Name: **Signature:** **Student Number:**

Dr. J. Nikiel : **Mathematics 218, Sections 1 and 2**

Final Examination, June 7, 2003, 15:00–17:00

Important Instructions:

1. This exam consists of two sets of problems. The ten "TRUE/FALSE answer" questions are numbered A, B, ..., J; each of them is worth 4 pts. The eight "workout" questions are numbered 1, 2, ..., 8; each of them is worth 8 pts. The maximum score is 104 points.

2. Do not separate these pages.

3. The pink booklets are merely a source of scrap paper. I will not read what is in them.

If you have any comments/requests write them here:

No calculators.

Good Luck!

Part I.

Write your answer TRUE or FALSE under each of the following problems/statements:

Problem A. If $\det(A) = 2$, then A is an orthogonal matrix.

Your answer:

Problem B. If A and B are square matrices of the same size, then $\det(A + B) = \det(A) + \det(B)$.

Your answer:

Problem C. If A and B are symmetric matrices of the same size, then $\alpha A + \beta B$ is a symmetric matrix for each choice of scalars α and β.

Your answer:

Problem D. If $\vec{v}$ is an eigenvector of a square matrix A, then the same $\vec{v}$ is an eigenvector of A^3.

Your answer:

Problem E. Let $\vec{v}_1 = (1,1,1,1)$, $\vec{v}_2 = (2,2,2,0)$, $\vec{v}_3 = (3,3,0,0)$ and $\vec{v}_4 = (4,0,0,0)$. Then $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a basis for $\mathbb{R}^4$.

Your answer:

Problem F. If A is a matrix, then each vector $\vec{v}$ belonging to the null-space of A must be orthogonal to each vector $\vec{w}$ belonging to the row space of A.

Your answer:

Problem G. If $\lambda = 1$ is an eigenvalue of a symmetric matrix A, then $\lambda = -1$ is an eigenvalue of A, too.

Your answer:

Problem H. If A and B are square matrices of the same size, then $(AB)^2 = A^2B^2$.

Your answer:

Problem I. If a square matrix A has a row of 1's or a column of 1's, then $\det(A) \neq 0$.

Your answer:

Problem J. If $\{u_1, u_2, \ldots, u_k\}$ is an orthogonal set of vectors in an inner product space V such that $\dim(V) = n$, then $n \leq k$.

Your answer:

Part II.

You must provide all essential details of your solution to each of the following problems $1, \ldots, 8$ on the page that contains the problem (continue on the reverse side of that page when needed).

<u>Problem 1.</u>

Use Cramer's Rule to solve the system $\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 1 & 0 & 1 \end{pmatrix} \cdot \vec{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$ Evaluate determinants in the denominators, but do <u>not</u> evaluate determinants in the nominators.

Problem 2.

Find a matrix P that diagonalizes the following matrix A. Then find $P^{-1}AP$.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

Problem 3.

Find the least squares solution of the linear system

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \cdot \vec{x} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$$

Problem 4.

Find an orthonormal basis for the row space of the following matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 2 & 4 & 2 \end{pmatrix}$$

Problem 5.

Let A be a matrix of size 6×5. Suppose that the null-space of A is spanned by the vectors $\vec{v}_1 = (1,1,1,0,0)$, $\vec{v}_2 = (0,0,1,1,1)$ and $\vec{v}_3 = (1,1,2,1,1)$. Use this information to find the rank of A.

Problem 6.

Let A be a square matrix of size $n \times n$. Write four conditions equivalent to "A is singular".

Problem 7.

Let V be an inner product space and W be a subspace of V. Let $T : V \to W$ denote the orthogonal projection of V onto W. Prove that $\ker(T) = W^{\perp}$.

Problem 8.

Let V_1 and V_2 be subspaces of $\mathbb{R}^7$. Let $V = V_1 \cap V_2$. Suppose that $\dim(V_1) = 4$ and $\dim(V_2) = 5$. Prove that V must be infinite.