INSTRUCTOR'S MANUAL TO ACCOMPANY

ELEMENTARY SURVEYING

AN INTRODUCTION TO GEOMATICS

THIRTEENTH EDITION

Charles D. Ghilani and Paul R. Wolf

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Preface

The Instructor's Manual has been prepared as a convenience for instructors who adopt, for use in their classes, the textbook ELEMENTARY SURVEYING (AN INTRODUCTION TO GEOMATICS), 13th Edition, by Charles D. Ghilani and Paul R. Wolf. As a benefit to the instructor, each problem consists of the book question and a derived solution. For most questions a reference to the equations, section, and/or paragraph containing the answer has been included together with a copy of the relevant material from the text. This is provided so that useful feedback and references to the appropriate parts of the book can be easily provided to the students. Every attempt has been made to provide correct responses to the questions, but most assuredly, a few mistakes may have occurred. Please accept our apologies for them and correct them in your printouts. Since there will be no reprints of the Instructors Manual, corrections will not be made to these files. However, any erratum that is found is the book will be graciously accepted by the author to update subsequent printings of this and future editions.

The companion website for this the book at http://www.pearsonhighered.com/ghilani contains the software WOLFPACK, MATRIX, and STATS. These software packages are discussed throughout the book were appropriate and can be used to check many of the solutions. These software packages are freeware, and can be freely downloaded and installed on your computer systems. However, they are only to be used for educational purposes and no support for using these packages is implied or given. Discussions in the book and the accompanying help file system should be sufficient for determining the proper procedures for using the software.

These software packages can be used by students to check their responses to numerical questions in the book, or to do some of the more numerically intensive problems in the book that would otherwise be extremely time-consuming. Additionally there is a software package called PROBLEM GENERATOR, which uses a normal random number generator and statistics to generate realistic data sets for a single observation, multiple observations, horizontal and vertical networks, traverse problems, GNSS networks, and two-dimensional conformal coordinate transformations. These software packages often only require coordinates of stations and the desired observations to be determined. The generated observations are randomly perturbed using a normal random number generator to create realistic data sets. This software can be used to create instructor-generated additional problems for the practice and examination.

Also on the companion website is a Mathcad® electronic book (e-book) to accompany most of the chapters in the book. Additionally, Mathcad worksheets have been developed for several map projections mentioned in the book, but not thoroughly discussed. For those who do not have Mathcad®, html files of these worksheets have also been placed on the companion website. Additionally, several MS Excel® spreadsheets which demonstrate these computations are on the companion website. All of the software on the companion website requires Windows 95 or later operating system and have accompanying help files, which discuss their use. Mathcad requires version 14.0 or higher.

Additionally, short videos of the solutions to some problems are available on the companion website for this book.

Charles D. Ghilani

PART I:
SOLUTIONS TO PROBLEMS

1 INTRODUCTION

1.1 Develop your personal definition for the practice of surveying.

Answers will vary by response. See Section 1.1 for book definitions.

1.2 Explain the difference between geodetic and plane surveys.

From Section 1.4,

In geodetic surveys the curved surface of the earth is considered by performing the computations on an ellipsoid (curve surface approximating the size and shape of the earth). In plane surveys, except for leveling, the reference base for fieldwork and computations is assumed to be a flat horizontal surface. The direction of a plumb line (and thus gravity) is considered parallel throughout the survey region, and all measured angles are presumed to be plane angles.

1.3 Describe some surveying applications in:

(a) Construction

In construction, surveying is used to locate the precise location of structures such as roads, buildings, bridges, and so forth. From the FIG definition of surveying, item 11: "The planning, measurement and management of construction works, including the estimation of costs. In application of the foregoing activities surveyors take into account the relevant legal, economic, environmental, and social aspects affecting each project."

(b) Mining

In mining, surveying is used to direct the locations of mining activities according to a systematic plan, to make sure mining occurs within the boundaries of the claim, to connect tunnels and shafts, and to provide legal records of mining activities.

(c) Agriculture

In agriculture, surveying is used to determine the acreage of fields, to locate lines of constant elevation for strip farming, to track harvesting machinery to enable the size of the harvest, and to track the position of the planting equipment to allow for precise applications of seeds and fertilizers. The field is known as high-precision agriculture.

1.4 List 10 uses for surveying other than property and construction surveying.

Some items students may lists include"

1. Establishing control for use in other surveys.
2. Mapping the surface of the Earth and other celestial objects with photogrammetry, laser scanning, or remote sensing.
3. Mapping archeological artifacts.

4. Mapping the bottom of oceans and waterways.
5. Creating Geographic and Land Information Systems for public use.
6. Performing ordinance surveys for the military.
7. Creating topographic maps.
8. Optical tooling.
9. Mapping of statues and other forms of artwork using terrestrial photogrammetry or laser scanning.
10. Mapping of accident sites in forensic surveying.

1.5 Why is it important to make accurate surveys of underground utilities?

To provide an accurate record of the locations of these utilities so they can be found if repairs or servicing is needed, and to prevent their accidental destruction during excavation for other projects.

1.6 Discuss the uses for topographic surveys.

Topographic surveys are used whenever elevation data is required in the end product. Some examples include (1) creating maps for highway design; (2) creating maps for construction surveys; (3) creating maps for flood plain delineation; (4) creating maps for site location of buildings; and so on.

1.7 What are hydrographic surveys, and why are they important?

From Section 1.6, hydrographic surveys define shorelines and depths of lakes, streams, oceans, reservoirs, and other bodies of water. *Sea surveying* is associated with port and offshore industries and the marine environment, including measurements and marine investigations made by ship borne personnel.

1.8 Name and briefly describe three different surveying instruments used by early Roman engineers.

From Section 1.3: (1) gromma, (2) libella, and (3) chorobates.

1.9 Briefly explain the procedure used by Eratosthenes in determining the Earth's circumference.

From Section 1-3, paragraph 8 of text: His procedure, which occurred about 200 B.C., is illustrated in Figure 1-2. Eratosthenes had concluded that the Egyptian cities of Alexandria and Syene were located approximately on the same meridian, and he had also observed that at noon on the summer solstice, the sun was directly overhead at Syene. (This was apparent because at that time of that day, the image of the sun could be seen reflecting from the bottom of a deep vertical well there.) He reasoned that at that moment, the sun, Syene, and Alexandria were in a common meridian plane, and if he could measure the arc length between the two cities, and the angle it subtended at the earth's center, he could compute the earth's circumference. He determined the angle by measuring the length of the shadow cast at Alexandria from a tall vertical staff of known length. The arc length was found from multiplying the number of caravan days between Syene and Alexandria by the average daily distance traveled. From these measurements Eratosthenes calculated the earth's circumference to be about 25,000 mi. Subsequent precise geodetic measurements using better instruments, but techniques similar geometrically to Eratosthenes', have shown his value, though slightly too large, to be amazingly close to the currently accepted one.

1.10 Describe the steps a land surveyor would need to do when performing a boundary survey.

Briefly, the steps should include (1) preliminary walking of property with owner; (2) courthouse research to locate deed of property and adjoiners to determine ownership, possible easements, right-of-ways, conflicts of interest, and so on; (3) location survey of property noting any encroachments; conflicting elements; and so on; (4) resolution of conflicting elements between deed and survey; (5) delivery of surveying report to owner.

1.11 Do laws in your state specify the accuracy required for surveys made to lay out a subdivision? If so, what limits are set?

Responses will vary

1.12 What organizations in your state will furnish maps and reference data to surveyors and engineers?

Responses will vary but some common organizations are the (1) county surveyor, (2) register of deeds, (3) county engineer, (4) Department of Transportation, (5) Department of Natural Resources of its equivalent, and so on.

1.13 List the legal requirements for registration as a land surveyor in your state.

Responses will vary. Contact with you licensing board can be found on the NCEES website at http://www.ncees.org/licensure/licensing_boards/.

1.14 Briefly describe the European Galileo system and discuss its similarities and differences with GPS.

See Section 13.10.2. Students can look this information and much more with a web search.

1.15 List at least five nonsurveying uses for GPS.

Responses may include (1) logistics in transportation; (2) hunting; (3) location of cell phone calls; (4) timing of telecommunications networks; (5) navigation in the boating industry; and so on.

1.16 Explain how aerial photographs and satellite images can be valuable in surveying.

Photogrammetry presently has many applications in surveying. It is used, for example, in land surveying to compute coordinates of section corners, boundary corners, or point of evidence that help locate these corners. Large–scale maps are made by photogrammetric procedures for many uses, one being subdivision design. Photogrammetry is used to map shorelines, in hydrographic surveying, to determine precise ground coordinates of points in control surveying, and to develop maps and cross sections for route and engineering surveys. Photogrammetry is playing an important role in developing the necessary data for modern Land and Geographic Information Systems.

1.17 Search the Internet and define a VLBI station. Discuss why these stations are important to the surveying community.

VLBI stands for *Very Long Baseline Interferometry*. Responses will vary. These stations provide extremely accurate locations on the surface of the Earth. The stations are used to develop world-wide reference frameworks such as ITRF00. They also may provide tracking information for satellites.

1.18 Describe how a GIS can be used in flood emergency planning.

Responses will vary but may mention the capabilities of a GIS to overlay soil type and their permeability with slopes, soil saturation, and watershed regions. A GIS can also be used to provide a list of business and residences that will be affected by possible flooding for evacuation purposes. It can provide "best" routes out of a flooded region.

1.19 Visit one of the surveying web sites listed in Table 1.1, and write a brief summary of its contents. Briefly explain the value of the available information to surveyors.

Responses will vary with time, but below are brief responses to the question

- NGS – control data sheets, CORS data, surveying software
- USGS – maps, software
- BLM – cadastral maps, software, ephemerides
- U.S. Coast Guard Navigation Center - GPS information
- U.S. Naval Observatory –Notice Advisory for NAVSTAR Users (NANU) and other GPS related links
- American Congress on Surveying and Mapping (ACSM) – professional organization for surveying and mapping profession
- American Society for Photogrammetry and Remote Sensing – professional organization for photogrammetry and remote sensing
- The Pennsylvania State University Surveying Program – Access to latest software that accompanies this book.

1.20 Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where GPS was used. Write a brief summary of the article.

Response will vary.

1.21 Same as Problem 1.20, except the article should be on safety as related to surveying.

Responses will vary.

2 UNITS, SIGNIFICANT FIGURES, AND FIELD NOTES

2.1 List the five types of measurements that form the basis of traditional plane surveying.

From Section 2.1, they are (1) horizontal angles, (2) horizontal distances, (3) vertical (altitude or zenith) angles, (4) vertical distances, and (5) slope (or slant) distances.

2.2 Give the basic units that are used in surveying for length, area, volume, and angles in
(a) The English system of units.

From Section 2.2:

length (U.S. survey ft or in some states m), area (sq. ft. or acres), volume (cu. ft. or cu. yd.), angle (sexagesimal)

(b) The SI system of units.

From Section 2.3:

length (m), area (sq. m. or hectare), volume (cu. m.), angle (sexagesimal, grad, or radian)

2.3 Why was the survey foot definition maintained in the United States?

From Section 2.2:

The survey foot definition was maintained in the United States because of the vast number of surveys performed prior to 1959. It would have been extremely difficult and confusing to change all related documents and maps that already existed. Thus the old standard, now call the U.S. survey foot, is still used today.

2.4 Convert the following distances given in meters to U.S. survey feet:

***(a)** 4129.574 m **<u>13,548.44 ft</u>**

(b) 738.296 m **<u>2422.23 ft</u>**

(c) 6048.083 m **<u>19,842.75 ft</u>**

2.5 Convert the following distances given in feet to meters:

***(a)** 537.52 ft **<u>163.836 m</u>**

(b) 9364.87 ft **<u>2854.418 m</u>**

(c) 4806.98 ft **<u>1465.170 m</u>**

2.6 Compute the lengths in feet corresponding to the following distances measured with a Gunter's chain:

***(a)** 10 ch 13 lk **<u>668.6 ft</u>**

(b) 6 ch 12 lk **<u>404 ft</u>**

(c) 24 ch 8 lk **<u>1589 ft</u>**

2.7 Express 95,748 ft^2 in:

*(a) acres <u>2.1981 ac</u>

(b) hectares <u>0.88953 ha</u>

(c) square Gunter's chains <u>21.981 sq. ch.</u>

2.8 Convert 5.6874 ha to:

(a) acres <u>14.054 ac</u>

(b) square Gunter's chains <u>140.54 sq. ch</u>

2.9 What are the lengths in feet and decimals for the following distances shown on a building blueprint:

(a) 30 ft 9-3/4 in. <u>30.81 ft</u>

(b) 12 ft 6-1/32 in. <u>12.50 ft</u>

2.10 What is the area in acres of a rectangular parcel of land measured with a Gunter's chain if the recorded sides are as follows:

*(a) 9.17 ch and 10.64 ch <u>9.76 ac</u>

(b) 12 ch 36 lk and 24 ch 28 lk <u>30.01 ac</u>

2.11 Compute the area in acres of triangular lots shown on a plat having the following recorded right-angle sides:

(a) 208.94 ft and 232.65 ft <u>0.55796 ac</u>

(b) 9 ch 25 lk and 6 ch 16 lk <u>2.85 ac</u>

2.12 A distance is expressed as 125,845.64 U.S. survey feet. What is the length in

*(a) international feet? <u>125,845.89 ft</u>

(b) meters? <u>38,357.828 m</u>

2.13 What are the radian and degree-minute-second equivalents for the following angles given in grads:

*(a) 136.00 grads <u>**122°24′**</u>

(b) 89.5478 grads <u>**80°35′35″**</u>

(c) 68.1649 grads <u>**61°20′54″**</u>

2.14 Give answers to the following problems in the correct number of significant figures:

*(a) sum of 23.15, 0.984, 124, and 12.5 <u>160.</u>

(b) sum of 36.15, 0.806, 22.4, and 196.458 <u>255.8</u>

(c) product of 276.75 and 33.7 <u>9330</u>

(d) quotient of 4930.27 divided by 1.29 <u>3820</u>

2.15 Express the value or answer in powers of 10 to the correct number of significant figures:

(a) 11,432 **$\underline{1.1432 \times 10^4}$**

(b) 4520 **$\underline{4.52 \times 10^3}$**

(c) square of 11,293 **$\underline{1.2753 \times 10^8}$**

(d) sum of (11.275 + 0.5 + 146.12) divided by 7.2 **$\underline{2.2 \times 10^1}$**

2.16 Convert the adjusted angles of a triangle to radians and show a computational check:

***(a)** 39°41′54″, 91°30′16″, and 48°47′50″ **0.692867, 1.59705,** and **0.851672**

0.6928666 + 1.597054 + 0.8516721 = 3.14059 check

(b) 82°17′43″, 29°05′54″, and 68°36′23″ **1.43632, 0.507862**, and **1.19741**

1.436324 + 0. 5078617 + 1.197407 = 3.14159 check

2.17 Why should a pen not be used in field notekeeping?

From Section 2.7: " Books so prepared will withstand damp weather in the field (or even a soaking) and still be legible, whereas graphite from a soft pencil, or ink from a pen or ballpoint, leaves an undecipherable smudge under such circumstances."

2.18 Explain why one number should not be superimposed over another or the lines of sketches.

From Section 2.7: This can be explained with the need for integrity since it would raise the issue of what are you hiding, legibility since the numbers are often hard to interpret when so written, or by clarity since the notes are being crowded.

***2.19** Explain why data should always be entered directly into the field book at the time measurements are made, rather than on scrap paper for neat transfer to the field book later.

From Section 2.7: Data should always be entered into the field book directly at the time of the measurements to avoid loss of data.

2.20 Why should a new day's work begin on a new page?

A new day's work should begin on a new page to provide a record of what work was accomplished each day and to document an changes in the field crew, weather, instrumentation, and so on.

2.21 Explain the reason for item 18 in Section 2.11 when recording field notes.

A zero should be placed before a decimal point for the sake of clarity.

2.22 Explain the reason for item 24 in Section 2.11 when recording field notes.

The need for a title, index, and cross-reference is to provide a clear path of where the work to find the notes for a specific project, even if some notes come from previous work.

2.23 Explain the reason for item 12 in Section 2.11 when recording field notes.

Explanatory notes are essential to provide office personnel with an explanation for something unusual and to provide a reminder in later reference to the project.

2.24 When should sketches be made instead of just recording data?

Sketches should be made instead of recording data anytime observations need to be clarified so that the personnel interpreting the notes can have a clear understanding of the field conditions. This also serves as a reminder of the work performed and any unusual conditions in later references to the project.

2.25 Justify the requirement to list in a field book the makes and serial numbers of all instruments used on a survey.

Listing the makes and serial numbers of the instruments used in the survey may help isolate instrumental errors later when reviewing the project.

2.26 Discuss the advantages of survey controllers that can communicate with several different types of instruments.

The ability of survey controllers to communicate with several different types of instruments allows the surveyor to match the specific conditions of the project with the instrument that this is ideally suited for the job. Thus total station, digital levels, and GNSS receivers can all be used in a single project.

2.27 Discuss the advantages of survey controllers.

From Section 2.15: " The major advantages of automatic data collection systems are that (**1**) mistakes in reading and manually recording observations in the field are precluded, and (**2**) the time to process, display, and archive the field notes in the office is reduced significantly. Systems that incorporate computers can execute some programs in the field, which adds a significant advantage. As an example, the data for a survey can be corrected for systematic errors and misclosures computed, so verification that a survey meets closure requirements is made before the crew leaves a site."

2.28 Search the Internet and find at least two sites related to

(a) Manufacturers of survey controllers.

(b) Manufacturers of total stations.

(c) Manufacturers of global navigation satellite system (GNSS) receivers.

<u>Answers should vary with student.</u>

2.29 What advantages are offered to field personnel if the survey controller provides a map of the survey?

This allows field personnel to view what has been accomplished and look for areas of the map that need more attention.

2.30 Prepare a brief summary of an article from a professional journal related to the subject matter of this chapter.

Answer should vary by student.

2.31 Describe what is meant by the phrase "field-to-finish."

From Section 2.15, " These field codes can instruct the drafting software to draw a map of the data complete with lines, curves and mapping symbols. The process of collecting field data with field codes that can be interpreted later by software is known as a *field-to-finish* survey. This greatly reduces the time needed to complete a project."

2.32 Why are sketches in field books not usually drawn to scale?

This is true since this would require an overwhelming amount of time. The sketches are simply to provide readers of the notes an approximate visual reference to the measurements.

2.33 Create a computational program that solves Problem 2.16.

Answers to this problem should vary with students.

3 THEORY OF ERRORS IN OBSERVATIONS

3.1 Explain the difference between *direct* and *indirect observations* in surveying. Give two examples of each.

From Section 3.2: A direct observation is made by applying a measurement instrument directly to a quantity to be measured and an indirect observation is made by computing a quantity from direct observations.

Examples should vary by student response.

3.2 Define the term *systematic error*, and give two surveying examples of a systematic error.

See Section 3.6

3.3 Define the term *random error*, and give two surveying examples of a random error.

See Section 3.6

3.4 Explain the difference between accuracy and precision.

See Section 3.7

3.5 Discuss what is meant by the precision of an observation.

See Section 3.7

A distance *AB* is observed repeatedly using the same equipment and procedures, and the results, in meters, are listed in Problems 3.6 through 3.10. Calculate **(a)** the line's most probable length, **(b)** the standard deviation and **(c)** the standard deviation of the mean for each set of results.

