Name: **Signature:** **Student Number:**

Mathematics 218, Section 3

Final Examination, June 25, 2002, 15:00–17:00

Important Instructions:

1. This exam consists of two sets of problems. The ten "TRUE/FALSE answer" questions are numbered A, B, ..., J; each of them is worth 4 pts. The eight "workout" questions are numbered 1, 2, ..., 8; each of them is worth 7 or 8 pts. The maximum score is 100 points.
2. Do not separate these pages.
3. The pink booklets are merely a source of scrap paper. I will not read what is in them.

If you have any comments/requests write them here: ***J. Nikiel***

Good Luck!

Part I.

Write your answer TRUE or FALSE under each of the following problems/statements:

Problem A. If $\vec{u}, \vec{v} \in \mathbb{R}^n$ are not orthogonal, then $\|\vec{u}\| \cdot \|\vec{v}\| \geq |\vec{u} \cdot \vec{v}|$.

Your answer:

Problem B. If A is a square matrix, then either the row vectors of A or the column vectors of A must be linearly independent.

Your answer:

Problem C. If $\det(A) = 1$, then A is an orthogonal matrix.

Your answer:

Problem D. If A is a matrix of size $n \times n$ and E is an elementary matrix of size $n \times n$, then A and EA must have the same nullspace.

Your answer:

Problem E. If A is a singular matrix, then A must have a row of 0's or a column of 0's.

Your answer:

Problem F. Each symmetric matrix can be diagonalized.

Your answer:

Problem G. If A and B are invertible matrices of the same size, then $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$.

Your answer:

Problem H. If the characteristic polynomial of a matrix A is $p(\lambda) = \lambda^n - 1$, then A is invertible.

Your answer:

Problem I. Two planes in $\mathbb{R}^4$ can have exactly one common point.

Your answer:

Problem J. If V is a subspace of $\mathbb{R}^n$ and W is a subspace of V, then $V^\perp$ is a subspace of $W^\perp$.

Your answer:

Problem 4. 7 pts.

Let V be a subspace of $\mathbb{R}^n$ with $\dim(V) \geq 2$. Let $c > 0$ be a constant. Show that V must contain infinitely many vectors of norm c.

Problem 3. 7 pts.

Let $T : V \to W$ be a linear transformation. Let $v_1, v_2, \ldots, v_k \in V$. Suppose that the vectors $T(v_1), T(v_2), \ldots, T(v_k)$ are linearly independent. Prove that then the vectors $v_1, v_2, \ldots, v_k$ are linearly independent.

Problem 2. 7 pts.

Let $A = [a_{ij}]_{n\times n}$ be an invertible matrix. Write down the formula for A^{-1} in terms of the cofactor expansion. You must explain all your notation.

Part II.

You must provide all essential details of your solution to each of the following problems 1, . . . , 8 on the page that contains the problem (continue on the reverse side of that page when needed).

Problem 1. 7 pts.

Prove that if λ is an eigenvalue of a square matrix $A = [a_{ij}]_{n\times n}$, then λ^5 is an eigenvalue of A^5.

Problem 5. 8 pts.

Find an orthonormal basis for the row space of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

Problem 6. 8 pts.

Find the vector form of the general solution of the following linear system $A\vec{x} = \vec{b}$. Then write the vector form of the general solution of $A\vec{x} = \vec{0}$.

$$\begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -1 & 3 \\ 2 & 3 & 2 & -1 \\ 1 & -3 & 5 & -4 \end{pmatrix} \cdot \vec{x} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ -3 \end{pmatrix}$$

Problem 7. 8 pts.

Find a matrix P that diagonalizes the following matrix A. Then find $P^{-1}AP$.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$