American University of Beirut

MATH 218

Elementary Linear Algebra

Spring 2004

Final Exam

Date: Wednesday, June 9, 2004 - 11:30 am to 1:30 pm
Instructor: Dr. Mohamed Kobeissi

Name: ______________________________

ID #: ______________________________

Section: ______

This is **NOT** an open-book exam. Your exam should have 11 pages including this one, and there are 6 questions totaling 100 points. You can continue each exercise on the reverse side of the paper if needed.

Question	Grade
1.	
2.	
3.	
4.	
5.	
6.	

Good luck

Exercise 1 Let $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & k & 1 \\ -k & 1 & k \end{pmatrix}$

a. (*7 points*) For what value of k the matrix A is invertible?

b. (*3 points*) For $k = 2$, solve the system $AX = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Exercise 2 Let $x = (1, -1, 1, 1), y = (-1, -1, 1, -1), z = (0, 1, 1, 0)$, and let W =span$\{x, y, z\}$ be a subspace of $\mathbb{R}^4$.

a. (*5 points*) Show that ~~S~~ W is an orthogonal set of vectors relative to the Euclidean inner product.

b. (*3 points*) What is the dimension of W? Justify.

c. (*8 points*) Find a basis for $W^{\perp}$, the orthogonal complement of W.

d. (*4 points*) Find a vector $t \in \mathbb{R}^4$ such that the $\{x, y, z, t\}$ form a basis of $\mathbb{R}^4$. Justify.

Exercise 3 *(10 points)* Let W be the subspace of $\mathbb{R}^3$ spanned by $\{(1,1,2),(1,-1,1)\}$, and let

$$b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Find $\text{proj}_W b$, the orthogonal projection of b on W.

Exercise 4 Consider the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

a. (*4 points*) Find the characteristic equation of A.

b. (*8 points*) Find the eigenvalues of A, and a basis for each eigenspace of A.

c. *(10 points)* Find an orthogonal matrix P that diagonalize A, and show the relation between A, P, P^{-1} and D.

d. (*3 points*) What is the rank of A? Justify.

Exercise 5 In an inner product space V, prove the following:

a. *(5 points)* $< u, v >= \frac{1}{4}||u+v||^2 - \frac{1}{4}||u-v||^2$.

b. *(5 points)* $||u+v|| \leq ||u|| + ||v||$.

(hint: you may use the Cauchy-Schwarz inequality, $| < u, v > | \leq ||u||.||v||$*)*

Exercise 6 (*25 points, 5 points each*) Prove (concisely) or disprove (by a counter example), the following statements.

a. Let A be a $n \times n$ matrix.

i) If A is invertible, then the orthogonal complement of the row space of A is reduced to 0.

ii) If 0 is an eigenvalue of A^4, then A is invertible.

iii) If λ is an eigenvalue of A with corresponding eigenvector v, then λ^3 is an eigenvalue of A^3 with the same corresponding eigenvector.

b. The Nullity of a 3×4 matrix A can be equal to zero.

c. If a set of 3 vectors in $\mathbb{R}^3$ have the property that each 2 of them are linearly independent, then this set is linearly independent.