***3.6** 65.401, 65.400, 65.402, 65.396, 65.406, 65.401, 65.396, 65.401, 65.405, and 65.404

(a) **<u>65.401</u>** $\sum 654.012$

(b) **<u>±0.003</u>** $\sum v^2 = 0.000104$

(c) **<u>±0.001</u>**

3.7 Same as Problem 3.6, but discard one observation, 65.396.

(a) **<u>65.402</u>** $\sum 588.616$

(b) **<u>±0.003</u>** $\sum v^2 = 0.000072$

(c) **<u>±0.001</u>**

3.8 Same as Problem 3.6, but discard two observations, 65.396 and 65.406.

(a) **65.402** $\sum 523.210$

(b) **±0.003** $\sum v^2 = 0.00007168$

(c) **±0.001**

3.9 Same as Problem 3.6, but include two additional observations, 65.398 and 65.408.

(a) **65.401** $\sum 784.818$

(b) **±0.004** $\sum v^2 = 0.000157$

(c) **±0.001**

3.10 Same as Problem 3.6, but include three additional observations, 65.398, 65.408, and 65.406.

(a) **65.402** $\sum 850.224$

(b) **±0.004** $\sum v^2 = 0.0001757$

(c) **±0.001**

In Problems 3.10 through 3.14, determine the range within which observations should fall **(a)** 90% of the time and **(b)** 95% of the time. List the percentage of values that actually fall within these ranges.

(a) $E_{90} = 1.6449\sigma$ (3.7) (b) $E_{95} = 1.9599\sigma$ (3.8)

3.11 For the data of Problem 3.6.

***(a)** **65.4012±0.0055 (65.3957, 65.4067), 100%**

(b) **65.4012±0.0066 (65.3946, 65.4078), 100%**

3.12 For the data of Problem 3.7.

(a) **65.4018±0.0049 (65.3968, 65.4067), 88.9%**

(b) **65.4018±0.00059 (65.6959, 65.4076), 100%**

3.13 For the data of Problem 3.8.

(a) **65.4012±0.0045 (65.3968, 65.4057), 87.5%**

(b) **65.4012±0.0053 (65.3959, 65.4066), 100%**

3.14 For the data of Problem 3.9.

(a) **65.4012±0.0062 (65.3968, 65.4077), 91.6%**

(b) **65.4012±0.0074 (65.3940, 65.4089), 100%**

In Problems 3.15 through 3.17, an angle is observed repeatedly using the same equipment and procedures. Calculate **(a)** the angle's most probable value, **(b)** the standard deviation, and **(c)** the standard deviation of the mean.

***3.15** $23°30'00''$, $23°29'40''$, $23°30'15''$, and $23°29'50''$.

(a) 23°29′56″

(b) ±14.9″

(c) ±7.5″

3.16 Same as Problem 3.15, but with three additional observations, $23°29'55''$, $23°30'05''$, and $23°30'20''$.

(a) 23°30′01″

(b) ±14.0″

(c) ±5.3″

3.17 Same as Problem 3.16, but with two additional observations, $23°30'05''$ and $23°29'55''$.

(a) 23°30′56″

(b) ±12.4″

(c) ±4.1″

***3.18** A field party is capable of making taping observations with a standard deviation of $\pm 0.010\ \text{ft}$ per 100-ft tape length. What standard deviation would be expected in a distance of 200 ft taped by this party?

By Equation (3.12): **±0.014 ft** $= 0.010\sqrt{2}$

3.19 Repeat Problem 3.18, except that the standard deviation per 30-m tape length is $\pm 0.003\ \text{m}$ and a distance of 120 m is taped. What is the expected 95% error in 120 m?

By Equation (3.12): **±0.006 m** $= 0.003\sqrt{4}$;

By Equation (3.8): **±0.018** $= 1.9559(0.006)$

3.20 A distance of 200 ft must be taped in a manner to ensure a standard deviation smaller than $\pm 0.04\ \text{ft}$. What must be the standard deviation per 100 ft tape length to achieve the desired precision?

±0.028 ft $= \pm 0.04/\sqrt{2}$ by Equation (3.12) rearranged.

3.21 Lines of levels were run requiring n instrument setups. If the rod reading for each backsight and foresight has a standard deviation σ, what is the standard deviation in each of the following level lines?

(a) $n = 26, \sigma = \pm 0.010\ \text{ft}$ By Equation (3.12): **±0.051 ft** $= 0.010\sqrt{26}$

(b) $n = 36, \sigma = \pm 3$ mm By Equation (3.12): **±18 m** = $3\sqrt{36}$

3.22 A line *AC* was observed in 2 sections *AB* and *BC*, with lengths and standard deviations listed below. What is the total length *AC*, and its standard deviation?

***(a)** $AB = 60.00 \pm 0.015$ ft; $BC = 86.13 \pm 0.018$ ft ; **146.13±0.023 ft** by Equation (3.11)

(b) $AB = 60.000 \pm 0.008$ m; 35.413 ± 0.005 m ; **95.413±0.0094 m** by Equation (3.11)

3.23 Line *AD* is observed in three sections, *AB, BC*, and *CD*, with lengths and standard deviations as listed below. What is the total length *AD* and its standard deviation?

(a) $AB = 572.12 \pm 0.02$ ft; $BC = 1074.38 \pm 0.03$ ft; $CD = 1542.78 \pm 0.05$ ft

3189.28 ± 0.062 ft by Equation (3.11)

(b) $AB = 932.965 \pm 0.009$ m; $BC = 945.030$ m ± 0.010 m; $CD = 652.250$ m ± 0.008 m

2530.245 ± 0.016 m by Equation (3.11)

3.24 A distance *AB* was observed four times as 236.39, 236.40, 236.36 and 236.38 ft. The observations were given weights of 2, 1, 3 and 2, respectively, by the observer. ***(a)** Calculate the weighted mean for distance *AB*. (**b**) What difference results if later judgment revises the weights to 2, 1, 2, and 3, respectively?

By Equation (3.17):

***(a)** **236.3775 ft**

(b) **236.3850 ft**

3.25 Determine the weighted mean for the following angles:

By Equation (3.17):

(a) 89°42′45″, wt 2; 89°42′42″, wt 1; 89°42′44″, wt 3 **89°42′44″**

(b) 36°58′32″ ± 3″; 36°58′28″ ± 2″; 36°58′26″ ± 3″ ; 36°58′30″ ± 1″ **36°58′29.5″**

3.26 Specifications for observing angles of an *n*-sided polygon limit the total angular misclosure to *E*. How accurately must each angle be observed for the following values of *n* and *E*?

By rearranged Equation (3.12):

(a) $n = 10, E = 8''$ **±2.5″**

(b) $n = 6, E = 14''$ **±5.7″**

3.27 What is the area of a rectangular field and its estimated error for the following recorded values:

By Equation (3.13):

***(a)** 243.89 ± 0.05 ft, by 208.65 ± 0.04 ft **50,887 ± 14 ft²** or **1.1682 ± 0.0003 ac**

(b) 725.33 ± 0.08 ft by 664.21 ± 0.06 ft **481,770 ± 70 ft²** or **11.060 ± 0.002 ac**

(c) 128.526 ± 0.005 m, by 180.403 ± 0.007 m **23,186.5 ± 1.3 m²** or **20.1865 ± 0.0001 ha**

3.28 Adjust the angles of triangle *ABC* for the following angular values and weights:

By Equation (3.17):

***(a)** $A = 49°24'22''$, wt 2; $B = 39°02'16''$, wt 1; $C = 91°33'00''$, wt 3

Misclosure = −22″

	Obs. Ang.	Wt	Corr.	Num. Cor.	Rnd. Cor.	Adj. Ang.
A	49°24′22″	2	3x	6″	6″	**49°24′28″**
B	39°02′16″	1	6x	12″	12″	**39°02′28″**
C	91°33′00″	3	2x	4″	4″	**91°33′04″**
	179°59′38″	6	11x			

11x = 22″ x = 2″

(b) $A = 80°14'04''$, wt 2; $B = 38°37'47''$, wt 1; $C = 61°07'58''$, wt 3

Misclosure = −11″

	Obs. Ang.	Wt	Corr.	Num. Cor.	Rnd. Cor.	Adj. Ang.
A	80°14′04″	2	3x	3″	3″	80°14′07″
B	38°37′47″	1	6x	6″	6″	38°37′53″
C	61°07′58″	3	2x	2″	2″	61°08′00″
	179°59′49″	6	11x			

11x =11″ x = 1″

3.29 Determine relative weights and perform a weighted adjustment (to the nearest second) for angles *A, B*, and *C* of a plane triangle, given the following four observations for each angle:

Angle *A*	Angle *B*	Angle *C*
38°47′58″	71°22′26″	69°50′04″
38°47′44″	71°22′22″	69°50′16″
38°48′12″	71°22′12″	69°50′30″
38°48′02″	71°22′12″	69°50′10″

A = 38°48′29″ ± 26″; ***B*** = 71°22′18″ ± 7″; ***C*** = 69°50′15″ ± 11″

Misclosure = +62″

	Obs. Ang.	Wt	Corr. Multiplier	Num. Cor.	Rnd. Cor.	Adj. Ang.
A	38°48′29″	0.001479	0.029952/wt = 20.25	−49.4″	−49″	38°47′40″
B	71°22′18″	0.020408	0.029952/wt = 1.47	−3.6″	−4″	71°22′14″
C	69°50′15″	0.008064	0.029952/wt= 3.71	−9.0″	−9″	69°50′06″
	180°01′02″	0.029952	25.43x			

25.43x = −62″ x = −2.44″

3.30 A line of levels was run from benchmarks *A* to *B*, *B* to *C*, and *C* to *D*. The elevation differences obtained between benchmarks, with their standard deviations, are listed below. What is the difference in elevation from benchmark *A* to *D* and the standard deviation of that elevation difference?

(a) BM *A* to BM $B = +34.65 \pm 0.10$ ft; BM *B* to BM $C = -48.23 \pm 0.08$ ft; and BM *C* to BM $D = -54.90 \pm 0.09$ ft

By Equation (3.11): **<u>−68.48 ± 0.16 ft</u>**

(b) BM *A* to BM $B = +27.823 \pm 0.015$ m; BM *B* to BM $C = +15.620 \pm 0.008$ m; and BM *C* to BM $D = +33.210 \pm 0.011$ m

By Equation (3.11): **<u>76.653 ± 0.020 m</u>**

(c) BM *A* to BM $B = -32.688 \pm 0.015$ m; BM *B* to BM $C = +5.349 \pm 0.022$ m; and BM C to BM $D = -15.608 \pm 0.006$ m

By Equation (3.11): **<u>42.947 ± 0.027 m</u>**

Solutions for the following problems should vary by student.

3.31 Create a computational program that solves Problem 3.9.

3.32 Create a computational program that solves Problem 3.17.

3.33 Create a computational program that solves Problem 3.29.

4 LEVELING THEORY, METHODS, AND EQUIPMENT

4.1 Define the following leveling terms: (**a**) vertical line, (**b**) level surface, and (**c**) leveling.

From Section 4.2:

(a) A vertical line is line that follows the local direction of gravity as indicated by a plumb line.

(b) A level surface is a curved surface that at every point is perpendicular to the local plumb line (the direction in which gravity acts).

(c) Leveling is the process of finding elevations of points or their differences in elevation.

4.2* How far will a horizontal line depart from the Earth's surface in 1 km? 5 km? 10 km? (Apply both curvature and refraction)

1 km? $C_m = 0.0675(1)^2 = 0.068$ m

5 km? $C_m = 0.0675(5)^2 = 1.688$ m

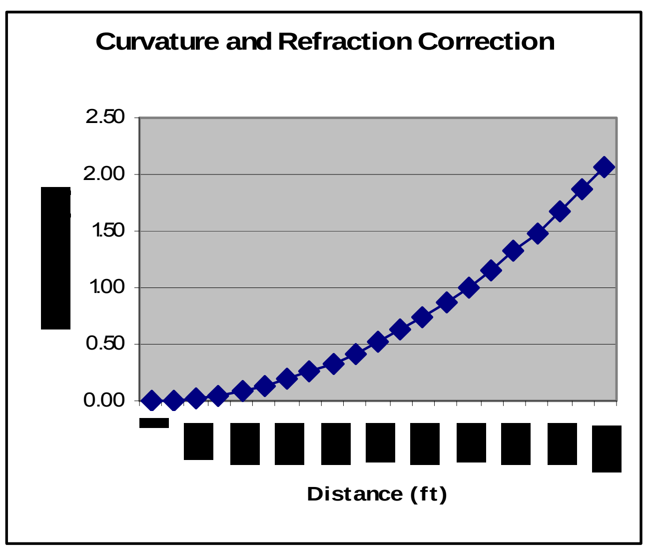
10 km? $C_m = 0.0675(10)^2 = 6.750$ m

4.3 Visit the website of the National Geodetic Survey, and obtain a data sheet description of a benchmark in your local area.

Solutions should vary.

4.4 Create plot of the curvature and refraction correction for sight lines going from 0 ft to 10,000 ft in 500 ft increments.

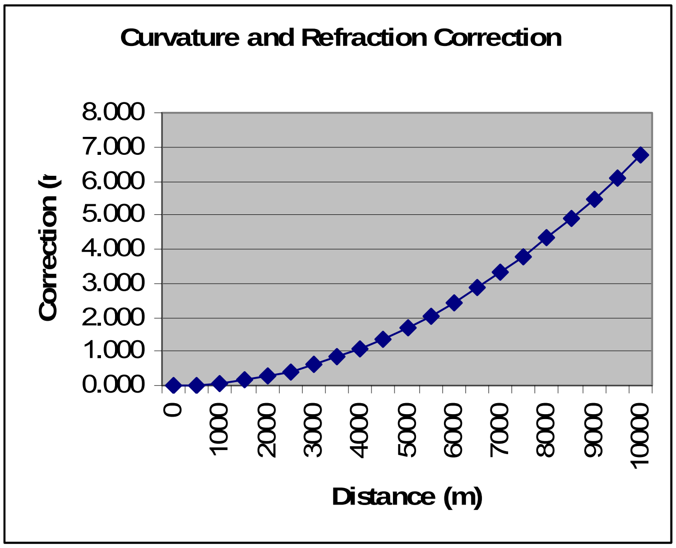
Sight (ft)	CR (ft)
0	0.00
500	0.01
1000	0.02
1500	0.05
2000	0.08
2500	0.13
3000	0.19
3500	0.25
4000	0.33
4500	0.42
5000	0.52
5500	0.62
6000	0.74



6500	0.87
7000	1.01
7500	1.16
8000	1.32
8500	1.49
9000	1.67
9500	1.86
10000	2.06

4.5 Create a plot of curvature and refraction corrections for sight lines going from 0 m to 10,000 m in 500 m increments.

Sight (m)	CR (m)
0	0.000
500	0.017
1000	0.068
1500	0.152
2000	0.270
2500	0.422
3000	0.608
3500	0.827
4000	1.080
4500	1.367
5000	1.688
5500	2.042
6000	2.430
6500	2.852
7000	3.308
7500	3.797
8000	4.320
8500	4.877
9000	5.468
9500	6.092
10000	6.750



4.6 Why is it important for a benchmark to be a stable, relatively permanent object?

It is important for a benchmark to be a stable, relatively permanent object so that is elevation will remain constant and useable over a long period of time.

4.7* On a large lake without waves, how far from shore is a sailboat when the top of its 30-ft mast disappears from the view of a person lying at the water's edge?

$$F = 1000\sqrt{\frac{30}{0.0206}} = 38{,}161 \text{ ft} = 7.228 \text{ mi}$$

4.8 Similar to Problem 4.7, except for a 8-m mast and a person whose eye height is 1.5 m above the water's edge.

$$K = \sqrt{\frac{1.5}{0.0675}} + \sqrt{\frac{8}{0.0675}} = 15.6 \text{ km}$$

4.9 Readings on a line of differential levels are taken to the nearest 0.01 ft. For what maximum distance can the Earth's curvature and refraction be neglected?

$$F = 1000\sqrt{\frac{0.005}{0.0206}} = 492.7 \text{ ft}$$

4.10 Similar to Problem 4.9 except readings are to the 3 mm.

$$m = 1000\sqrt{\frac{0.0015}{0.0675}} = 149 \text{ m}$$

4.11 Describe how readings are determined in a digital level when using a bar coded rod.

From Section 4.11:

"At the press of a button, the image of bar codes in the telescope's field of view is captured and processed. This processing consists of an on-board computer comparing the captured image to the rod's entire pattern, which is stored in memory. When a match is found, which takes about 4 sec, the rod reading is displayed digitally."

Successive plus and minus sights taken on a downhill line of levels are listed in Problems 4.12 and 4.13. The values represent the horizontal distances between the instrument and either the plus or minus sights. What error results from curvature and refraction?

4.12* 20, 225; 50, 195; 40, 135; 30, 250 ft.

Plus	CR	Minus	CR
20	0.00000824	225	0.001043
50	0.0000515	195	0.000783
40	0.00003296	135	0.000375
30	0.00001854	250	0.001288
Sum	0.00011124		0.003489

Combined **−0.003 ft**

4.13 5, 75; 10, 60; 10, 55; 15, 70 m.

Plus	CR	Minus	CR
5	1.6875E-06	75	0.00038
10	0.00000675	60	0.000243
10	0.00000675	55	0.000204
15	1.5188E-05	70	0.000331
	3.0375E-05		0.001158

Combined −1.1 mm

4.14 What error results if the curvature and refraction correction is neglected in trigonometric leveling for sights: (**a**) 3000 ft long (**b**) 500 m long (**c**) 5000 ft long?

(a) $h_f = 0.0206\left(\frac{3000}{1000}\right)^2 = 0.185 \text{ ft}$

(b) $h_m = 0.0675\left(\frac{500}{1000}\right)^2 = 0.017 \text{ m} = 16.9 \text{ mm}$

(c) $h_f = 0.0206\left(\frac{5000}{1000}\right)^2 = 0.515 \text{ ft}$

4.15* The slope distance and zenith angle observed from point *P* to point *Q* were 2013.875 m and 95°13′04″, respectively. The instrument and rod target heights were equal. If the elevation of point *P* is 88.988 m, above datum, what is the elevation of point *Q*?

$$Elev_Q = 188.988 + 2013.875\cos(95°13'04'') + 0.0675\left(\frac{2013.875\sin(95°13'04'')}{1000}\right)^2$$

$$= 188.988 - 183.145 + 0.271$$

$$= 6.114 \text{ m}$$

4.16 The slope distance and zenith angle observed from point *X* to point *Y* were 5401.85 ft and 83°53′16″. The instrument and rod target heights were equal. If the elevation of point *X* is 2045.66 ft above datum, what is the elevation of point *Y*?

$$Elev_Y = 2045.66 + 5401.85\cos(83°53'16'') + 0.0206\left(\frac{5401.85\sin(83°53'16'')}{1000}\right)^2$$

$$= 2045.66 + 575.168 + 0.594 = 2621.42 \text{ ft}$$

$$= 6.114 \text{ m}$$

4.17 Similar to Problem 4.15, except the slope distance was 854.987 m, the zenith angle was 82°53′48″, and the elevation of point *P* was 354.905 m above datum.

$$Elev_Q = 354.905 + 854.987\cos(82°53'48'') + 0.0675\left(\frac{854.987\sin(82°53'48'')}{1000}\right)^2$$

$$= 354.905 + 105.727 + 0.0486$$

$$= 460.681 \text{ m}$$

4.18 In trigonometric leveling from point *A* to point *B*, the slope distance and zenith angle measured at *A* were 2504.897 m and 85°08′54″ At *B* these measurements were 2504.891 m and 94°52′10″ respectively. If the instrument and rod target heights were equal, calculate the difference in elevation from *A* to *B*.

$$Z_{avg} = \frac{85°08'54'' + 180° - 94°52'10''}{2} = 85°08'22''$$

$$S_{avg} = \frac{2504.897 + 2504.891}{2} = 2504.894$$

$$\Delta Elev = 2504.894\cos(85°08'22'') = 212.242$$

4.19 Describe how parallax in the viewing system of a level can be detected and removed.

From Section 4.7:

"After focusing, if the cross hairs appear to travel over the object sighted when the eye is shifted slightly in any direction, *parallax* exists. The objective lens, the eyepiece, or both must be refocused to eliminate this effect if accurate work is to be done."

