<u>Mathematics 218 Final Exam. (2 hrs.)</u>

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1. (24%) Prove (concisely) or disprove (by a counter example) in an inner product space.

(a) $4(u,v) = \|u+v\|^2 - \|u-v\|^2$

(b) If $\|u+v\|^2 = 4(u,v)$, then $u = v$.

(c) Non-zero orthogonal vectors are linearly independent.

(d) If two $m \times n$ matrices have the same null space, then they have the same row space.

(e) If two $m \times n$ matrices have the same null space, then they have the same column space.

(f) (For any square matrix A). A and A^t have the same null space.

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2. (10%) (a) What do we mean precisely by a least square solution of a non-consistent system $AX = b$? **AND** <u>(ii) how do we find them?</u>

(b) What do we know about symmetric $n \times n$ matrices regarding eigenvalues & diagonalization ?

(c) Apply the Cauchy-Schwarz inequality on functions in C[a, b] & vectors in R^n.

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3. (15%) Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(i) Find the eigen values of A <u>and a basis for each eigen space of A.</u>

(ii) Show that A is diagonalizable <u>and find the exact relation between A</u> and its diagonalization D. (Yes, calculate P^{-1}).

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4. (6%) Let T: V→W be a linear transformation of vector spaces. If $T(a_1), T(a_2), \dots, T(a_n)$ are linearly independent, show that $\{a_1, a_2, \dots, a_n\}$ are linearly independent.

5. (6%) Let T: V→W be a linear transformation of vector spaces. If T is 1-1 and dimV=dimW=n, show that T is onto. (Hint: You may use the rank-nullity theorem.

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6. (7%) Let $\{w_1, w_2, \dots, w_n, a_1, a_2, \dots, a_t\}$ be an o.n basis of an inner product space V. Let W= span $\{w_1, w_2, \dots, w_n\}$ and let A=span $\{a_1, a_2, \dots, a_t\}$. Show that $W^\perp = A$. Then deduce that $V = W \oplus W^\perp$.

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7. (6%) If {a, b, c} is a basis of a vector space V, show that {a+b, a+2b, b+c} is also a basis of V.

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8. (15%) Let $A = \begin{bmatrix} 1 & 2 & 1 & 3 & 6 \\ 2 & 4 & 0 & 2 & 2 \\ 0 & 0 & 1 & 2 & 5 \end{bmatrix}$

(i) Show that rank A=2

(ii) Prove or disprove that the first 2 rows of A form a basis for the row space of A.

(iii) Does the system $AX = B$ have a solution for every B in R^3? **Justify.**

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9. (6%) Suppose dimV=dimA+dimB and $A \cap B = 0$ where A and B are subspaces of V. Show that V=A+B.

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10. (5%) For any rectangular matrix A, show that A^t and $A^t A$ have the same column space.