

Name:

Signature:

Student Number:

Mathematics 218, Section 3

Final Examination, June 25, 2002, 15:00–17:00

Important Instructions:

1. This exam consists of two sets of problems. The ten "TRUE/FALSE answer" questions are numbered A, B, . . . , J; each of them is worth 4 pts. The eight "workout" questions are numbered 1, 2, . . . , 8; each of them is worth 7 or 8 pts. The maximum score is 100 points.
2. Do not separate these pages.
3. The pink booklets are merely a source of scrap paper. I will not read what is in them.

If you have any comments/requests write them here:

J. Nikiel

Good Luck!

Part I.

Write your answer TRUE or FALSE under each of the following problems/statements:

Problem A. If $\vec{u}, \vec{v} \in \mathbb{R}^n$ are not orthogonal, then $\|\vec{u}\| \cdot \|\vec{v}\| \geq |\vec{u} \cdot \vec{v}|$.

Your answer:

Problem B. If A is a square matrix, then either the row vectors of A or the column vectors of A must be linearly independent.

Your answer:

Problem C. If $\det(A) = 1$, then A is an orthogonal matrix.

Your answer:

Problem D. If A is a matrix of size $n \times n$ and E is an elementary matrix of size $n \times n$, then A and EA must have the same nullspace.

Your answer:

Problem E. If A is a singular matrix, then A must have a row of 0's or a column of 0's.

Your answer:

Problem F. Each symmetric matrix can be diagonalized.

Your answer:

Problem G. If A and B are invertible matrices of the same size, then

$$(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}.$$

Your answer:

Problem H. If the characteristic polynomial of a matrix A is $p(\lambda) = \lambda^n - 1$, then A is invertible.

Your answer:

Problem I. Two planes in \mathbb{E}^3 can have exactly one common point.

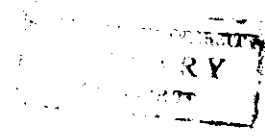
Your answer:

Problem J. If V is a subspace of \mathbb{R}^n and W is a subspace of V , then V^\perp is a subspace of W^\perp .

Your answer:

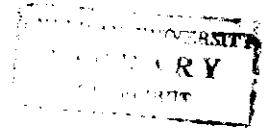
Problem 4. 7 pts.

Let V be a subspace of \mathbb{R}^n with $\dim(V) \geq 2$. Let $c > 0$ be a constant. Show that V must contain infinitely many vectors of norm c .



Problem 3. 7 pts.

Let $T : V \rightarrow W$ be a linear transformation. Let $v_1, v_2, \dots, v_k \in V$. Suppose that the vectors $T(v_1), T(v_2), \dots, T(v_k)$ are linearly independent. Prove that then the vectors v_1, v_2, \dots, v_k are linearly independent.



Problem 2. 7 pts.

Let $A = [a_{ij}]_{n \times n}$ be an invertible matrix. Write down the formula for A^{-1} in terms of the cofactor expansion. You must explain all your notation.

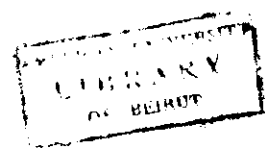


Part II.

You must provide all essential details of your solution to each of the following problems 1....8 on the page that contains the problem (continue on the reverse side of that page when needed).

Problem 1. 7 pts.

Prove that if λ is an eigenvalue of a square matrix $A = [a_{ij}]_{n \times n}$, then λ^5 is an eigenvalue of A^5 .



Problem 5. 8 pts.

Find an orthonormal basis for the row space of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

Problem 6. 8 pts.

Find the vector form of the general solution of the following linear system $A\vec{x} = \vec{b}$. Then write the vector form of the general solution of $A\vec{x} = \vec{0}$.

$$\begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -1 & 3 \\ 2 & 3 & 2 & -1 \\ 1 & -3 & 5 & -4 \end{pmatrix} \cdot \vec{x} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ -3 \end{pmatrix}$$



Problem 7. 8 pts.

Find a matrix P that diagonalizes the following matrix A . Then find $P^{-1}AP$.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$