4.20 What is the sensitivity of a level vial with 2-mm divisions for: (**a**) a radius of 40 m (**b**) a radius of 10 m?

(**a**) $\theta = \left[\dfrac{2}{40(1000)}\right]206264.8 = 10.3''$

(**b**) $\theta = \left[\dfrac{2}{10(1000)}\right]206264.8 = 41.3''$

4.21* An observer fails to check the bubble, and it is off two divisions on a 250-ft sight. What error in elevation difference results with a 10-sec bubble?

angular error = 2(10) = 20 sec

Error = 250 tan(20) = **<u>0.024 ft</u>**

4.22 An observer fails to check the bubble, and it is off two divisions on a 100-m sight. What error results for a 20-sec bubble?

angular error = 2(20) = 40 sec

Error = 100 tan(40) = **<u>0.019 m</u>**

4.23 Similar to Problem 4.22, except a 10-sec bubble is off three divisions on a 130-m sight.

angular error = 3(10) = 30 sec

Error = 130 tan(30) = **<u>0.019 m</u>**

4.24 With the bubble centered, a 150-m-length sight gives a reading of 1.208 m. After moving the bubble four divisions off center, the reading is 1.243 m. For 2-mm vial divisions, what is: (**a**) the vial radius of curvature in meters (**b**) the angle in seconds subtended by one division?

Δrdg = 1.243 − 1.208 = 0.035 m

$\text{angle} = \tan^{-1}\left(\dfrac{0.035}{150}\right) = 9.6''$

(**a**) $R = \dfrac{0.002}{\tan(9.6'')} = 42.857 \text{ m}$

(**b**) $9.6''/4 = 2.4''$

4.25 Similar to Problem 4.24, except the sight length was 300 ft, the initial reading was 4.889 ft, and the final reading was 5.005 ft.

$\Delta \text{rdg} = 5.005 - 4.889 = 0.116 \text{ ft}$

$$\text{angle} = \tan^{-1}\left(\frac{0.116}{300}\right) = 79.8''$$

(a) $R = \dfrac{0.002}{\tan(79.8'')} = 5.172 \text{ m}$

(b) $79.8''/4 = 20''$

4.26 Sunshine on the forward end of a $20''/2 \text{ mm}$ level vial bubble draws it off two divisions, giving a plus sight reading of 1.632 m on a 100-m shot. Compute the correct reading.

Correction = 100tan(40″) = 0.0194 m

Correct reading = 1.632 − 0.019 = 1.613 m

Note: the correction is subtracted since the bubble was drawn off on the forward end of the level, thus raising the line of sight.

4.27 List in tabular form, for comparison, the advantages and disadvantages of an automatic level versus a digital level.

See Section 4.10 and 4.11.

4.28* If a plus sight of 3.54 ft is taken on BM *A*, elevation 850.48 ft, and a minus sight of 7.84 ft is read on point *X*, calculate the HI and the elevation of point *X*.

HI = 850.48 + 3.54 = **<u>854.02 ft</u>**

Elev = 854.02 − 7.84 = **<u>846.18 ft</u>**

4.29 If a plus sight of 2.486 m is taken on BM *A*, elevation 605.348 m, and a minus sight of 0.468 m is read on point *X*, calculate the HI and the elevation of point *X*.

HI = 605.348 + 2.486 = **<u>607.834 m</u>**

Elev = 607.834 − 0.468 = **<u>607.366 m</u>**

4.30 Similar to Problem 4.28, except a plus sight of 7.44 ft is taken on BM *A* and a minus sight of 1.55 ft read on point *X*.

HI = 850.48 + 7.44 = **<u>857.92 ft</u>**

Elev = 857.92 − 1.55 = **<u>856.37 ft</u>**

4.31 Describe the procedure used to test if the level vial is perpendicular to the vertical axis of the instrument.

See Section 4.15.5

4.32 A horizontal collimation test is performed on an automatic level following the procedures described in Section 4.15.5. With the instrument setup at point 1, the rod reading at *A* was 4.886 ft, and to *B* it was 4.907 ft. After moving and leveling the instrument at point 2, the rod reading to *A* was 5.094 ft and to *B* was 5.107 ft. What is the collimation error of the instrument and the corrected reading to *B* from point 2?

$$\varepsilon = \frac{4.907 - 4.886 - 5.107 + 5.094}{2} = 0.004 \text{ ft}$$

Correct reading at B = 5.107 − 0.004 = **5.103 ft**

4.33 The instrument tested in Problem 4.32 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was −30.36 ft. The sum of the plus sight distances between the benchmarks was 1800 ft and the sum of the minus sight distances was 1050 ft. What is the corrected elevation difference between the two benchmarks?

−30.33 ft; = −30.36 + 0.004/100(1800 − 1050)

4.34 Similar to Problem 4.32 except that the rod readings are 0.894 m and 0.923 m to *A* and *B*, respectively, from point 1, and 1.083 m and 1.100 m to *A* and *B*, respectively, from point 2. The distances between the points in the test was 50 m.

$$\varepsilon = \frac{0.923 - 0.894 - 1.100 + 1.083}{2} = 0.006 \text{ m}$$

Correct reading at B = 1.100 − 0.006 = **1.094 m**

4.35 The instrument tested in Problem 4.34 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was 28.024 m. The sum of the plus sight distances between the benchmarks was 1300 m and the sum of the minus sight distances was 3200 m. What is the corrected elevation difference between the two benchmarks?

28.252 m; = 28.024 + 0.006/50(1300 − 3200)

5 LEVELING — FIELD PROCEDURES AND COMPUTATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

5.1 What proper field procedures can virtually eliminate Earth curvature and refraction errors in differential leveling?

From Section 5.4: Balancing plus and minus sights will eliminate errors due to instrument maladjustment and the combined effect of Earth curvature and refraction.

5.2 Why is it advisable to set up a level with all three tripod legs on, or in, the same material (concrete, asphalt, soil), if possible?

From Section 5.12.2: " Therefore setups on spongy ground, blacktop, or ice should be avoided if possible, but if they are necessary, unusual care is required to reduce the resulting errors. This can include taking readings in quick order, using two rods and two observers to preclude walking around the instrument, and alternating the order of taking plus and minus sights. Additionally whenever possible, the instrument tripod’s legs can be set on long hubs that are driven to refusal in the soft material."

Simply stated, to avoid uneven settling of the tripod.

5.3 Explain how a stable set up of the level may be achieved on soft soil such as in a swamp.

From Section 5.12.2: "This can include taking readings in quick order, using two rods and two observers to preclude walking around the instrument, and alternating the order of taking plus and minus sights. Additionally whenever possible, the instrument tripod’s legs can be set on long hubs that are driven to refusal in the soft material."

5.4 Discuss how errors due to lack of instrument adjustment can be practically eliminated in running a line of differential levels.

From Section 5.4: Balancing plus and minus sights will eliminate errors due to instrument maladjustment and the combined effect of Earth curvature and refraction.

5.5 Why is it preferable to use a rod level when plumbing the rod?

From Section 5.3 and 5.13: A rod level ensures fast and correct rod plumbing.

5.6 Why are double-rodded lines of levels recommended for precise work?

From Section 5.4: *Double-rodded* lines of levels are sometimes used on important work. In this procedure, plus and minus sights are taken on two turning points, using two rods from each setup, and the readings carried in separate note form columns. A check on each instrument setup is obtained if the HI agrees for both lines. This same result can be accomplished using just one set of turning points, and reading both sides of a single rod that has two-faces, i.e., one side in feet and the other in meters. These rods are often used in precise leveling."

5.7 List four considerations that govern a rodperson's selection of TPs and BMs.

1. From Chapter 4: BMs must be permanent.
2. From Section 5.4: "Turning points should be solid objects with a definite high point."
3. From Section 5.6: "...it is recommended that some turning points or benchmarks used in the first part of the circuit be included again on the return run. This creates a multi-loop circuit, and if a blunder or large error exists, its location can be isolated to one of the smaller loops."
4. From Section 5.12.2: "It (settlement) can be avoided by selecting firm, solid turning points or, if none are available, using a steel turning pin set firmly in the ground."
5. Find turning points that aid in the balancing of plus and minus sight distances.

5.8* What error is created by a rod leaning 10 min from plumb at a 5.513 m reading on the leaning rod?

Error = 0.000 m

Correct reading = 5.513cos(10') = 5.12977; So error is 0.00023m, or 0.000 m

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

5.9 Similar to Problem 5.6, except for a 12 ft reading.

Error is 0.00 ft

Correct reading = 12cos(10') = 11.99995 ft, so error is 0.00005, again error is negligible.

5.10 What error results on a 50-m sight with a level if the rod reading is 1.505 m but the top of the 3 m rod is 0.3 m out of plumb?

Angle = asin(0.3/3) = 5°44'21"

Correct reading = 1.505cos(5°44'21") = 1.4975 m;

Error = 0.0075 m

5.11 What error results on a 200-ft sight with a level if the rod reading is 6.307 ft but the top of the 7-ft rod is 0.2 ft out of plumb?

Angle = asin(0.2/7) = 1°38'14"

Correct reading = 6.307cos(1°38'14") = 6.3044

Error = 0.0026 ft

5.12 Prepare a set of level notes for the data listed. Perform a check and adjust the misclosure. Elevation of BM 7 is 852.045 ft. If the total loop length is 2000 ft, what order of leveling is represented? (Assume all readings are in feet)

POINT	+S (BS)	–S (FS)
BM 7	9.432	
TP 1	6.780	8.363
BM 8	7.263	9.822
TP 2	3.915	9.400
TP 3	7.223	5.539
BM 7		1.477

```
Problem 5.12
       Sta    +sight         HI     -sight       Elev    Adj.
Elev
-----------------------------------------------------------
---
      BM_7      9.432                                852.045
852.045
                          861.477
       TP1      6.780                      8.363    853.114
853.112
                          859.894
      BM_8      7.263                      9.822    850.072
850.067
                          857.335
       TP2      3.915                      9.400    847.935
847.928
                          851.850
       TP3      7.223                      5.539    846.311
846.301
                          853.534
      BM_7                                 1.477    852.057
852.045
-----------------------------------------------------------
---
       sum    34.613              34.601

Misclosure: 852.045 - 852.057 = -0.012
Page Check: 852.045+34.613-34.601=852.057
```

Misclosure = 0.012*12/39.37 = 3.66 mm; K = 2000*12/39.37 = 0.6096

From Equation 5.3: $m = 3.66/\sqrt{0.6096} = 4.7$ mm , **first order, class 2**

5.13* Similar to Problem 5.12, except the elevation of BM 7 is 306.928 m and the loop length 2 km. (Assume all readings are in meters)

```
       Sta    +sight         HI     -sight       Elev    Adj.
```

Elev

BM_7	9.432			306.928	306.928
		316.360			
TP1	6.780		8.363	307.997	307.995
		314.777			
BM_8	7.263		9.822	304.955	304.950
		312.218			
TP2	3.915		9.400	302.818	302.811
		306.733			
TP3	7.223		5.539	301.194	301.184
		308.417			
BM_7			1.477	306.940	306.928
sum	34.613		34.601		

Misclosure: 306.928 - 306.940 = -0.012
Page Check: 306.928+34.613-34.601=306.940

From Equation 5.3: $m = 12/\sqrt{2} = 8.5$ mm, Third order

5.14 A differential leveling loop began and closed on BM Tree (elevation 654.07 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 5.06 (+S) on BM Tree, 8.99 (−S) and 7.33 (+S) on TP1, 2.52 (−S) and 4.85 (+S) on BM *X*, 3.61 (−S) and 5.52 (+S) on TP2, and 7.60 (−S) on BM Tree. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev
BM_Tree	5.06			654.07	654.07
		659.13			
TP1	7.33		8.99	650.14	650.13
		657.47			
BM_X	4.85		2.52	654.95	654.93
		659.80			

TP2	5.52		3.61	656.19	656.16
		661.71			
BM_Tree			7.60	654.11	654.07
sum	22.76		22.72		

Misclosure: 654.07 - 654.11 = -0.04
Page Check: 654.07+22.76-22.72=654.11

5.15 A differential leveling circuit began on BM Hydrant (elevation 1823.65 ft) and closed on BM Rock (elevation 1841.71 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) given in the order taken are 8.04 (+S) on BM Hydrant, 5.63 (−S) and 6.98 (+S) on TP1, 2.11 (−S) and 9.05 (+S) on BM 1, 3.88 (−S) and 5.55 (+S) on BM 2, 5.75 (−S) and 10.44 (+S), on TP2, and 4.68 (−S) on BM Rock. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev
BMHY	8.04			1823.65	1823.65
		1831.69			
TP1	6.98		5.63	1826.06	1826.07
		1833.04			
BM1	9.05		2.11	1830.93	1830.95
		1839.98			
BM2	5.55		3.88	1836.10	1836.13
		1841.65			
TP2	10.44		5.75	1835.90	1835.94
		1846.34			
BMROCK			4.68	1841.66	1841.71
sum	40.06		22.05		

Misclosure: 1841.71 - 1841.66 = 0.05
Page Check: 1823.65+40.06-22.05=1841.66

5.16 A differential leveling loop began and closed on BM Bridge (elevation 103.895 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 1.023 (+S) on BM Bridge, 1.208 (−S) and 0.843 (+S) on TP1, 0.685 (−S) and 0.982 (+S) on BM *X*, 0.944 (−S) and 1.864 (+S) on TP2, and 1.879 (−S) on BM Bridge. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev
BM_Bridge	1.023			103.895	103.895
		104.918			
TP1	0.843		1.208	103.710	103.711
		104.553			
BM_X	0.982		0.685	103.868	103.870
		104.850			
TP2	1.864		0.944	103.906	103.909
		105.770			
BM_Bridge			1.879	103.891	103.895
sum	4.712		4.716		

Misclosure: 103.895 - 103.891 = 0.004
Page Check: 103.895+4.712-4.716=103.891

5.17 A differential leveling circuit began on BM Rock (elevation 243.897 m) and closed on BM Manhole (elevation 240.100 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 0.288 (+S) on BM Rock, 0.987 (−S) and 0.305 (+S) on TP1, 1.405 (−S) and 0.596 (+S) on BM 1, 1.605 (−S) and 0.661 (+S) on BM 2, 1.992 (−S) and 1.056 (+S) on TP2, and 0.704 (−S) on BM Manhole. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev
BM_Rock	0.288			243.897	243.897
		244.185			
TP1	0.305		0.987	243.198	243.196
		243.503			
BM1	0.596		1.405	242.098	

```
242.094
                       242.694
       BM2       0.661                      1.605    241.089
241.083
                       241.750
       TP2       1.056                      1.992    239.758
239.750
                       240.814
     BM_MH                                  0.704    240.110
240.100
-----------------------------------------------------------
---
       sum     2.906              6.693

Misclosure: 240.100 - 240.110 = -0.010
Page Check: 243.897+2.906-6.693=240.110
```

5.18 A differential leveling loop started and closed on BM Juno, elevation 5007.86 ft. The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 3.00 (+S) on BM Juno, 8.14 (−S) and 5.64 (+S) on TP1, 3.46 (−S) and 6.88 (+S) on TP2, 10.27 (−S) and 8.03 (+S) on BM1, 4.17 (−S) and 7.86 (+S) on TP3, and 5.47 (−S) on BM Juno. Prepare, check, and adjust the notes.

```
        Sta     +sight          HI     -sight        Elev    Adj.
Elev
-------------------------------------------------------------------
---
   BM_Juno        3.00                                5007.86
5007.86
                         5010.86
        TP1        5.64                     8.14     5002.72
5002.74
                         5008.36
        TP2        6.88                     3.46     5004.90
5004.94
                         5011.78
        BM1        8.03                    10.27     5001.51
5001.57
                         5009.54
        TP3        7.86                     4.17     5005.37
5005.45
                         5013.23
   BM_Juno                                  5.47     5007.76
5007.86
-------------------------------------------------------------------
---
        sum     31.41               31.51

Misclosure: 5007.86 - 5007.76 = 0.10
Page Check: 5007.86+31.41-31.51=5007.76
```

5.19* A level setup midway between *X* and *Y* reads 6.29 ft on *X* and 7.91 ft on *Y*. When moved within a few feet of *X*, readings of 5.18 ft on *X* and 6.76 ft on *Y* are recorded. What is the true elevation difference, and the reading required on *Y* to adjust the instrument?

Correct ΔElev = 7.91 − 6.29 = 1.62 ft
Unbalanced ΔElev = 6.76 − 5.18 = 1.58 ft
Error at Y = 0.04 ft
Corrected reading at Y = 6.76 + 0.04 = 6.80 ft

5.20 To test its line of sight adjustment, a level is setup near *C* (elev 193.436 m) and then near *D*. Rod readings listed in the order taken are *C* = 1.256 m, *D* = 1.115 m, *D* = 1.296 m, and *C* = 1.151 m. Compute the elevation of *D*, and the reading required on *C* to adjust the instrument.

ΔElev from C = 1.256 − 1.115 = 0.141

ΔElev from D = 1.296 − 1.151 = 0.145
ΔElev = (0.141 + 0.145) = 0.143 m
$Elev_D$ = 193.436 + 0.143 = **193.579 m**
Corrected reading at *C* = 1.151 + 0.004/2 = **1.153 m**

5.21* The line of sight test shows that a level's line of sight is inclined downward 3 mm/50 m. What is the allowable difference between BS and FS distances at each setup (neglecting curvature and refraction) to keep elevations correct within 1 mm?

$$\frac{50}{3} = \frac{X}{1} \rightarrow X = 16.7 \text{ m}$$

5.22 Reciprocal leveling gives the following readings in meters from a set up near *A:* on *A*, 2.558; on *B,* 1.883, 1.886, and 1.885. At the setup near *B:* on *B,* 1.555; on *A,* 2.228, 2.226, and 2.229. The elevation of *A* is 158.618 m. Determine the misclosure and elevation of *B.*

$$\text{From A: } \Delta Elev = 2.558 - \frac{1.883+1.886+1.885}{3} = 0.6733$$

$$\text{From B: } \Delta Elev = \frac{2.228+2.226+2.229}{3} - 1.555 = 0.6727$$

$$Elev_B = 158.618 + \frac{0.6733+0.6727}{2} = 159.291 \text{ m}$$

5.23* Reciprocal leveling across a canyon provides the data listed (in meters). The elevation of *Y* is 2265.879 ft. The elevation of *X* is required. Instrument at *X*: +3.182, $-\text{S} = 9.365, 9.370,$ and 9.368. Instrument at *Y*: $+\text{S} = 10.223; -\text{S} = 4.037, 4.041,$ and 4.038.

$$\text{From } X\text{: } \Delta Elev = 3.182 - \frac{9.365+9.370+9.368}{3} = -6.1857$$

$$\text{From } Y\text{: } \Delta Elev = \frac{4.037+4.041+4.038}{3} - 10.223 = -6.1843$$

$$Elev_B = 2265.879 - \frac{6.1857+6.1843}{2} = 2259.694 \text{ m}$$

5.24 Prepare a set of three-wire leveling notes for the data given and make the page check. The elevation of BM *X* is 106.101 m. Rod readings (in meters) are (*H* denotes upper cross-wire readings, *M* middle wire, and *L* lower wire): +S on BM *X: H* = 0.965, *M* = 0.736, *L* = 0.507; −S on TP 1: $H = 1.594, M = 1.341, L = 1.088$; +S on TP 1: *H* = 1.876, *M* = 1.676, $L = 1.476$; −S on BM *Y:* $H = 1.437, M = 1.240, L = 1.043$.

