American University of Beirut — Fall 2007-2008
Elementary Linear Algebra (Math 218)
Lecturer: Dr. Wassim Daher
Section 3, MWF: 1:00 pm - 2:00 pm

**Final Exam, January 23 2008**

**Time: 2 hours**

*The quality of your writing, the clarity and the precision of your reasoning, all together will be taken into account in the correction process.* **Open book and Calculators are not allowed for this Final**

**Exercise 1.(10 Points)**

Let $L : \mathcal{M}_{22} \to \mathcal{M}_{22}$ be the linear operator defined by

$$L(M) = M^T$$

Find the matrix for $L$ with respect to the standard basis for $\mathcal{M}_{22}$

**Exercise 2.(10 Points)**

Find $(s,t)$ so $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} s \\ t \end{pmatrix}$ is close as possible to $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

**Exercise 3.(10 Points)**

Find an orthogonal basis for the subspace $w + 2x + 3y + 4z = 0$ of $\mathbf{R}^4$.

**Exercise 4.(15 Points)**

Let

$$A = \begin{pmatrix} -5 & 3 & 4 \\ -2 & 2 & 2 \\ -6 & 3 & 5 \end{pmatrix}$$

a) Find the eigenvalues of $A$? Is A diagonalizable?.

b) Find a basis to each eigenspace corresponding to the eigenvalues found in part a) and give its dimension.

c) calculate $A^5$.

**Exercise 5.(15 Points)**

Consider the two subspaces of $\mathbf{R}^3$:

$$E_1 = \{(x, y, z) \in \mathbf{R}^3, 3x - 2y + 3z = 0\},$$
$$E_2 = \{(x, y, z) \in \mathbf{R}^3, x + 3y - z = 0 \text{ and } 3x + 3y + z = 0\}$$

a) Find the bases $B_1$ and $B_2$ for $E_1$ and $E_2$ respectively.
b) Prove that $\mathbf{R}^3 = E_1 \oplus E_2$ i.e. **1)** $E_1 \cap E_2 = \{\overrightarrow{\mathbf{0}}_{\mathbf{R}^3}\}$ and **2)** $\mathbf{R}^3 = E_1 + E_2$

**Exercise 6.(10 Points)**

Consider the bases for $\mathcal{P}_1$, $B = \{p_1 = 6 + 3x, p_2 = 10 + 2x\}$ and $B' = \{q_1 = 2, q_2 = 3 + 2x\}$

a) Find the transition matrix from $B'$ to $B$
b) Find the transition matrix from $B$ to $B'$
c) Compute the coordinate vector $[p]_B$, where $p = -4 + x$ and then find $[p]_{B'}$

**Exercise 7.(20 Points)**

Let

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

a) Show that $A$ is diagonalizable

b) Find the matrix $P$ that orthogonally diagonalizes $A$.

c) Find an orthogonal basis for $\mathbf{R}^3$

**Exercise 8.(10 Points)**

Let $E$ be a finite dimensional vector space and let $f$ be a linear operator from $E$ to $E$. Show the following statement

$$Ker(f) = Ker(f \circ f) \Rightarrow Range(f) = Range(f \circ f)$$

**Good Luck**