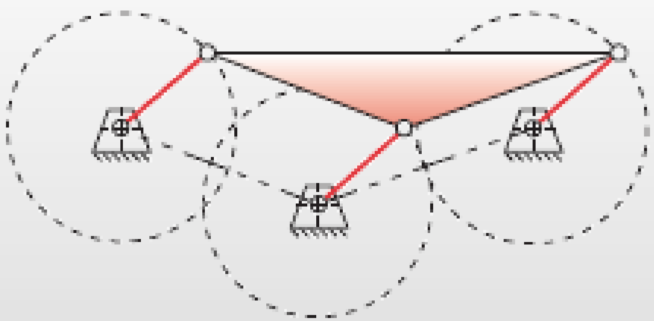
Special case Grashof



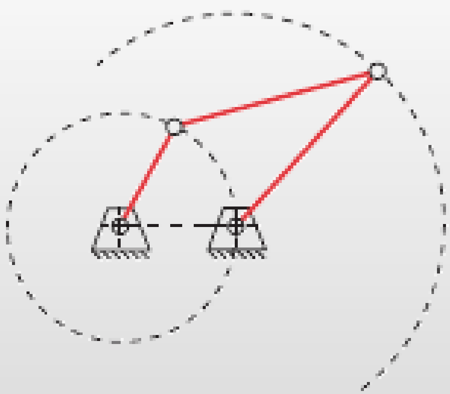
(a) Parallelogram form



(b) Antiparallelogram form



(c) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points



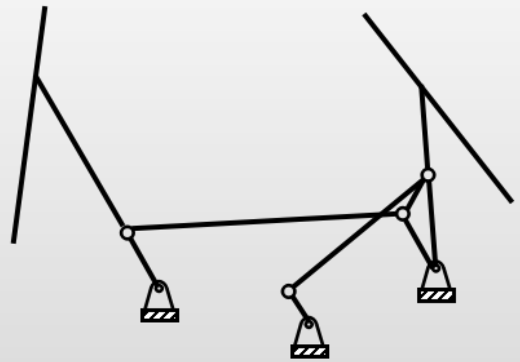
(d) Deltoid or kite form

Example 3.1

- A Fourbar Chain with the following Link Proportions: 30^{mm}, 70^{mm}, 90^{mm}, and 120^{mm}. Check the **Grashof** Condition
- S = 30 mm, L = 120 mm, P = 70 mm, and, Q = 90 mm
- S+L = 150 < P+Q = 160 thus the Linkage is a Grashof Four bar:
  1. If ground is the shortest → crank-crank-crank
  2. If the input is the shortest → crank-rocker-rocker
  3. If the coupler is the shortest → rocker-crank-rocker
  4. Output is the shortest → rocker-rocker-crank

Do we always need Grashof Condition?

- There is nothing either bad or good about the Grashof condition.
- If, for **example**, your need is for a motor driven windshield wiper linkage, you may want a **Type I** Grashof crank-rocker linkage in order to have a rotating link for the motor's input, plus a **Type III** parallelogram stage to couple the two sides together.



http://d2vlcm61l7u1fs.cloudfront.net/media%2Fff0%2Fff0c007d-bff3-4b92-84f6-031e19fcdd72%2FphpUC6raD.png

Do we always need Grashof Condition?

- If our need is to control the wheel motions of a car over bumps, you may want a non-Grashof triple-rocker linkage for short stroke oscillatory motion.

http://www.carbibles.com/images/coilspring2.jpg

Do we always need Grashof Condition?

- If you want to exactly duplicate some input motion at a remote location, you may want a special-case Grashof parallelogram linkage, as used in a drafting machine.

http://moziru.com/images/drawn-planks-kuhlmann-12.jpg