Station	+S	stadia	HI	−S	Stadia	Elev (m)
BM X	0.965					106.101
	0.736	0.229				
	0.507	0.229				
	2.208	0.458				
	0.736✓					
			106.837			
TP1	1.876			1.594		105.496
	1.676	0.200		1.341	0.253	
	1.476	0.200		1.088	0.253	
	5.028	0.400		4.023	0.506	
	1.676✓			1.341✓		
			107.172			
BMY				1.437		105.932
				1.240	0.197	
				1.043	0.197	
				3.720	0.394	(cont.)

				1.240✓		
	2.412	0.858		2.581	0.900	

Page check: 106.101 + 2.412 − 2.581 = 105.932✓

5.25 Similar to Problem 5.24, except the elevation of BM *X* is 638.437 ft, and rod readings (in feet) are: +S on BM *X:* H = 4.329, M = 3.092, L = 1.855; −S on TP 1: H = 6.083, $M = 4.918, L = 3.753$; +S on TP 1: $H = 7.834, M = 6.578, L = 5.321$; −S on BM *Y:* $H = 4.674, M = 3.367, L = 2.060$.

Station	+S	stadia	HI	−S	Stadia	Elev (ft)
BM X	4.329					638.437
	3.092	1.237				
	<u>1.855</u>	<u>1.237</u>				
	9.276	2.474				
	3.092✓					
			641.529			
TP1	7.834			6.083		636.611
	6.578	1.256		4.918	1.165	
	<u>5.321</u>	<u>1.257</u>		<u>3.753</u>	<u>1.165</u>	
	19.733	2.513		14.754	2.330	
	6.578✓			4.918✓		
			643.189			
BMY				4.674		639.822
				3.367	1.307	
				<u>2.060</u>	<u>1.307</u>	
				10.101	2.614	
				3.367✓		
	9.670	4.987		2.581	4.944	

Page check: 638.437 + 9.670 − 8.285 = 639.822✓

5.26 Assuming a stadia constant of 99.987, what is the distance leveled in Problem 5.24?

D = 99.987(0.858 + 0.900) = <u>**175.8 m**</u>

5.27 Assuming a stadia constant of 101.5, what is the distance leveled in Problem 5.25?

D = 101.5(4.987 + 4.944) = <u>**1008 ft**</u>

5.28 Prepare a set of profile leveling notes for the data listed and show the page check. All data is given in feet. The elevation of BM *A* is 1364.58, and the elevation of BM *B* is 1349.26. Rod readings are: +S on BM *A,* 2.86 intermediate foresight (IFS) on 11+00, 3.7; –S on TP1, 10.56; +S on TP 1, 11.02; intermediate foresight on 12+00, 8.7; on 12+50, 6.5; on 13+00, 5.7; on 14+00, 6.3; −S on TP 2, 9.15, +S on TP 2, 4.28; intermediate foresight on 14+73, 3.5; on 15+00, 4.2; on 16+00, 6.4; −S on TP3, 8.77; +S on TP3, 4.16; −S on BM *B*, 9.08.

Sta	+S	HI	−S	−IFS	Elev (ft)	Adj. Elev
BM A	2.86				1364.58	1364.58
		~~1367.44~~				
11+00		1367.4		3.7		1363.7
TP1	11.02		10.56		1356.88	1356.86
		~~1367.90~~				
12+00		1367.9		8.7		1359.2
12+50				6.5		1361.4
13+00				5.7		1362.2
14+00				6.3		1361.6
TP2	4.28		9.15		1358.75	1358.71
		~~1363.03~~				
14+73		1363.0		3.5		1359.5
15+00				4.2		1358.8
16+00				6.4		1356.6
TP3	4.16		8.77		1354.26	1354.20
		1358.42				
BM *B*			9.08		1349.34	1349.26
	22.32		37.56			

Page check: 1364.58 + 22.32 − 37.56 = 1349.34✓

Misclosure = 1349.34 − 1349.26 = 0.08

5.29 Same as Problem 5.28, except the elevation of BM *A* is 438.96 ft, the elevation of BM *B* is 427.32 ft, and the +S on BM *A* is 6.56 ft.

Sta	+S	HI	−S	−IFS	Elev (ft)	Adj. Elev
BM A	6.56					438.96
		~~445.52~~				
11+00		445.5		3.7		441.8
TP1	11.02		10.56		434.96	434.94
		~~445.98~~				
12+00		445.9		8.7		437.2
12+50				6.5		439.4
13+00				5.7		440.2
14+00				6.3		439.6
TP2	4.28		9.15		436.83	436.78
		~~441.11~~				
14+73		441.0		3.5		437.5
15+00				4.2		436.8

16+00				6.4		434.6
TP3	4.16		8.77		432.34	432.26
		436.50				
BM *B*			9.08			
	26.02		37.56		427.42	427.32

Page check: 438.96 + 26.02 − 37.56 = 427.42✓
Misclosure = 427.42 − 427.32 = 0.10

5.30 Plot the profile Problem 5.28 and design a grade line between stations 11 + 00 and 16 + 00 that balances cut and fill areas.

Plot of profile from 5.28. Response should vary, but cut and fill sections should visually balance.

5.31* What is the percent grade between stations 11+00 and 16 + 00 in Problem 5.28?

$$\frac{-7.1}{500}100\% = -1.4\%$$

5.32 Differential leveling between BMs *A, B, C, D*, and *A* gives elevation differences (in meters) of −6.352, +12.845, +9.241, and −15.717, and distances in km of 0.6, 1.0, 1.3, and 0.5, respectively. If the elevation of *A* is 886.891, compute the adjusted elevations of BMs *B, C,* and *D,* and the order of leveling.

Overall length of loop = 3.4 km.

Misclosure = −6.532 + 12.845 + 9.241 − 15.717 = 0.017 m

$$m = \frac{17\text{ mm}}{\sqrt{3.4}} = 9.2\text{ mm},\ \text{Third order leveling}$$

Sta	ΔElev (m)	Length (km)	Correction	Adj. ΔElev.	Adj. Elev.
A					886.891
	−6.352	0.6	−0.017(0.6/3.4) = −0.003	−6.355	
B					**880.536**
	12.845	1.0	−0.017(1.0/3.4) = −0.005	12.840	
C					**893.376**
	9.241	1.3	−0.017(1.3/3.4) = −0.0065	9.234	
D					**899.610**
	−15.717	0.5	−0.017(0.5/3.4) = −0.0025	−15.719	
A					883.891
	−0.017	3.4		0.0000	

5.33 Leveling from BM *X* to *W,* BM *Y* to *W,* and BM *Z* to *W* gives differences in elevation (in feet) of −30.24, +26.20, and +10.18, respectively. Distances between benchmarks are $XW = 3500, YW = 2700,$ and $ZW = 4500.$ True elevations of the benchmarks are

$X = 460.82, Y = 404.36,$ and $Z = 420.47.$ What is the adjusted elevation of *W?* (Note: All data are given in feet.)

5.34 A 3-m level rod was calibrated and its graduated scale was found to be uniformly expanded so that the distance between its 0 and 3.000 marks was actually 3.006 m. How will this affect elevations determined with this rod for (**a**) circuits run on relatively flat ground (**b**) circuits run downhill (**c**) circuits run uphill?

(**a**) On level ground the plus and minus sight readings will be approximately the same. Since the plus readings are added and minus readings subtracted, the net effect will tend to cancel the errors.

(**b**) In level downhill, the minus readings will tend to be higher on the rod than the plus readings. Thus the minus readings will tend to have more error than the plus readings. Thus the elevations will tend to be too low.

(**c**) In level uphill, the plus readings will be tend to be higher on the rod than the minus readings. Thus the plus readings will tend to have more error than the minus readings. Thus the elevations will tend to be too high.

5.35* A line of levels with 42 setups (84 rod readings) was run from BM Rock to BM Pond with readings taken to the nearest 3.0 mm; hence any observed value could have an error of ± 1.5 mm. For reading errors only, what total error would be expected in the elevation of BM Pond?

$$1.5\sqrt{84} = \pm 13.7 \text{ mm}$$

5.36 Same as Problem 5.35, except for 64 setups and readings to the nearest 0.01 ft with possible error of ± 0.005 ft each.

$$0.005\sqrt{64} = \pm 0.04 \text{ ft}$$

5.37 Compute the permissible misclosure for the following lines of levels: (**a**) a 10-km loop of third-order levels (**b**) a 20-km section of second-order class I levels (**c**) a 40-km loop of first-order class I levels.

(**a**) $C = 12\sqrt{10} = 37.9$ mm

(**b**) $C = 6\sqrt{20} = 26.8$ mm

(**c**) $C = 4\sqrt{40} = 25.2$ mm

5.38 Create a computational program that solves Problem 5.12.

Programming problem. Responses should vary.

6 DISTANCE MEASUREMENT

Asterisks (*) indicate problems that have answers given in Appendix G.

6.1 What distance in travel corresponds to 1 millisecond of time for electromagnetic energy?

299.792 m = 299,792,458(0.001)

6.2* A student counted 92, 90, 92, 91, 93, and 91 paces in six trials of walking along a course of 200 ft known length on level ground. Then 85, 86, 86, and 84 paces were counted in walking four repetitions of an unknown distance AB. What is (a) the pace length and (b) the length of AB?

(a) pace length = 200(6)/(92+90+92+91+93+91) = **2.18 ft/pace**

(b) *AB* = (85+86+86+84)2.18/4 = **186 ft**

6.3 What difference in temperature from standard, if neglected in use of a steel tape, will cause an error of 1 part in 5000?

31° F or **17.2° C**

$$1 = 0.00000645(\Delta T)5000 \qquad \text{or} \qquad 1 = 0.0000116(\Delta T)5000$$

$$\Delta T = \frac{1}{0.00000645(5000)} = 31°\text{ F} \qquad \Delta T = \frac{1}{0.0000116(5000)} = 17.2°\text{ C}$$

6.4 An add tape of 101 ft is incorrectly recorded as 100 ft for a 200-ft distance. What is the correct distance?

202 ft

6.5* List five types of common errors in taping. (See Section 6.14)

6.6 List the proper procedures taping a horizontal distance of about 123 ft down a 4% slope.

See Section 6.12 and 6.13.

6.7 For the following data, compute the horizontal distance for a recorded slope distance AB,

(a) AB = 385.29 ft, slope angle = $6°03'26''$ $H = 385.29\cos 6°03'26''$ = **383.14 ft**

(b) AB = 186.793 m, difference in elevation A to B = −8.499 m.

$$H = \sqrt{186.793^2 - 8.499^2} = 186.600 \text{ m}$$

A 100 ft steel tape of cross-sectional area 0.0025 in.2, weight 2.3 lb, and standardized at 68°F is 99.992 ft between end marks when supported throughout under a 12-lb pull. What is the true horizontal length of a recorded distance AB for the conditions given in Problems 6.8 through 6.11? (Assume horizontal taping and all full tape lengths except the last.)

Recorded Distance AB	Average Temperature	Means of Support	Tension (lb)

	(ft)	(°F)		
6.8*	86.06	68	Thoughout	12
6.9	124.73	85	Thoughout	15
6.10	86.35	50	Ends only	22
6.11	94.23	75	Ends only	25

6.8 **86.05 ft**; $C_L = \frac{99.992 - 100}{100} 86.06 = -0.007 \text{ ft}$

6.9 **124.74 ft**;

$$C_L = \frac{99.992 - 100}{100} 124.73 = -0.010;\ C_T = 0.00000645(85 - 68)124.73 = 0.014 \text{ ft}$$

$$C_P = (15 - 12)\frac{124.73}{0.0025(29{,}000{,}000)} = 0.005 \text{ ft}$$

6.10 **86.33 ft**;

$$C_L = \frac{99.992 - 100}{100} 86.35 = -0.007;\ C_T = 0.00000645(50 - 68)86.35 = -0.010 \text{ ft}$$

$$C_P = (22 - 12)\frac{86.35}{0.0025(29{,}000{,}000)} = 0.012 \text{ ft};\ C_S = -\frac{0.023^2(86.35)^3}{24(22)^2} = -0.012 \text{ ft}$$

6.11 **94.23 ft**;

$$C_L = \frac{99.992 - 100}{100} 94.23 = -0.008;\ C_T = 0.00000645(75 - 68)94.23 = 0.004 \text{ ft}$$

$$C_P = (25 - 12)\frac{94.23}{0.0025(29{,}000{,}000)} = 0.017 \text{ ft};\ C_S = -\frac{0.023^2(94.23)^3}{24(25)^2} = -0.017 \text{ ft}$$

For the tape of Problems 6.8 through 6.11, determine the true horizontal length of the recorded slope distance BC for the conditions shown in Problems 6.12 through 6.13. (Assume the tape was fully supported for all measurements.)

	Recorded Slope Distance BC (ft)	Average Temperature Per 100 ft (°F)	Tension (lb)	Elevation Difference (ft)
6.12	95.08	48	15	2.45
6.13	65.86	88	20	3.13

6.12 **95.03 ft**;

$$C_L = \frac{99.992 - 100}{100} 95.08 = -0.008;\ C_T = 0.00000645(48 - 68)95.08 = -0.012 \text{ ft}$$

$$C_P = (25 - 12)\frac{95.08}{0.0025(29{,}000{,}000)} = 0.004 \text{ ft}; H = \sqrt{95.064^2 - 2.45^2} = 95.032 \text{ ft}$$

6.13 **65.08 ft**;

$$C_L = \frac{99.992 - 100}{100} 65.86 = -0.005;\ C_T = 0.00000645(88-68)65.86 = -0.008 \text{ ft}$$

$$C_P = (25-12)\frac{65.86}{0.0025(29{,}000{,}000)} = 0.007 \text{ ft}; H = \sqrt{65.870^2 - 3.13^2} = 65.796 \text{ ft}$$

A 30-m steel tape measured 29.991 m when standardized fully supported under a 5.500-kg pull at a temperature of 20°C. The tape weighed 1.22 kg and had a cross-sectional area of 0.016 cm^2. What is the corrected horizontal length of a recorded distance AB for the conditions given in Problems 6.14 through 6.15?

	Recorded Distance AB (m)	Average Temperature (°C)	Tension (kg)	Means of Support
6.14	28.056	18	8.3	Throughout
6.15	16.302	25	7.9	Ends only

6.14 **<u>28.047 m</u>**;

$$C_L = \frac{29.991 - 30}{30} 28.056 = -0.008;\ C_T = 0.0000116(18-20)28.056 = -0.0006 \text{ m}$$

$$C_P = (8.3-5.5)\frac{28.056}{0.016(2{,}000{,}000)} = 0.0024 \text{ m};$$

6.15 **<u>16.299 m</u>**;

$$C_L = \frac{29.991 - 30}{30} 16.302 = -0.005;\ C_T = 0.0000116(25-20)16.302 = 0.0009 \text{ m}$$

$$C_P = (7.9-5.5)\frac{16.302}{0.016(2{,}000{,}000)} = 0.0012 \text{ m};$$

For the conditions given in Problems 6.14 through 6.16, determine the horizontal length of CD that must be laid out to achieve required true horizontal distance CD. Assume a 100-ft steel tape will be used, with cross-sectional area 0.0025 $in.^2$, weight 2.4 lb, and standardized at 68°F to be 100.008 ft between end marks when supported throughout with a 12-lb pull. (Assume horizontal taping and all full tape lengths except the last.)

	Required Horizontal Distance CD (ft)	Average Temperature (°F)	Means of Support	Tension (lb)
6.16	97.54	68	Throughout	12
6.17	68.96	54	Throughout	20
6.18	68.78	91	Throughout	18

6.16 **<u>97.53 ft</u>**;

$$C_L = \frac{100.008 - 100}{100} 97.54 = 0.008;\ C_T = 0.000 \text{ ft}$$

$$C_P = 0.000 \text{ ft}$$

6.17 **68.95 ft**;

$$C_L = \frac{100.008 - 100}{100} 68.96 = 0.006;\ C_T = 0.00000645(54-68)68.96 = -0.006 \text{ ft}$$

$$\mathrm{C}_P = (20-12)\frac{68.96}{0.0025(29{,}000{,}000)} = 0.008 \text{ ft}$$

6.18 **68.76 ft**;

$$C_L = \frac{100.008 - 100}{100} 68.78 = 0.006;\ C_T = 0.00000645(91-68)68.78 = 0.010 \text{ ft}$$

$$\mathrm{C}_P = (18-12)\frac{68.78}{0.0025(29{,}000{,}000)} = 0.006 \text{ ft}$$

6.19* When measuring a distance AB, the first taping pin was placed 1.0 ft to the right of line AB and the second pin was set 0.5 ft left of line AB. The recorded distance was 236.89 ft. Calculate the corrected distance. (Assume three taped segments, the first two 100 ft each.)

236.87 ft

A-Pin1: $\sqrt{100^2 - 1^2} = 99.995$ ft ; Pin1-Pin2: $\sqrt{100^2 - (1+0.5)^2} = 99.989$ ft

Pin1-B: $\sqrt{36.89^2 - 0.5^2} = 36.887$ ft ; $AB = 99.995 + 99.989 + 36.887 = 236.870$ ft

6.20 List the possible errors that can occur when measuring a distance with an EDM.

See Section 6.24

6.21 Briefly describe how a distance can be measured by the method of phase comparison.

See Section 6.18

6.22 Describe why the sight line for electronic distance measurement should be at least 0.5 m off the edge of a parked vehicle.

Due to the fact the that index of refraction will be different near the vehicle due a microclimate that will surround the vehicle.

6.23* Assume the speed of electromagnetic energy through the atmosphere is 299,784,458 m/sec for measurements with an EDM instrument. What time lag in the equipment will produce an error of 800 m in a measured distance?

0.0027 ms; $t = \dfrac{800}{299{,}784{,}458} = 0.0000027$ sec

6.24 What is the length of the partial wavelength for electromagnetic energy with a frequency of the 15 MHz and a phase shift of 263°?

14.6 m; $\lambda = \dfrac{299{,}784{,}458}{15\times10^6}\dfrac{263°}{360} = 14.601$ m

6.25 What "actual" wavelength results from transmitting electromagnetic energy through an atmosphere having an index of refraction of 1.0006, if the frequency is:

(a)* 29.988 MHz; **9.991 m**; $\lambda = \dfrac{299,784,458/1.0006}{29.988 \times 10^6} = 9.9908$ m

(b) 2.988 MHz; **100.269 m**; $\lambda = \dfrac{299,784,458/1.0006}{2.988 \times 10^6} = 100.2693$ m

6.26 Using the speed of electromagnetic energy given in Problem 6.23, what distance corresponds to each nanosecond of time?

0.300 m; $D = 299,784,458\left(10^{-9}\right) = 0.2998$ m

6.27 To calibrate an EDM instrument, distances AC, AB, and BC along a straight line were observed as 216.622 m, 130.320 m, and 86.281 m, respectively. What is the system measurement constant for this equipment? Compute the length of each segment corrected for the constant.

0.021 m; From Equation (6.16): $K = 216.622 - 130.320 - 86.281 = 0.021$ m

6.28 Which causes a greater error in a line measured with an EDM instrument? **(a)** A disregarded 10° C temperature variation from standard or **(b)** a neglected atmospheric pressure difference from standard of 20 mm of mercury?

(a) temperature; From Figure 6.17, a 10° C difference in temperature will cause a 10 ppm error in the distance whereas a 20 mm of Hg difference in pressure will cause less than 10 ppm error in the distance. So the temperature difference will cause the largest error.

6.29* In Figure 6.15, h_e, h_r, elev$_A$, elev$_B$ and the measured slope length L were 5.32, 5.18, 1215.37, 1418.68, and 2282.74 ft, respectively. Calculate the horizontal length between A and B.

2273.68 ft; From Equation (6.13) $d = (1215.37 + 5.32) - (1418.68 + 5.18) = -203.17$ ft

By Equation (6.2): $H = \sqrt{2282.74^2 - 203.17^2} = 2273.68$ ft

6.30 Similar to Problem 6.32, except that the values were 1.535, 1.502, 334.215, 386.289, and 1925.461 m, respectively.

1924.758 m; From Equation (6.13) $d = (334.215 + 1.535) - (386.289 + 1.502) = -52.041$ m

By Equation (6.2): $H = \sqrt{1925.461^2 - 52.041^2} = 1924.7576$ m

6.31 In Figure 6.15, h_e, h_r, z, and the measured slope length L were 5.25 ft, 5.56 ft, $86°30'46''$ and 1598.27 ft, respectively. Calculate the horizontal length between A and B if a total station measures the distance.

1595.31 ft; $H = 1598.27 \sin(86°30'46'') = 1595.311$ ft

6.32* Similar to Problem 6.31, except that the values were 1.45 m, 1.55 m, $96°05'33''$ and 1663.254 m, respectively.

1653.860 m; $H = 1663.254 \sin(96°05'33'') = 1653.8597$ m

6.33 What is the actual wavelength and velocity of a near-infrared beam $(\lambda = 0.899\ \mu\text{m})$ of light modulated at a frequency of 330 MHz through an atmosphere with a dry bulb temperature, T, of 24° C, a relative humidity, h, of 69%, and an atmospheric pressure of 933 hPa?

0.908 m

$$N_g = 287.6155 + \frac{4.88660}{0.899^2} + \frac{0.06800}{0.899^4} = 293.76587$$

$$a = \frac{7.5(24)}{237.3 + 24} + 0.7858 = 1.474663375$$

$$E = 10^a = 29.8307$$

$$e = E\frac{69}{100} = 20.58318$$

$$n_a = 1 + \left(\frac{273.15}{1013.25}\frac{293.76587(933)}{24 + 273.15} - \frac{11.27e}{24 + 273.15}\right)10^{-6} = 1.0002479$$

$$V = \frac{299,792,458}{1.00001} = 299,718,166 \text{ m/s}$$

$$\lambda = \frac{299,718,166}{330 \cdot 10^6} = 0.908237 \text{ m}$$

6.34 If the temperature and pressure at measurement time are 35°C and 760 mm Hg, what will be the error in electronic measurement of a line 3 km long if the temperature at the time of observing is recorded 10°C too low? Will the observed distance be too long or too short?

From Figure 6.17: ppm_T = 5 ppm; error = $5/10^6(3000)$ = 0.015 m, which is too long

6.35 Determine the most probable length of a line *AB*, the standard deviation, and the 95% error of the measurement for the following series of taped observations made under the same conditions: 632.088, 632.087, 632.089, 632.083, 632.093, 632.088, 632.083, 632.088, 632.092, and 632.091 m.

632.088 ± 0.008 m = 632.088 ± 1.9599(0.004) = 632.088 ± 0.0078 m

6.36* The standard deviation of taping a 30 m distance is ±5 mm. What should it be for a 90-m distance?

8.7 mm; $5\sqrt{90/30} = 8.66$ mm

6.37 If an EDM instrument has a purported accuracy capability of ±(3 mm + 3 ppm), what error can be expected in a measured distance of **(a)** 30 m **(b)** 1586.49 ft **(c)** 975.468 m? (Assume that the instrument and target miscentering errors are equal to zero.)

(a) <u>**±3.0 mm**</u>; $\sqrt{3^2+\left[\dfrac{3(30000)}{10^6}\right]^2}=\pm 3.001\text{ mm}$

(b) <u>**±0.011 ft**</u>; $\sqrt{0.01^2+\left[\dfrac{3(1586.49)}{1000000}\right]^2}=\pm 0.0111\text{ ft}$

(c) <u>**±4.2 mm**</u>; $\sqrt{3^2+\left[\dfrac{3(975468)}{1000000}\right]^2}=\pm 4.19\text{ mm}$

6.38 The estimated error for both instrument and target miscentering errors is ±3 mm. For the EDM in Problem 6.37, what is the estimated error in the observed distances?

(a) <u>**±5.2 mm**</u>; $\sqrt{3^2+3^2+3^2+\left[\dfrac{3(30000)}{10^6}\right]^2}=\pm 3.001\text{ mm}$

(b) <u>**±0.018 ft**</u>; $\sqrt{0.01^2+0.01^2+0.01^2+\left[\dfrac{3(1586.49)}{1000000}\right]^2}=\pm 0.01797\text{ ft}$

(c) <u>**±6.0 mm**</u>; $\sqrt{3^2+3^2+3^2+\left[\dfrac{3(975468)}{1000000}\right]^2}=\pm 5.96\text{ mm}$

6.39 If a certain EDM instrument has an accuracy capability of ±(1 mm + 2 ppm), what is the precision of measurements, in terms of parts-per-million, for line lengths of: **(a)** 30.000 m **(b)** 300.000 m **(c)** 3000.000 m? (Assume that the instrument and target miscenteringerrors are equal to zero.)

(a) <u>**33 ppm**</u>; $E = \pm 1.002$; ppm = (1.0/30,000)1,000,000 = 33.39 ppm

(b) <u>**3.9 ppm**</u>; $E = \pm 1.166$; ppm = (1.166/300,000)1,000,000 = 3.887 ppm

(c) <u>**2 ppm**</u>; $E = \pm 6.08$; ppm = (6.08/3,000,000)1,000,000 = 2.027 ppm

6.40 The estimated error for both instrument and target miscentering errors is ±3 mm. For the EDM and distances listed in Problem 6.39, what is the estimated error in each distance? What is the precision of the measurements in terms of part-per-million?

(a) <u>**145 ppm**</u>; $E = \pm 4.36$; ppm = (4.36/30,000)1,000,000 = 145.31 ppm

(b) <u>**14.7 ppm**</u>; $E = \pm 4.4$; ppm = (4.4/300,000)1,000,000 = 14.667 ppm

(c) <u>**2.5 ppm**</u>; $E = \pm 7.42$; ppm = (7.42/3,000,000)1,000,000 = 2.47 ppm

6.41 Create a computational program that solves Problem 6.29. Response should vary.

6.42 Create a computational program that solves Problem 6.38. Response should vary.

7 ANGLES, AZIMUTHS, AND BEARINGS

Asterisks (*) indicate problems that have answers given in Appendix G.

7.1 List the different reference meridians that can be used for the direction of a line and describe the advantages and disadvantages of each system.

See Section 7.4. Geodetic, astronomic, magnetic, grid, record, and assumed.

7.2 What are the disadvantages of using an assumed meridian for the starting course in a traverse?

From Section 7.4, paragraph 7:

"Disadvantages of using an assumed meridian are the difficulty, or perhaps impossibility, of reestablishing it if the original points are lost, and its nonconformance with other surveys and maps in an area."

7.3 What is meant by an angle to the right?

From Section 7.3, paragraph 3: "Angles to the right are measured clockwise from the rear to the forward station."

7.4 By means of a sketch describe **(a)** interior angles, **(b)** angles to the right, and **(c)** deflection angles.

See Section 7.3.

7.5 Convert: ***(a)** 203°26′48″ to grads **(b)** 339.0648 grads to degrees, minutes, and seconds **(c)** 207°18′45″ to radians.

(a) **<u>226.0518 grad</u>**

(b) **<u>305°09'30"</u>**

(c) **<u>3.6183 rad</u>**

In Problems 7.6 through 7.7, convert the azimuths from north to bearings, and compute the angles, smaller than 180° between successive azimuths.

7.6 68°06′42″, 133°15′56″, 217°44′05″, and 320°35′18″

Bearings	Angles
N 68°06′42″ E	65°09″14″
S46°44′04″E	84°28′09″
S37°44'05″W	102°51′13″
N39°24′42″W	107°31′24″

7.7 $65°12'55'', 146°27'39'', 233°56'12'',$ and $348°52'11''$

Bearings	Angles
N65°12′55″E	81°14′44″
S33°32′21″E	87°28′33″
S53°56′12″W	114°55′59″
N11°07′49″W	76°20′44″

Convert the bearings in Problems 7.8 through 7.9 to azimuths from north and compute the angle, smaller than 180°, between successive bearings.

7.8 N27°50′05″E, S38°12′44″E, S23°16′22″W, and N73°14″30″W

Azimuths	Angles
27°50′05″	113°57′11″
141°47′16″	61°29′06″
203°16′22″	83°29′08″
286°45′30″	101°04′35″

7.9 N12°18′38″E, S14°32′12″E, S82°12′10″W, and N02°15′41″W

Azimuths	Angles
12°18′38″	153°09′10″
165°27′48″	96°44′22″
262°12′10″	95°32′09″
357°44′19″	14°34′19″

Compute the azimuth from north of line CD in Problems 7.10 through 7.12. (Azimuths of AB are also from north.)

***7.10** Azimuth AB = 68°26′32″; angles to the right $ABC = 45°07'08''$, $BCD = 36°26'48''$.

Az_{CD} = **150°00'28"**; Az_{BC} = 293°33'40"

7.11 Bearing $AB = S14°26'12''E$; angles to the right $ABC = 133°20'46''$, $BCD = 54°31'28''$.

Brg_{CD} = **N6°33'58"W**; Brg_{BC} = S61°05'26"E

7.12 Azimuth $AB = 195°12'07''$; angles to the right $ABC = 10°36'09''$, $BCD = 32°16''14''$.

Az_{CD} = **238°04'30"**; Az_{BC} = 25°48'16"

***7.13** For a bearing $DE = N08°53'56''$W and angles to the right, compute the bearing of FG if angle $DEF = 88°12''29''$ and $EFG = 40°20'30''$.

Brg_{FG} = **S60°20'57"E**; Brg_{EF} = S79°18'33"W

7.14 Similar to Problem 7.13, except the azimuth of DE is $132°22'48''$ and angles to the right DEF and EFG are $101°34'02''$ and $51°09'01''$, respectively.

Az_{FG} = **285°05'51"**; Az_{EF} = 53°56'50"

Course AB of a five-sided traverse runs due north. From the given balanced interior angles to the right, compute and tabulate the bearings and azimuths from north for each side of the traverses in Problems 7.15 through 7.17.

7.15 $A = 77°23'26'', B = 125°58''59'', C = 105°28'32'', D = 116°27'02'', E = 114°42'01''$

Course	Bearing	Azimuth
AB	Due North	0°00'00"
BC	N54°01'01"W	305°58'59"
CD	S51°27'31"W	231°27'31"
DE	S12°05'27"E	167°54'33"
EA	S77°23'26"E	102°36'34"

***7.16** $\text{A} = 90°29'18''$, $B = 107°54'36''$, $C = 104°06'37''$, $D = 129°02'57''$, $E = 108°26'32''$

Course	Bearing	Azimuth
AB	Due North	0°00'00"
BC	N72°05'24"W	287°54'36"
CD	S32°01'13"W	212°01'13"
DE	S18°55'50"E	161°04'10"
EA	N89°30'42"	89°30'42"

7.17 $\text{A} = 98°12'18''$, $B = 126°08'30''$, $C = 100°17'44''$, $D = 110°50'40''$, $E = 104°30'48''$

Course	Bearing	Azimuth
AB	Due North	0°00'00"
BC	N53°51'30"W	306°08'30"
CD	S46°26'14"W	226°26'14"
DE	S22°43'06"E	157°16'54"
EA	N81°47'42"E	81°47'42"

In Problems 7.18 and 7.19, compute and tabulate the azimuths of the sides of a regular hexagon (polygon with six equal angles), given the starting direction of side AB.

7.18 Bearing of $AB = \text{N}56°27'13''\text{W}$ (Station C is westerly from B.)

Course	Azimuths
AB	303°32'47"
BC	243°32'47"
CD	183°32'47"
DE	123°32'47"
EF	63°32'47"
FA	3°32'47"

7.19 Azimuth of $AB = 87°14'26''$ (Station C is westerly from B.)

Course	Azimuths
AB	87°14'26"
BC	27°14'26"
CD	327°14'26"
DE	267°14'26"
EF	207°14'26"
FA	147°14'26"

7.20 Describe the relationship between forward and back azimuths.

A back azimuth is simply the line viewed from the other end. Its numerical values is 180° different from the forward azimuth.

Compute azimuths of all lines for a closed traverse ABCDEFA that has the following balanced angles to the right, using the directions listed in Problems 7.21 and 7.22. $FAB = 118°26'59''$, $ABC = 123°20'28''$, $BCD = 104°10'32''$, $CDE = 133°52'50''$, $DEF = 108°21'58''$, $EFA = 131°47'13''$.

7.21 Bearing $AB = \text{S}28°18'42''\text{W}$.

Course	Azimuths
AB	208°18'42"
BC	151°39'10"
CD	75°49'42"
DE	29°42'32"
EF	318°04'30"
FA	269°51'43"

7.22 Azimuth $DE = 116°10'20''$.

Course	Azimuths
AB	294°46'30"
BC	238°06'58"
CD	162°17'30"
DE	116°10'20"
EF	44°32'18"
FA	356°19'31"

7.23 Similar to Problem 7.21, except that bearings are required, and fixed bearing $AB = \text{N}33°46'25''\text{E}$.

Course	Bearings
AB	N33°46′25″E
BC	N22°53'07"W
CD	S81°17'25"W
DE	S35°10'15"W
EF	S36°27'47"E
FA	S84°40'34"E

7.24 Similar to Problem 7.22, except that bearings are required, and fixed azimuth $DE = 286°22'40''$ (from north).

Course	Bearings
AB	S75°01'10"E
BC	N48°19'18"E
CD	N27°30'10"W
DE	N73°37'20"W
EF	S34°44'38"W
FA	S13°28'09"E

7.25 Geometrically show how the sum of the interior angles of a pentagon (five sides) can be computed using the formula $(n-2)180°$?

A sketch showing that a pentagon can be divided into three triangles each of which as a sum of angles of 180°.

7.26 Determine the predicted declinations on January 1, 2010 using the WMM-10 model at the following locations.

(a)* latitude = 42°58′28″N, longitude = 77°12′36″W, elevation = 310.0 m; **11.9° W**

(b) latitude = 37°56′44″N, longitude = 110°50′40″W, elevation = 1500 m; **11.5°E**

(c) latitude = 41°18′15″N, longitude = 76°00′26″W, elevation = 240 m **12.1°W**

7.27 Using Table 7.4, what was the total difference in magnetic declination between Boston, MA and San Francisco, CA on January 1, 2000?

31°14'; 15°26'E – 15°48'W

7.28 The magnetic declination at a certain place is 12°06′W. What is the magnetic bearing there: **(a)** of true north **(b)** of true south **(c)** of true west?

(a) **N12°06'E**

(b) **S12°06'W**

(c) **N77°54'W**

7.29 Same as Problem 7.28, except the magnetic declination at the place is 3°30′E.

(a) **N3°30'W**

(b) **S3°30'E**

(c) **S86°30'W**

For Problems 7.30 through 7.32 the observed magnetic bearing of line AB and its true magnetic bearing are given. Compute the amount and direction of local attraction at point A.

	Observed Magnetic Bearing	True Magnetic Bearing	**Local Attraction**
7.30*	N28°15′E	N30°15′E	**2°00'W**
7.31	S13°25′W	S10°15′W	**3°10'W**
7.32	N11°56′W	N8°20′E	**20°16'W**

What magnetic bearing is needed to retrace a line for the conditions stated in Problems 7.33 through 7.36?

	1875 Magnetic Bearing	1875 Declination	Present Declination	**Present Magnetic Bearing**
7.33*	N32°45′E	8°12′W	2°30′E	**N22°03'E**
7.34	S63°40′W	3°40′E	2°20′W	**S69°40'W**
7.35	N69°20′W	1°20′W	3°30′W	**N67°10'W**
7.36	S24°30′E	12°30′E	22°30′E	**S59°30'E**

In Problems 7.37 through 7.38 calculate the magnetic declination in 1870 based on the following data from an old survey record.

	1870 Magnetic Bearing	Present Magnetic Bearing	Present Magnetic Declination	1870 Magnetic Declination
7.37	S14°20′E	S15°50′E	1°15′W	**2°45'W**
7.38	S40°30′E	S52°35′E	8°30′E	**20°35'W**

7.39 An angle APB is measured at different times using various instruments and procedures. The results, which are assigned certain weights, are as follows: 46°13′28″, wt 1; 46°13′32″, wt 2; and 43°13′30″, wt 3. What is the most probable value of the angle?

46°13'30"; $s = \dfrac{28(1)+32(2)+30(3)}{1+2+3} = 30.3''$

7.40 Similar to Problem 7.39, but with an additional measurement of 43°13′32″, wt 4.

46°13'31"; $s = \dfrac{28(1)+32(2)+30(3)+32(4)}{1+2+3+4} = 31''$

7.41 Explain why the letters E and W on a compass [see Figure 7.9(b)] are reversed from their normal positions.

It allows the direction of bearings to be read directly from the face of the instrument.

7.42 Create a computational program that solves Problem 7.21. Solutions will vary.

7.43 Create a computational program that solves Problem 7.22. Solutions will vary.

8 TOTAL STATION INSTRUMENTS; ANGLE OBSERVATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

8.1 At what step should the instrument be mounted on the tripod when setting up over a point?

From Section 8.5, the instrument should be mounted no earlier than at step 4 after the tripod has been leveled roughly over the point.

8.2 List the four axes of a total station and their relationship with each other.

From Section 8.2, three of the axes are the (1) *axis of sight*, which defines the vertical plane, (2) *horizontal axis* about which the telescope revolves, and the (3) *vertical axis* about which the telescope can also be rotated in any azimuth. The fourth axis is defined in Section 8.19, which is the (4) *axis of the plate level vial*.

Relationships:

1. Axis of the plate level vial should be perpendicular to the vertical axis.
2. The vertical axis should be perpendicular to the horizontal axis.
3. The axis of the line of sight should be perpendicular to the horizontal axis.

8.3 Describe a systematic error that can be present in an angle and describe how it is removed by field procedure.

The systematic errors can be caused by any misadjustment of the items in the solution to 8.2. All errors are handled by the principle of reversion Specifically, each misadjustment is handled as listed below:

1. Identify the error by rotating the instrument 180° from its leveled positions. Bring the bubble halfway back and check around the circle. If the bubble stays in the same location, the instrument is level.
2. Handled by averaging measurements taken in both the direct and reversed positions.
3. Handled by averaging measurements taken in both the direct and reversed positions, or by locating averaging the direct and reversed positions located when prolonging a line.

8.4 Name and briefly describe the three main components of a total station.

From Section 8.2: "an electronic distance measuring (EDM) instrument, an electronic angle measuring component, and a computer or microprocessor"

8.5 What is the purpose of dual-axis compensation in a total station instrument?

It is to mathematically correct for the nonperpendicularity of the horizontal and vertical axes.

8.6 What is the purpose of the jog/shuttle mechanism on a servo-driven total station?

From Section 8.4.4: This device actuates internal servo-drive motors that rotate the telescope about its horizontal and vertical axes.

8.7 Why is it important to remove any parallax from an optical plummet?

If parallax is not removed, the centering of the instrument will appear to shift dependent on the position of the observer's eye. Thus it will be impossible to center over the point.

8.8 Describe the steps used in setting up a total station with an adjustable leg tripod over a point.

This procedure will vary slightly dependent on the type of plummet, however, the basic procedure is outlined in Section 8.5, paragraph 2, which states:

"The setup process using an instrument with an optical plummet, tribrach mount with circular bubble, and adjustable-leg tripod is accomplished most easily using the following steps: (1) extend the legs so that the scope of the instrument will be at an appropriate elevation for view and then adjust the position of the tripod legs by lifting and moving the tripod as a whole until the point is roughly centered beneath the tripod head (beginners can drop a stone from the center of the tripod head, or use a plumb bob to check nearness to the point); (2) firmly place the legs of the tripod in the ground and extend the legs so that the head of the tripod is approximately level; repeat step (1) if the tripod head is not roughly centered over the point; (3) roughly center the tribrach leveling screws on their posts; (4) mount the tribrach approximately in the middle of the tripod head to permit maximum translation in step (9) in any direction; (5) focus the plummet properly on the point, making sure to check for parallax; (6) manipulate the leveling screws to aim the plummet's pointing device at the point below; (7) center the circular bubble by adjusting the lengths of the tripod extension legs; (8) and level the instrument using the plate bubble and leveling screws; and (9) if necessary, loosen the tribrach screw and translate the instrument (do not rotate it) to carefully center the plummet's pointing device on the point; (10) repeat steps (8) and (9) until precise leveling and centering are accomplished. With total stations that have their plummets in the tribrach, the instrument can and should be left in the case until step (8)."

8.9 What is meant by an angular position?

From Section 8.10, paragraph 2: "A set of readings around the horizon in both the direct and reversed modes constitutes a so-called *position*."

8.10 Why are the bases of total station instruments designed to be interchanged with other accessories?

This enables accessories and the total station to be swapped during field work, which speeds up the setup process.

8.11 Why is it important to keep the circular bubble of a sighting rod in adjustment?

A maladjusted circular bubble will cause the target on the rod to be off of the point.

8.12 Determine the angles subtended for the following conditions:

***(a)** a 2-cm diameter pipe sighted by total station from 100 m.

41"; $\frac{0.020}{100}206{,}264.8 \approx 41.2''$

(b) a 1/4-in. stake sighted by total station from 400 ft.

11"; $\frac{{}^{1/4}\!/_{(12)}}{400}206{,}264.8 \approx 10.8''$

(c) a 1/4-in. diameter chaining pin observed by total station from 50 ft.

86"; $\frac{{}^{1/4}\!/_{12}}{50}206{,}264.8 \approx 86''$

8.13 What is the error in an observed direction for the situations noted?

(a) setting a total station 3 mm to the side of a tack on a 50-m sight.

$\frac{0.0015}{50}206264.8 \approx 6.2''$

(b) lining in the edge (instead of center) of an 1/4-in. diameter chaining pin at 100 ft.

22"; $\frac{{}^{1/4}\!/_{(12\times 2)}}{100} \approx 21.5''$

(c) sighting the edge (instead of center) of a 2-cm diameter range pole 100 m.

21"; $\frac{0.010}{100}206{,}264.8 \approx 20.6''$

(d) sighting the top of a 6-ft range pole that is $3'$ off-level on a 300-ft sight.

3.6";
$$S = 6\left(\frac{180''}{206{,}264.8}\right) = 0.005 \text{ ft}$$
$$\frac{0.005}{300}206264.8 = 3.6''$$

***8.14** Intervening terrain obstructs the line of sight so only the top of a 6-ft long pole can be seen on a 250-ft sight. If the range pole is out of plumb and leaning sideways 0.025-ft per vertical foot what maximum angular error results?

$S = 0.025(6) = 0.15$ ft

124"; $\frac{0.15}{250} 206,264.8 \approx 123.8''$

8.15 Same as Problem 8.14, except that it is a 2-m pole that is out of plumb and leaning sideways 2 cm per meter on a 100 m sight.

$S = 0.02(2) = 0.040$ m

82"; $\frac{0.04}{100} 206,264.8 \approx 82.5''$

8.16 Discuss the advantages of a robotic total station instrument.

From Section 8.6: "The computer retrieves the direction to the point from storage or computes it and activates a servomotor to turn the telescope to that direction within a few seconds. This feature is particularly useful for construction stakeout, but it is also convenient in control surveying when multiple observations are made in observing angles. In this instance, final precise pointing is done manually."

In essence, it speeds the field operations.

8.17 Explain why the level bubble should be shaded when leveling an instrument in bright sun.

From Section 8.5, paragraph 2: "A solid tripod setup is essential, and the instrument must be shaded if set up in bright sunlight. Otherwise, the bubble will expand and run toward the warmer end as the liquid is heated."

8.18 How is a total station with a level bubble off by 2 graduations leveled in the field?

The bubble should be leveled in all positions by bringing the bubble back by 1 graduation. From Section 8.19.1: "If the level vial is out of adjustment, it can be adjusted by bringing the bubble *halfway back* to the centered position by turning the screw. Repeat the test until the bubble remains centered during a complete revolution of the telescope."

8.19 An interior angle *x* and its explement *y* were turned to close the horizon. Each angle was observed once direct and once reversed using the repetition method. Starting with an initial backsight setting of $0°00'00''$ for each angle, the readings after the first and second turnings of angle *x* were $49°36'24''$ and $99°13'00''$ and the readings after the first and second turnings of angle *y* were $310°23'28''$ and $260°46'56''$. Calculate each angle and the horizon misclosure.

$x = 99°13'00''/2 = 49°36'30''$

49°36'30", 310°23'28", 2"; $y = (360° + 260°46'56'')/2 = 310°23'28''$

$\text{misclosure} = 360° - (49°36'30'' + 310°23'28'') = 2''$

*** 8.20** A zenith angle is measured as $284°13'56''$ in the reversed position. What is the equivalent zenith angle in the direct position?

75°46'04" = 360° – 284°13'56"

8.21 What is the average zenith angle given the following direct and reversed readings

Direct: $94°23'48'', 94°23'42'', 94°23'44''$
Reversed: $265°36'20'', 265°36'24'', 265°36'22''$

94°23'41"

$\sum z_D = 283°11'14''; \qquad \sum z_R = 796°49'06''$

By Equation (8.3): $94°23'41.3'' = \dfrac{283°11'14''}{3} + \dfrac{3(360) - (283°11'14'' + 796°49'06'')}{2(3)}$

In Figure 8.9(c), direct and reversed directions observed with a total station instrument from A to points B, C, and D are listed in Problems 8.23 and 8.24. Determine the values of the three angles, and the horizon misclosure.

8.22 Direct: $0°00'00'', 191°13'36'', 245°53'44'', 0°00'02''$

Reversed: $0°00'00'', 191°13'42'', 245°53'46'', 0°00'00''$

$\mathbf{191°13'39'', 54°40'06'', 114°06'15''}$
$\sum \measuredangle = \mathbf{360°00'00''}$; **misclosure = 0″**

8.23 Direct: $0°00'00'', 43°11'12'', 121°36'42'', 0°00'02''$

Reversed: $359°59'58'', 43°11'16'', 121°36'48'', 359°59'56''$

$\mathbf{43°11'15'', 78°25'31'', 238°23'14''}$
$\sum \measuredangle = \mathbf{360°}$, **misclosure = 0″**

***8.24** The angles at point X were observed with a total station instrument. Based on 4 readings, the standard deviation of the angle was $\pm 5.6''$. If the same procedure is used in observing each angle within a six-sided polygon, what is the estimated standard deviation of closure at a 95% level of probability?

27"; $1.9599(5.6)\sqrt{6} = 26.9''$

8.25 The line of sight of a total station is out of adjustment by $5''$.

(a) In prolonging a line by plunging the telescope between backsight and foresight, but not double centering, what angular error is introduced?

10"; 2(5")

(b) What off-line linear error results on a foresight of 300 m?

14.5 mm; 300tan(10") = 0.0145 m

8.26 A line PQ is prolonged to point R by double centering. Two foresight points R' and R'' are set. What angular error would be introduced in a single plunging based on the following lengths of QR and $R'R''$ respectively?

(a)* 650.50 ft and 0.35 ft.

56"; $\tan^{-1}\left(\frac{0.35/2}{650.05}\right) = 55.5''$

(b) 253.432 m and 23 mm.

9"; $\tan^{-1}\left(\frac{0.023/2}{253.432}\right) = 9.4''$

8.27 Explain why the "principal of reversion" is important in angle measurement.

From Section 8.15 and 8.20.1: The principle of reversion is applied when angles are measured in both the direct and reversed positions. The procedure negates the effect of the horizontal axis not being perpendicular to the vertical axis. It is also used in detecting a maladjusted level and in prolonging a line of sight.

8.28 What is indexing error, and how can its value be obtained and eliminated from observed zenith angles?

From Section 8.13, paragraph 4 and Section 8.20.1, paragraph 4:

An index error exists if 0° on the vertical circle is not truly at the zenith with the instrument in the direct mode. The error ca be eliminated by computing the mean from equal numbers of vertical angles read in the direct and reversed modes.

***8.29** A total station with a $20''/\text{div.}$ level bubble is one divisions out of level on a point with an altitude angle of $38°15'44''$. What is the error in the horizontal pointing?

16"; By Equation (8.4): $E_H = (20'')\tan(38°15'44'') = 15.8''$

8.30 What is the equivalent altitude angle for a zenith angle of $86°02'06''$?

3°57'54"

8.31 What error in horizontal angles is consistent with the following linear precisions?

(a) 1/5000, 1/10,000, 1/20,000, and 1/100,000

41", 20.6", 10.3", 2"

(b) 1/300, 1/800, 1/1000, 1/3000, and 1/8000

11'28", 4'18", 3'26", 68.8", 25.8"

8.32 Why is it important to check if the shoes on a tripod are tight?

If the shoes are not tight, an unstable setup will result.

8.33 Describe the procedure to adjust an optical plummet on a total station.

From Section 8.19.4: "To adjust a plummet contained in the alidade, set the instrument over a fine point and aim the line of sight exactly at it by turning the leveling screws. Carefully adjust for any existing parallax. Rotate the instrument 180° in azimuth. If the plummet reticle moves off the point, bring it *halfway* back by means of the adjusting screws provided. These screws are similar to those shown in Figure 8.19. As with any adjustment, repeat the test to check the adjustment and correct if necessary.

For the second case where the optical plummet is part of the tribrach, carefully lay the instrument, with the tribrach attached, on its side (horizontally) on a stable base such as a bench or desk, and clamp it securely. Fasten a sheet of paper on a vertical wall at least six feet away, such that it is in the field of view of the optical plummet's telescope. With the horizontal lock clamped, mark the position of the optical plummet's line of sight on the paper. Release the horizontal lock and rotate the tribrach 180°. If the reticle of the optical plummet moves off the point, bring it *halfway* back by means of the adjusting screws. Center the reticle on the point again with the leveling screws, and repeat the test."

8.34 List the procedures for "wiggling-in" a point.

See Section 8.16

8.35 A zenith angle was read twice direct giving values of 86°34′12″ and 86°36′16″, and twice reversed yielding readings of 273°25′32″ and 273°25′36″. What is the mean zenith angle? What is the indexing error?

By Equation 8.3: **86°34′20″**; **indexing error = 6″**

8.36 Write a review of an article on total station instruments written in a professional journal.

Solutions will vary.

8.37 Create a computational program that takes the directions in Figure 8.12 and computes the average angles, their standard deviations, and the horizon misclosure.

Solutions will vary.

9 TRAVERSING

Asterisks (*) indicate problems that have answers given in Appendix G.

9.1 Discuss the differences and similarities between a polygon and link traverse.

From Section 9.1: Polygon and link traverses are both closed mathematically, which means the observations can be checked. A polygon traverse commences from and ends at the same station whereas a link traverse starts at one station with known position and end on another. The polygon traverse is closed geometrically whereas the link traverse is an open figure.

9.2 Discuss the differences between an open and closed traverse.

From Section 9.1, a closed traverse provides a mathematical method of checking the observations whereas an open traverse has no geometric nor mathematical checks on the observations.

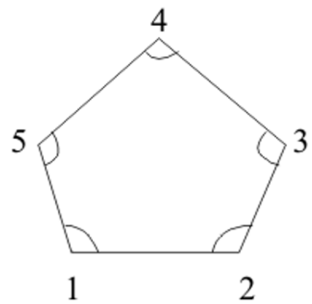
9.3 How can an angular closure be obtained on a link traverse?

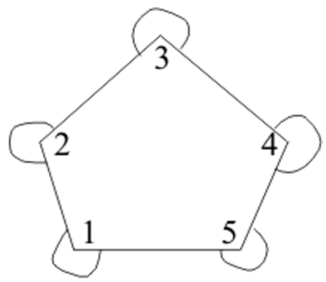
From Section 9.7, paragraph 4: "A closed-polygon azimuth traverse is checked by setting up on the starting point a second time, after having occupied the successive stations around the traverse, and orienting by back azimuths. The azimuth of the first side is then obtained a second time and compared with its original value. Any difference is the misclosure. If the first point is not reoccupied, the interior angles computed from the azimuths will automatically check the proper geometric total, even though one or more of the azimuths may be incorrect."

9.4 What similarities and differences exist between interior angles and angles to the right in a polygon traverse?

From Section 9.2.2: "Depending on the direction that the traversing procedures, angles to the right may be interior or exterior angles in a polygon traverse." … "From the foregoing definitions of interior angles and angles to the right, it is evident that in a polygon traverse the only difference between the two types of observational procedures may be ordering of the backsight and foresight stations since both procedures observe clockwise angles."

9.5 Draw two five-sided closed polygon traverses with station labels 1 to 5. The first traverse should show angles to the right that are interior angles, and the second should show angles to the right that are exterior angles.





9.6 Discuss the importance of reconnaissance in establishing traverse stations.

From Section 9.4, paragraph 2: "Often the number of stations can be reduced and the length of the sight lines increased by careful reconnaissance. It is always wise to “walk” the area being surveyed and find ideal locations for stations before the traverse stakes are set and the observation process is undertaken."

9.7 How should traverse stations be referenced?

See Section 9.5.

9.8 Discuss the advantages and dangers of radial traversing.

From Section 9.9, the advantages are the several stations with known positions can be laid out quickly from one setup. However, the disadvantage is that these spur stations have no geometric or mathematical checks and thus should be resurveyed from a second station, which has coordinates derived from the first occupied station.

9.9 What should be the sum of the interior angles for a closed-polygon traverse that has: ***(a)** 6 sides **(b)** 7 sides **(c)** 10 sides.

From Equation (9.1):

(a) 720°

(b) 900°

(c) 1440°

9.10 What should the sum of the exterior angles for a closed-polygon traverse that are listed in Problem 9.9.

From Equation (9.2):

(a) 1440°

(b) 1620°

(c) 2160°

9.11 Four interior angles of a five-sided polygon traverse were observed as; $A = 98°33'26''$, $B = 111°04'37''$, $C = 123°43'58''$, and $D = 108°34''25''$. The angle at E was not observed. If all observed angles are assumed to be correct, what is the value of angle E?

<u>98°03'34"</u>

9.12 Similar to Problem 9.11, except the traverse had seven sides with observed angles of $A = 138°55'04''$, $B = 125°05'16''$, $C = 104°14'49''$, $D = 129°13'13''$, $E = 138°48'37''$ and $F = 128°08'25''$. Compute the angle at G, which was not observed.

<u>135°34'36"</u>

9.13 What is the angular misclosure of a five-sided polygon traverse with observed angles of $83°07'23''$, $105°23'01''$, $124°56'48''$, $111°51'31''$, and $114°41'27''$.

<u>10"</u>

9.14 Show that the sum of the exterior angles for a closed-polygon traverse is $(n+2)180°$.

Sum of all (interior and exterior) angles about the traverse = $n360°$; subtract off the sum of the interior angles (Equation 9.1) to get:
$n360° - (n - 2)180° = 360n - 180n + 360 = 180n + 360 = (n + 2)180°$

***9.15** According to FGSC standards, what is the maximum acceptable angular misclosure for a second order, class I traverse having 20 angles?

<u>13"</u>; by Equation (9.3) using $K = 3''$

***9.16** What is the angular misclosure for a five-sided polygon traverse with observed exterior angles of $252°26'37''$, $255°55'13''$, $277°15'53''$, $266°35'02''$, and $207°47'05''$?

<u>10"</u>

9.17 What is the angular misclosure for a six-sided polygon traverse with observed interior angles of $121°36'06''$, $125°16'04''$, $123°21'44''$, $121°09'58''$, $120°30'12''$ and $108°06'08''$?

<u>12"</u>

9.18 Discuss how a data collector can be used to check the setup of a total station in traversing.

From Section 9.8, paragraph 4: "Mistakes in orientation can be minimized when a data collector is used in combination with a total station. In this process, the coordinates of each backsight station are checked before proceeding with the angle and distance observations to the next foresight station. For example in Figure 9.1(a), after the total station is leveled and oriented at station *B,* an observation is taken “back” on *A*. If the newly computed coordinates of *A* do not closely match their previously stored values, the instrument setup, leveling, and orientation should be rechecked, and the problem resolved before proceeding with any further measurements. This procedure often takes a minimal amount of time and typically identifies most field mistakes that occur during the observational process."

***9.19** If the standard error for each measurement of a traverse angle is $\pm 3.3''$, what is the expected standard error of the misclosure in the sum of the angles for a eight-sided traverse?

9.3"; by Equation (3.12)

9.20 If the angles of a traverse are turned so that the 95% error of any angle is $\pm 2.5''$, what is the 95% error in a twelve-sided traverse?

17"; by Equation (3.12) using an E_{95} multiplier of 1.96

9.21 What criteria should be used when making reference ties to traverse stations?

From Section 9.5, paragraph 2: "As illustrated, these ties consist of distance observations made to nearby fixed objects. Short lengths (less than 100 ft) are convenient if a steel tape is being used, but, of course, the distance to definite and unique points is a controlling factor. Two ties, preferably at about right angles to each other, are sufficient, but three should be used to allow for the possibility that one reference mark may be destroyed. Ties to trees can be observed in hundredths of a foot if nails are driven into them. However, *permission must be obtained from the landowner before driving nails into trees*."

***9.22** The azimuth from station A of a link traverse to an azimuth mark is $212°12'36''$. The azimuth from the last station of the traverse to an azimuth mark is $192°12'15''$. Angles to the right are observed at each station: $A = 136°15'41''$, $B = 119°15'37''$, $C = 93°48'55''$, $D = 136°04'17''$, $E = 108°30'10''$, $F = 42°48'03''$, and $G = 63°17'17''$. What is the angular misclosure of this link traverse?

21"

$Az_{A\text{-}Mk}$	212°12'36"	DC	21°32'49"
$+A$	+136°15'41"	$+D$	+136°04'17"
AB	348°28'17"	DE	157°37'06"
	–180°		+180°
BA	168°28'17"	ED	337°37'06"
$+B$	+119°15'37"	$+E$	+108°30'10"
BC	287°43'54"	EF	446°07'16"
	–180°		–180°
CB	107°43'54"	FE	266°07'16"
$+C$	+93°48'55"	$+F$	+42°48'03"
CD	201°32'49"	FG	308°55'19"
	–180°		–180°
DC	21°32'49"	GF	128°55'19"
		$+G$	+63°17'17"
		$Az_{G\text{-}Mk}$	192°12'36"

Misclosure = 192°12'36" − 192°12'15" = 21"

9.23 What FGCS order and class does the traverse in Problem 9.22 meet?

Third order, Class I; By Equation (9.3): $K = \frac{21''}{\sqrt{7}} = 7.9''$

***9.24** The interior angles in a five-sided closed-polygon traverse were observed as $A = 104°28'36''$, $B = 110°26'54''$, $C = 106°25'58''$, $D = 102°27'02''$, and $E = 116°11'15''$. Compute the angular misclosure. For what FGCS order and class is this survey adequate?

−15"; Third order, Class I; By Equation (9.3): $K = \frac{15''}{\sqrt{5}} = 6.7''$

9.25 Similar to Problem 9.24, except for a six-sided traverse with observed exterior angles of $A = 244°28'36''$, $B = 238°26'54''$, $C = 246°25'58''$, $D = 234°27'02''$, $E = 235°08'55''$, and $F = 241°02'45''$.

10"; Second order, Class II, By Equation (9.3): $K = \frac{10''}{\sqrt{6}} = 4.1''$

9.26 In Figure 9.6, what is the average interior angle with the instrument at station 101.

82°18'18"; $\frac{82°18'19'' + (262°18'18'' - 180°)}{2} = 82°18'18.5''$

9.27 Same as Problem 9.26 except at instrument station 103.

49°33'46"; $\frac{49°33'46'' + (229°33'47'' - 180°)}{2} = 49°33'46.5''$

9.28 Explain why it is advisable to use two instrument stations, as O and O' in Figure 9.7(b), when running radial traverses.

From Section 9.9, paragraph 2: To provide checks in computed positions for observed stations.

9.29 Create a computational program that computes the misclosure of interior angles in a closed polygon traverse. Use this program to solve Problem 9.24.

Solutions will vary.

9.30 Create a computational program that computes the misclosure of angles in a closed link traverse. Use this program to solve Problem 9.22.

Solutions will vary.

10 TRAVERSE COMPUTATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

10.1 What are the usual steps followed in adjusting a closed traverse?

From Section 10.1, paragraph 3: "The usual steps followed in making elementary traverse computations are (1) adjusting angles or directions to fixed geometric conditions, (2) determining preliminary azimuths (or bearings) of the traverse lines, (3) calculating departures and latitudes and adjusting them for misclosure, (4) computing rectangular coordinates of the traverse stations, and (5) calculating the lengths and azimuths (or bearings) of the traverse lines after adjustment."

***10.2** The sum of seven interior angles of a closed-polygon traverse each read to the nearest $3''$ is $899°59'39''$. What is the misclosure, and what correction would be applied to each angle in balancing them by method 1 of Section 10.2?

Misclosure = −21"; Apply +3" correction per angle

10.3 Similar to Problem 10.2, except the angles were read to the nearest $3''$ and their sum was $720°00'18''$ for a six-sided polygon traverse.

Misclosure = +18"; Apply −3" correction per angle.

10.4 Similar to Problem 10.2, except the angles were read to the nearest $1''$ and their sum for a nine-sided polygon traverse was $1259°59'44''$.

Misclosure = −16"; Apply +1.8" correction per angle.

***10.5** Balance the angles in Problem 9.22. Compute the preliminary azimuths for each course.

Preliminary computations are in the solution of Problem 9.22. The misclosure was 21".

Balanced angles (Correction −3" per angle):
$A = \mathbf{136°15'38''}$, $B = \mathbf{119°15'34''}$, $C = \mathbf{93°48'52''}$, $D = \mathbf{136°04'14''}$, $E = \mathbf{108°30'07''}$, $F = \mathbf{42°48'00''}$, **and** $G = \mathbf{63°17'14''}$.

Preliminary azimuths: ***AB* = 348°28'14"; *BC* = 287°43'48"; *CD* = 201°32'40"; *DE* = 157°36'54"; *EF* = 86°07'01"; *FG* = 308°55'01"**

10.6 Balance the following interior angles (angles-to-the-right) of a five-sided closed polygon traverse using method 1 of Section 10.2. If the azimuth of side *AB* is fixed at 74°31′17″, calculate the azimuths of the remaining sides. A = 105°13′14″; B = 92°36′06″; C = 67°15′22″; D = 217°24′30″; E = 57°30′38″. (Note: line BC bears NW.)

Misclosure = −10"; Correction = +2" per angle

Balanced angles: A = 105°13′16″; B = 92°36′08″; C = 67°15′24″; D = 217°24′32″; E = 57°30′40″.

Azimuths: *AB* = 74°31'17"; *BC* = 347°07'25"; *CD* = 234°22'49"; *DE* = 271°47'21"; *EA* 149°18'01"

10.7 Compute departures and latitudes, linear misclosure, and relative precision for the traverse of Problem 10.6 if the lengths of the sides (in feet) are as follows: *AB* = 2157.34; *BC* = 1722.58; *CD* = 1318.15; *DE* = 1536.06; and *EA* = 1785.58. (Note: Assume units of feet for all distances.)

Course	Length	Dep	Lat
AB	2,157.34	2079.094	575.748
BC	1,722.58	-383.874	1679.262
CD	1,318.15	-1071.53	-767.694
DE	1,536.06	-1535.31	47.959
EA	1,785.58	911.608	-1535.34
∑ =	8,519.71	-0.009	-0.065

LEC = 0.065 ft; Relative precision = 1:130,000

10.8 Using the compass (Bowditch) rule, adjust the departures and latitudes of the traverse in Problem 10.7. If the coordinates of station *A* are *X* = 20,000 ft and *Y* = 15,000 ft, calculate (a) coordinates for the other stations, (b) lengths and azimuths of lines *AD* and *EB*, and (c) the final adjusted angles at stations *A* and *C*.

(a) Balanced departures and latitudes and coordinates.

Course	Dep	Lat	Point	X	Y
AB	2079.1	575.764	*A*	20,000.00	15,000.00
BC	−383.87	1679.28	*B*	22,079.10	15,575.76
CD	−1071.5	−767.68	*C*	21,695.22	17,255.04
DE	−1535.3	47.97	*D*	20,623.70	16,487.36
EA	911.61	−1535.3	*E*	19,088.39	16,535.33

(b) *AD* = 1612.84 ft, Az_{AD} = 22°45'00"; *EB* = 3140.88 ft, Az_{EB} = 107°47'20"

(c) Adjusted angles at *A* and *C*.

Point	Angle
A	105°13'16"
C	67°15'25"

10.9 Balance the following interior angles-to-the-right for a polygon traverse to the nearest 1″ using method 1 of Section 10.2. Compute the azimuths assuming a fixed azimuth of 277°00′04″ for line *AB*. $A = 119°37'10''$; $B = 106°12'58''$; $C = 104°39'22''$; $D = 130°01'54''$; $E = 79°28'16''$. (Note: Line *BC* bears *SW*.)

Station	Obs. Angle	**Adj. Angle**
A	119°37'10.0"	**119°37'14"**
B	106°12'58.0"	**106°13'02"**
C	104°39'22.0"	**104°39'26"**
D	130°01'54.0"	**130°01'58"**
E	79°28'16.0"	**79°28'20"**
misclosure		**−20"**

Course	**Azimuth**
AB	277°00'04.0"
BC	203°13'06.0"
CD	127°52'32.0"
DE	77°54'30.0"
EA	337°22'50.0"

10.10 Determine departures and latitudes, linear misclosure, and relative precision for the traverse of Problem 10.9 if lengths of the sides (in meters) are as follows: $AB = 223.011$; $BC = 168.818$; $CD = 182.358$; $DE = 229.024$; and $EA = 207.930$.

Course	**Length**	**Dep**	**Lat**
AB	223.011	−221.348	27.183
BC	168.818	−66.554	−155.145
CD	182.358	143.944	−111.958
DE	229.024	223.943	47.975
EA	207.930	−79.972	191.936
	∑ = 1011.141	0.0124	-0.0101

Linear misclosure = 0.016; Relative precision = 1:63,000

10.11 Using the compass (Bowditch) rule adjust the departures and latitudes of the traverse in Problem 10.10. If the coordinates of station *A* are $X = 310{,}630.892$ m and $Y = 121{,}311.411$ m, calculate (a) coordinates for the other stations and, from them, (b) the lengths and bearings of lines *CA* and *BD*, and (c) the final adjusted angles at *B* and *D*.

(a)

Course	**Dep**	**Lat**	**Point**	**X**	**Y**
AB	−221.351	27.185	*A*	310,630.892	121,311.411
BC	−66.556	−155.144	*B*	310,409.541	121,338.596
CD	143.941	−111.957	*C*	310,342.985	121,183.452
DE	223.940	47.977	*D*	310,486.926	121,071.496
EA	−79.974	191.938	*E*	310,710.866	121,119.473

(b)

Course	**Distance**	**Bearing**
CA	315.062	N66°02'15"E
BD	278.084	S16°09'27"E

(c) *B* = **106°13'03"**; *D* = **130°01'56"**

10.12 Same as Problem 10.9, except assume line AB has a fixed azimuth of 147°36′25″ and line *BC* bears *NE*.

Station	Obs. Angle	**Adj. Angle**
A	119°37'10.0"	**119°37'14"**
B	106°12'58.0"	**106°13'02"**
C	104°39'22.0"	**104°39'26"**
D	130°01'54.0"	**130°01'58"**
E	79°28'16.0"	**79°28'20"**
misclosure		**−20"**

Course	**Azimuth**
AB	147°36'25"
BC	73°49'27"
CD	358°28'53"
DE	308°30'51"
EA	207°59'11"

10.13 Using the lengths from Problem 10.10 and azimuths from Problem 10.12, calculate departures and latitudes, linear misclosure, and relative precision of the traverse.

Course	**Azimuth**	**Length**	**Dep**	**Lat**
AB	147°36'25"	223.011	119.4724	−188.309
BC	73°49'27"	168.818	162.1347	47.0303
CD	358°28'53"	182.358	−4.8328	182.2940

DE	308°30'51"	229.024	−179.2008	142.6151
EA	207°59'11"	207.93	−97.5736	−183.6145
		∑1011.141	0.0000	0.0160

Linear misclosure = 0.016; Relative precision = 1:63,000

10.14 Adjust the departures and latitudes of Problem 10.13 using the compass (Bowditch) rule, and compute coordinates of all stations if the coordinates of station *A* are $X = 243,605.596$ m and $Y = 25,393.201$ m. Compute the length and azimuth of line *AC*.

Course	Dep	Lat	Point	X	Y
AB	119.4725	−188.312	*A*	243,605.596	25,393.201
BC	162.1347	47.0277	*B*	243,725.068	25,204.889
CD	−4.8328	182.2911	*C*	243,887.203	25,251.916
DE	−179.201	142.6115	*D*	243,882.370	25,434.207
EA	−97.5736	−183.618	*E*	243,703.170	25,576.819

Course	Length	Azimuth
AC	315.0618	116°38'36"

10.15 Compute and tabulate for the following closed-polygon traverse: (a) preliminary bearings (b) unadjusted departures and latitudes (c) linear misclosure and (d) relative precision. (Note: line *BC* bears *NE*.)

Course	Bearing	Length (m)	Interior Angle (Right)
AB	S50°54′23″E	329.722	$A = 120°07'10''$
BC		210.345	$B = 59°39'10''$
CD		279.330	$C = 248°00'57''$
DE		283.426	$D = 86°51'04''$
EF		433.007	$E = 102°09'16''$
FA		307.625	$F = 103°12'41''$

(a) Preliminary bearings are listed below.

(b) Unadjusted latitudes and departures are listed below.

(c) **0.0235 m**

(d) **1:78,000**

From WolfPack:

Station	Obs. Angle	Adj. Angle
A	120°07'10"	120°07'07"
B	59°39'10"	59°39'07"
C	248°00'57"	248°00'54"
D	86°51'04"	86°51'01"
E	102°09'16"	102°09'13"
F	103°12'41"	103°12'38"

Misclosure 18"

Course	Length	Bearing	Dep (m)	Lat (m)
AB	329.722	S50°54'23"E	255.9028	−207.9192
BC	210.345	N8°44'44"E	31.9823	207.8994
CD	279.330	N76°45'38"E	271.9058	63.9725
DE	283.426	N16°23'21"W	−79.9715	271.9096
EF	433.007	S85°45'52"W	−431.8244	−31.9806
FA	307.625	S8°58'30"W	−47.9906	−303.8586
Sum	1843.455		0.0044	0.0231

Linear	**misclosure**	**0.0235**
Relative	**Precision**	**1:78,000**

10.16* In Problem 10.15, if one side and/or angle is responsible for most of the error of closure, which is it likely to be?

The bearing of the misclosure line is N10°47'03"E. The line most closely matching this bearing is *BC*. Thus *BC* is the line most likely course with a distance blunder.

10.17 Adjust the traverse of Problem 10.15 using the compass rule. If the coordinates in meters of point *A* are 6521.951 E and 7037.072 N, determine the coordinates of all other points. Find the length and bearing of line *AE*.

Course	Dep	Lat	Station	X	Y
AB	255.9020	−207.9233	*A*	6521.951	7037.072
BC	31.9818	207.8967	*B*	6777.853	6829.149
CD	271.9052	63.9690	*C*	6809.835	7037.045
DE	−79.9722	271.9061	*D*	7081.740	7101.014
EF	−431.8254	−31.9861	*E*	7001.768	7372.921
FA	−47.9913	−303.8624	*F*	6569.942	7340.934

***AE* = 585.678 m; Brg$_{AE}$ = N30°59'25"E**

For the closed-polygon traverses given in Problem 10.18 through 10.19 (lengths in feet), compute and tabulate: (a) unbalanced departures and latitudes (b) linear misclosure (c) relative precision and (d) preliminary coordinates if $X_A = 10{,}000.00$ and $Y_A = 5000.00$. Balance the traverses by coordinates using the compass rule.

	Course	*AB*	*BC*	*CD*	*DA*
10.18	Bearing	N54°07′19″W	S38°52′55″W	S30°38′15″E	N44°47′31″E
	Length	305.55	239.90	283.41	373.00
10.19	Azimuth	124°09′35″	61°57′48″	298°13′52″	238°20′54″
	Length	541.17	612.41	615.35	524.18

Solutions from WolfPack:

10.18

```
Angle Summary
 Station   Unadj. Angle     Adj. Angle
 ---------------------------------------
```

A	78° 5' 0.0"	78°05'10.0"
B	96° 0' 4.0"	96°00'14.0"
C	110°28'40.0"	110°28'50.0"
D	75°25'36.0"	75°25'46.0"

Angular misclosure (sec): -40"

Course	Length	Bearing	Unbalanced Dep	Lat
AB	305.55	N57°07'19.0"W	-256.609	165.869
BC	239.90	S38°52'55.0"W	-150.589	-186.748
CD	283.41	S30°38'15.0"E	144.427	-243.848
DA	373.00	N44°47'31.0"E	262.791	264.707
Sum =	1,201.86		0.020	-0.021

Balanced Dep	Lat	Point	Coordinates X	Y
-256.614	165.874	1	7,193.66	7,308.95
-150.593	-186.744	2	6,937.05	7,474.82
144.422	-243.843	3	6,786.45	7,288.08
262.785	264.713	4	6,930.87	7,044.24

Linear misclosure = 0.029
Relative Precision = 1 in 42,000

Area: 87,600 sq. ft.
2.011 acres {if distance units are feet}

Adjusted Observations

Course	Distance	Bearing	Point	Angle
AB	305.56	N57°07'18"W	1	78°05'16"
BC	239.90	S38°53'00"W	2	96°00'18"
CD	283.40	S30°38'14"E	3	110°28'46"
DA	373.00	N44°47'26"E	4	75°25'40"

10.19

Angle Summary

Station	Unadj. Angle	Adj. Angle
A	65°48'44.0"	65°48'41.0"
B	117°48'16.0"	117°48'13.0"
C	56°16' 7.0"	56°16'04.0"
D	120° 7' 5.0"	120°07'02.0"

Angular misclosure (sec): 12"

Course	Length	Azimuth	Unbalanced Dep	Lat
AB	541.17	124°09'35.0"	447.805	-303.868
BC	612.41	61°57'48.0"	540.542	287.855
CD	615.35	298°13'52.0"	-542.152	291.079
DA	524.18	238°20'54.0"	-446.210	-275.065

```
            ----------                  ---------  ---------
   Sum =   2,293.11                      -0.016     0.000

              Balanced                            Coordinates
          Dep         Lat     Point          X               Y
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
      447.809   -303.868     A      10,000.00       5,000.00
      540.546    287.855     B      10,447.81       4,696.13
     -542.148    291.078     C      10,988.35       4,983.99
     -446.207   -275.065     D      10,446.21       5,275.07

     Linear misclosure  = 0.016
     Relative Precision = 1 in 145,800
     Area: 286,100 sq. ft.
            6.568 acres {if distance units are feet}

Adjusted Observations
~~~~~~~~~~~~~~~~~~~~~
    Course    Distance   Azimuth     Point      Angle
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
       AB       541.17  124°09'34"      A   65°48'41"
       BC       612.41   61°57'49"      B  117°48'14"
       CD       615.35  298°13'53"      C   56°16'04"
       DA       524.18  238°20'53"      D  120°07'01"
```

10.20 Compute the linear misclosure, relative precision, and adjusted lengths and azimuths for the sides after the departures and latitudes are balanced by the compass rule in the following closed-polygon traverse.

Course	Length (m)	Departure (m)	Latitude (m)
AB	399.233	−367.851	+155.150
BC	572.996	+129.550	−558.158
CA	640.164	+497.420	+403.018

		Unbalanced		**Balanced**		**Adjusted**	
Course	**Length**	**Departure**	**Latitude**	**Departure**	**Latitude**	**Length**	**Azimuth**
AB	399.233	−367.851	155.150	−367.856	155.146	399.234	292°52'05"
BC	572.996	−129.550	−558.152	−129.557	−558.158	572.996	193°04'04"
CA	640.164	497.420	403.018	497.412	403.012	640.186	50°59'06"
	1612.39	0.019	0.016	0.000	0.000		

10.21 The following data apply to a closed link traverse [like that of Figure 9.1(b)]. Compute preliminary azimuths, adjust them, and calculate departures and latitudes, misclosures in departure and latitude, and traverse relative precision. Balance the departures and latitudes using the compass rule, and calculate coordinates of points *B*, *C*, and *D*. Compute the final lengths and azimuths of lines *AB*, *BC*, *CD*, and *DE*.

Station	Measured Angle (to the right)	Adjusted Azimuth	Measured Length (ft)	X (ft)	Y (ft)
$AzMk_1$					
		342°09′28″			
A	258°12′17″			2,521,005.86	379,490.84
			200.55		
B	215°02′53″				
			253.84		
C	128°19′11″				
			205.89		
D	237°34′05″			2,521,575.16	379,714.76
		101°18′31″			
$AzMk_2$					

From WolfPack:

```
Angle Summary
Station   Unadj. Angle     Adj. Angle
-------------------------------------
    A      258°12'18"      258°12'27"
    B      215° 2'53"      215°03'02"
    C      128°19'11"      128°19'20"
    D      237°34'05"      237°34'14"
Angular misclosure (sec): -36"
```

```
                                          Unbalanced
Course      Length       Azimuth        Dep          Lat
-------------------------------------------------------
 AB         200.55     60°21'55.0"   174.317      99.166
 BC         253.84     95°24'57.0"   252.707     -23.958
 CD         205.89     43°44'17.0"   142.345     148.757
         ----------                 ---------  --------
  Sum =     660.28                   569.369    223.965

Misclosure in Departure = 569.369 - 569.300 = 0.069
Misclosure in Latitude  = 223.965 - 223.920 = 0.045

     Linear misclosure  = 0.082
     Relative Precision = 1 in 8,100

            Balanced                       Coordinates
        Dep         Lat     Point         X             Y
------------------------------------------------------------
      174.296     99.152     A   2,521,005.86     379,490.84
      252.680    -23.975     B   2,521,180.16     379,589.99
      142.323    148.743     C   2,521,432.84     379,566.02
```

```
                         D   2,521,575.16     379,714.76

Adjusted Observations
    Course    Distance    Azimuth  Station      Angle
-----------------------------------------------------
       AB       200.53   60°21'57"      A  258°12'29"
       BC       253.82   95°25'13"      B  215°03'16"
       CD       205.87   43°44'11"      C  128°18'58"
                                        D  237°34'14"
```

10.22 Similar to Problem 10.21, except use the following data:

Station	Measured Angle (to right)	Adjusted Azimuth	Measured Length (m)	X (m)	Y (m)
$AzMk_1$					
		330°40′42″			
A	82°57′54″			185,435.380	24,947.460
			285.993		
B	261°21′42″				
			275.993		
C	149°31′27″				
			318.871		
D	118°33′32″				
			236.504		
E	215°00′51″			184,539.770	24,880.286
		258°05′38″			
$AzMk_2$					

From WolfPack:

```
          Angle Summary
          Station   Unadj. Angle     Adj. Angle
          ------------------------------------
              A       82°57'54"      82°57'48"
              B      261°21'42"     261°21'36"
              C      149°31'27"     149°31'21"
              D      118°33'32"     118°33'26"
              E      215° 0'51"     215°00'45"
          Angular misclosure (sec): 30"
                                               Unbalanced
Course      Length       Azimuth        Dep         Lat
-------------------------------------------------------
 AB        285.993    233°38'30.0" -230.3174  -169.5462
 BC        275.968    315°00'06.0" -195.1332   195.1445
 CD        318.871    284°31'27.0" -308.6805    79.9691
 DE        236.504    223°04'53.0" -161.5409  -172.7388
         ----------                 ---------  ---------
  Sum =  1,117.336                 -895.6719   -67.1713

Misclosure in Departure = -895.6719 - -895.6110 = -0.0609
Misclosure in Latitude  = -67.1713 - -67.1740 = 0.0027

     Linear misclosure  = 0.0610
     Relative Precision = 1 in 18,300
```

```
              Balanced                              Coordinates
          Dep          Lat    Station          X               Y
------------------------------------------------------------
    -230.3018  -169.5469      A    185,435.381     24,947.460
    -195.1181   195.1439      B    185,205.079     24,777.913
    -308.6631    79.9684      C    185,009.961     24,973.057
    -161.5280  -172.7393      D    184,701.298     25,053.025
                              E    184,539.770     24,880.286

Adjusted Observations
    Course    Distance   Azimuth   Station      Angle
-----------------------------------------------------
       AB      285.981  233°38'23"      A   82°57'41"
       BC      275.957  315°00'14"      B  261°21'51"
       CD      318.854  284°31'29"      C  149°31'16"
       DE      236.496  223°04'44"      D  118°33'15"
                                        E  215°00'45"
```

The azimuths (from north of a polygon traverse are $AB = 38°17'02''$, $BC = 121°26'30''$, $CD = 224°56'59''$, and $DA = 308°26'56''$. If one observed distance contains a mistake, which course is most likely responsible for the closure conditions given in Problems 10.23 and 10.24? Is the course too long or too short?

10.23* Algebraic sum of departures = 5.12 ft latitudes = −3.13 ft.

Az_{LEC} = 121°26'19", which closely matches course ***BC***

10.24 Algebraic sum of departures = −3.133 m latitudes = +2.487 m.

Az_{LEC} = 308°26'34", which closely matches course ***DA***

10.25 Determine the lengths and bearings of the sides of a lot whose corners have the following *X* and *Y* coordinates (in feet): *A* (5000.00, 5000.00); *B* (5289.67, 5436.12); *C* (4884.96, 5354.54); *D* (4756.66,5068.37).

Course	Length	Azimuth
AB	523.55	33°35'31"
BC	412.85	258°36'12"
CD	313.61	204°08'54"
DA	252.76	223°10'58"

10.26 Compute the lengths and azimuths of the sides of a closed-polygon traverse whose corners have the following X and Y coordinates (in meters): A (8000.000, 5000.000); B (2650.000, 4702.906); C (1752.028, 2015.453); D (1912.303, 1511.635).

Course	Length	Azimuth
AB	5358.24	266°49'18"
BC	2833.51	198°28'35"
CD	528.70	162°21'11"
DA	7016.32	231°40'28"

10.27 In searching for a record of the length and true bearing of a certain boundary line which is straight between *A* and *B*, the following notes of an old random traverse were found (survey by compass and Gunter's chain, declination 4°45′W). Compute the true bearing and length (in feet) of *BA*.

Course	***A*-1**	**1-2**	**2-3**	**3-*B***
Magnetic bearing	Due North	N20°00′E	Due East	S46°30′E
Distance (ch)	11.90	35.80	24.14	12.72

Course	**BA**
Distance (ch)	58.60
Bearing	S55°51'50"W

Convert direction to true north and then compute departures and latitudes shown below.

Course	***A*-1**	**1-2**	**1-3**	**3-*B***	**Total**
Departure	0.985	14.988	24.057	8.470	48.501
Latitude	11.859	32.512	-1.999	-9.490	32.882

10.28 Describe how a blunder may be located in a traverse.

From Section 10.16: If a single blunder in a distance exists, the azimuth of the misclosure line will closely approximate the azimuth of the course with the blunder. If the blunder is in an angle, the perpendicular bisector of the misclosure line will come close to bisecting the angle with the blunder.

Instructor's Note: The Mathcad worksheet C10.xmcd and its equivalent html file demonstrate a traverse with a single angle blunder. A graphic of the traverse contained in the files shows how the perpendicular bisector of the misclosure line points at the station with the angular blunder.

10.29 Create a computational program that solves Problem 10.19.

Solutions will vary.

10.30 Create a computational program that solves Problem 10.23.

Solutions will vary.

11 COORDINATE GEOMETRY IN SURVEYING CALCULATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

11.1 The *X* and *Y* coordinates (in meters) of station Shore are 379.241 and 819.457, respectively, and those for station Rock are 437.854 and 973.482, respectively. What are the azimuth, bearing, and length of the line connecting station Shore to station Rock?

Az = 20°50'02"; Brg = N 20°50'02" E; Distance = 164.800 m

11.2 Same as Problem 11.1, except that the *X* and *Y* coordinates (in feet) of Shore are 3875.17 and 5678.15, respectively, and those for Rock are 1831.49 and 3849.61, respectively.

Az = 41°46'39"; Brg = N 41°46'39" E; Distance = 2706.20 ft

***11.3** What are the slope, and y-intercept for the line in Problem 11.1?

m = **2.62783; *b* = −177.124 m**

11.4 What are the slope, and the y-intercept for the line in Problem 11.2?

m = **1.116325**; b = **−2506.08 ft**

***11.5** If the slope (*XY* plane) of a line is 0.800946, what is the azimuth of the line to the nearest second of arc? (*XY* plane)

51°18'26"

11.6 If the slope (*XY* plane) of a line is -0.240864, what is the azimuth of the line to the nearest second of arc? (*XY* plane)

102°32'33" or **283°32'33**

***11.7** What is the perpendicular distance of a point from the line in Problem 11.1, if the *X* and *Y* coordinates (in meters) of the point are 422.058 and 932.096, respectively?

0.044 m

$Az_{SR} = 20°48'47''$; $SR = 120.502\,\text{m}$

By (11.11): $\alpha = 0°01'15''$

By (11.12):

$SP = 120.502 \sin 0°01'15'' = 0.044$

11.8 What is the perpendicular distance of a point from the line in Problem 11.2, if the X and Y coordinates (in feet) of the point are 2698.98 and 2408.61, respectively?

$$Az_{SR} = 41°48'38''; \ SR = 1163.862 \text{ m}$$

0.67 ft

By (11.11): $\alpha = 0°01'59''$

By (11.12): $SP = 1163.862 \sin 0°01'59'' = 0.67$

***11.9** A line with an azimuth of $105°46'33''$ from a station with X and Y coordinates of 5885.31 and 5164.15, respectively, is intersected with a line that has an azimuth of $200°31'24''$ from a station with X and Y coordinates of 7337.08 and 5949.99, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

(6932.18, 4868.39)

D_{12} = 1650.81 ft; Az_{12} = 61°34'24"; $\angle 1$ = 44°12'09"; $\angle P$ = 94°44'51" D_{2P} = 1154.90 ft

11.10 A line with an azimuth of $74°39'34''$ from a station with X and Y coordinates of 1530.66 and 1401.08, respectively, is intersected with a line that has an azimuth of $301°56'04''$ from a station with X and Y coordinates of 1895.53 and 1348.16, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

(1725.06, 1454.41)

D_{12} = 368.688 ft; Az_{12} = 98°15'09"; $\angle 1$ = 23°35'35"; $\angle 2$ = 23°40'55" D_{1P} = 201.58 ft

11.11 Same as Problem 11.9 except that the bearing of the first line is S $50°22'44''$ E and the bearing of the second line is S $28°42'20''$ W.

(6588.12, 4582.30)

D_{12} = 1650.81 ft; Az_{12} = 61°34'24"; $\angle 1$ = 61°03'09"; $\angle P$ = 79°05'00" D_{2P} = 912.32 ft

11.12 In the accompanying figure, the X and Y coordinates (in meters) of station A are 5084.274 and 8579.124, respectively, and those of station B are 6012.870 and 6589.315, respectively. Angle BAP was measured as $315°15'47''$ and angle ABP was measured as $41°21'58''$. What are the coordinates of station P?

(6448.921, 8075.795)

D_{12} = 2195.821 m; Az_{12} = 154°58'58"; $\angle 1$ = 44°44'13"; $\angle P$ = 93°53'49" D_{1P} = 1454.511 m; Az_{1P} = 110°14'44.6"

***11.13** In the accompanying figure, the X and Y coordinates (in feet) of station A are 1248.16 and 3133.35, respectively, and those of station B are 1509.15 and 1101.89, respectively. The length of BP is 2657.45 ft, and the azimuth of line AP is $98°25'00''$. What are the coordinates of station P?

(3560.56, 2791.19)

Az_{AB} = 172°40'45"; D_{AB} = 2048.157; D_{AP} = 2337.576 or −1226.525; ∠PAB = 74°15'45";

a = 1; b = 1111.051; c = −2,867,094.99;

11.14 In the accompanying figure, the X and Y coordinates (in feet) of station A are 7593.15 and 9971.03, respectively, and those of station B are 8401.78 and 7714.63, respectively. The length of AP is 1987.54 ft, and angle ABP is 30°58′26″. What are the possible coordinates for station P?

<u>(9107.22, 11258.63) or (8498.75, 8201.79)</u>

Az_{BA} = 340°17'01.5"; Az_{BP} = 11°15'27.5"; D_{AB} = 2396.92; D_{BP} = 3613.528 or 496.719;
a = 1; b = 4110.247; c = 1,794,908.19;

***11.15** A circle of radius 798.25 ft, centered at point A, intersects another circle of radius 1253.64 ft, centered at point B. The X and Y coordinates (in feet) of A are 3548.53 and 2836.49, respectively, and those of B are 4184.62 and 1753.52, respectively. What are the coordinates of station P in the figure?

<u>(4330.13, 2998.69) or (3026.23, 2232.83)</u>

AB = 1255.96; Az_{AB} = 149°34'18.7"; ∠PAB = 41°17'43.6"; Az_{AP} = 78°16'35" or 220°52'02"

11.16 The same as Problem 11.15, except the radii from A and B are 787.02 ft and 1405.74 ft, respectively, and the X and Y coordinates (in feet) of A are 4058.74 and 6311.32, respectively, and those of station B are 4581.52 and 4345.16, respectively.

<u>(4610.99, 5750.59) or (3857.91, 5550.36)</u>

AB = 2034.474; Az_{AB} = 165°06'36.7"; ∠PAB = 29°40'25.9"; Az_{AP} = 135°26'10.8" or 194°47'02.7"

11.17 For the subdivision in the accompanying figure, assume that lines AC, DF, GI, and JL are parallel, but that lines BK and CL are parallel to each other, but not parallel to AJ. If the X and Y coordinates (in feet) of station A are (1000.00, 1000.00), what are the coordinates of each lot corner shown?

Station	X	Y	Method
A	1000.00	1000.00	Given
B	1149.99	997.99	Forward
C	1299.97	995.99	Forward
D	1013.14	1078.91	Forward
E	1162.53	1076.92	Direction-Direction
F	1312.52	1074.91	Direction-Distance
G	1026.27	1157.83	Forward
H	1175.08	1155.84	Direction-Direction
I	1325.07	1153.83	Direction-Distance
J	1039.40	1236.74	Forward
K	1187.63	1234.76	Direction-Direction
L	1337.61	1232.75	Direction-Distance

11.18 If the *X* and *Y* coordinates (in feet) of station *A* are (5000.00, 5000.00), what are the coordinates of the remaining labeled corners in the accompanying figure?

Station	X	Y	Method
A	5000.00	5000.00	Given
B	5000.00	5400.01	Forward
C	5430.00	5400.01	Forward
D	5430.00	5000.00	Direction-Direction
E	5235.58	5193.82	Forward or Direction-Distance
F	5194.42	5193.82	Forward or Direction-Distance
G	5215.00	5171.99	Forward or Direction-Direction
H	5200.00	5146.01	Direction-Distance
I	5230.00	5146.01	Direction-Distance
J	5200.00	5000.00	Forward
K	5230.00	5000.00	Forward

***11.19** In Figure 11.8, the *X* and *Y* coordinates (in feet) of *A* are 1234.98 and 5415.48, respectively, those of *B* are 3883.94 and 5198.47, respectively, and those of *C* are 6002.77 and 5603.25, respectively. Also angle x is $36°59'21''$ and angle y is $44°58'06''$. What are the coordinates of station *P*?

(4538.67, 2940.13)

Az_{BA} = 274°41'00.1"; Az_{BC} = 79°11'04.4"; BC(a) = 2157.148; AB(c) = 2657.834;
α = 195°29'55.7"; A+C = 82°32'37.3"; A = 32°09'34.9"; C = 50°23'02.4";
α_1 = 110°51'04.1"; *AP* = 4128.165; Az_{AP} = 126°50'35.1"

11.20 In Figure 11.8, the X and Y coordinates (in feet) of *A* are 4371.56 and 8987.63, those of *B* are 8531.05 and 8312.57, and those of *C* are 10,240.98 and 8645.07, respectively. Also angle x is $50°12'45''$ and angle y is $44°58'06''$. What are the coordinates of station *P*?

(8891.54, 6582.68)

Az_{BA} = 279°13'06.2"; Az_{BC} = 78°59'45.7"; BC(a) = 1741.958; AB(c) = 4213.913;
α = 200°13'20.5"; A+C = 64°35'48.5"; A = 18°47'52.1"; C = 45°47'56.4";
α_1 = 110°59'22.9"; *AP* = 5119.963; Az_{AP} = 118°00'58.3"

11.21 In Figure 11.9, the following *EN* and *XY* coordinates for points *A* through *C* are given. In a 2-D conformal coordinate transformation, to convert the *XY* coordinates into the *EN* system, what are the

(a)* Scale factor? **0.31006** AB = 1373.231 m; ab = 4428.90 ft

(b) Rotation angle? **356°28'31.7"**; Az_{AB} = 105°24'06.6"; Az_{ab} = 101°52'38.2"

(c) Translations in X and Y? **Tx = 639,168.753 m** and **Ty = 640,542.347 m**

(d) Coordinates of points *C* in the *EN* coordinate system?

***E* = 640,860.384 m** and ***N* = 642,300.726**

	State Plane Coordinates (m)		Arbitrary Coordinates (ft)	
Point	**E**	**N**	**X**	**Y**
A	639,940.832	642,213.266	2154.08	5531.88
B	641,264.746	641,848.554	6488.16	4620.34
C			5096.84	5995.73

11.22 Do Problem 11.21 with the following coordinates.

	State Plane Coordinates (m)		Arbitrary Coordinates (m)	
Point	**E**	**N**	**X**	**Y**
A	588,933.451	418,953.421	5492.081	3218.679
B	588,539.761	420,185.869	6515.987	4009.588
C			4865.191	3649.031

(a) Scale factor? **0.9999998** AB = 1293.8006; ab = 1293.8008

(b) Rotation angle? **70°01'53.0"**; Az_{AB} = 342°17'04.5"; Az_{ab} = 52°18'57.5"

(c) Translations in X and Y? **Tx = 590,083.047** and **Ty = 412,692.330**

(d) Coordinates of points *C* in the *EN* coordinate system?

***E* = 588,314.886** and ***N* = 418,511.187**

11.23 In Figure 11.12, the elevations of stations *A* and *B* are 403.16 ft, and 410.02 ft, respectively. Instrument heights hi_A and hi_B are 5.20 ft, and 5.06 ft, respectively. What is the average elevation of point P if the other field observations are:

$$AB = 256.79 \text{ ft}$$

$$A = 52°30'08'' \quad B = 40°50'51''$$

$$v_1 = 24°38'15'' \quad v_2 = 22°35'42''$$

Elev = 880.61 ft

AI = 168.240; BI = 204.080; IP_A = 77.160; IP_B = 84.929

11.24 In Problem 11.23, assume station *P* is to the left of the line *AB*, as viewed from station *A*. If the *X* and *Y* coordinates (in feet) of station *A* are 1245.68 and 543.20, respectively, and the azimuth of line *AB* is 55°23′44″, what are the *X* and *Y* coordinates of the inaccessible point?

(1254.17, 711.22); Az_{AI} = 2°53'36"

11.25 In Figure 11.12, the elevations of stations *A* and *B* are 1106.78 ft, and 1116.95 ft, respectively. Instrument heights hi_A and hi_B are 5.14 and 5.43 ft, respectively. What is the average elevation of point P if the other field observations are:

$$AB = 438.18 \text{ ft}$$
$$A = 49°31'00'' \quad B = 52°35'26''$$
$$v_1 = 27°40'57'' \quad v_2 = 27°20'51''$$

Elev = 1298.67 ft

AI = 355.971; BI = 340.859; IP_A = 186.750; IP_B = 176.288

11.26 In Problem 11.25, assume station *P* is to the left of line *AB* as viewed from station *A*. If the *X* and *Y* coordinates (in feet) of station *A* are 8975.18 and 7201.89, respectively, and the azimuth of line *AB* is 347°22′38″, what are the *X* and *Y* coordinates of the inaccessible point?

(8673.83, 7361.18); Az_{AI} = 297°51'38"

11.27 In Figure 11.13, the *X*, *Y*, and *Z* coordinates (in feet) of station *A* are 1816.45, 987.39, and 1806.51, respectively, and those of *B* are 1633.11, 1806.48, and 1806.48, respectively. Determine the three-dimensional position of the occupied station *P* with the following observations:

$$v_1 = 30°06'22'' \quad PA = 228.50 \text{ ft} \quad hr_A = 5.68 \text{ ft} \quad \gamma = 72°02'28''$$
$$v_2 = 29°33'02'' \quad PB = 232.35 \text{ ft} \quad hr_B = 5.68 \text{ ft} \quad hi_P = 5.34 \text{ ft}$$

(1626.05, 1039.18, 1692.23)

AB = 235.139; Az_{AB} = 231°14'02.5"; PC = 197.675; PD = 202.126; $\angle DCP$ = 54°51'25.3"

P: (1626.048, 1039.175); AC = 114.616; BC = 114.593

11.28 Adapt Equations (11.43) and (11.47) so they are applicable for zenith angles.

(11.43): $PC = PA \sin(z_1)$; $PD = PB \sin(z_2)$

(11.47): $PA = PA \cos(z_1)$; $BD = PB \cos(z_2)$

11.29 In Figure 11.13, the *X*, *Y*, and *Z* coordinates (in meters) of station *A* are 135.461, 211.339, and 98.681, respectively, and those of *B* are 301.204, 219.822, and 100.042, respectively. Determine the three-dimensional position of occupied station *P* with the following observations:

$$z_1 = 119°22'38'' \quad PA = 150.550 \text{ m} \quad hr_A = 1.690 \text{ m} \quad \gamma = 79°05'02''$$
$$z_2 = 120°08'50'' \quad PB = 149.770 \text{ m} \quad hr_B = 1.690 \text{ m} \quad hi_P = 1.685 \text{ m}$$

<u>(88.531, 88.829, 24.831)</u>

AB = 111.268; Az_{AB} = 137°00'16.2"; *PC* = 131.191; *PD* = 129.511; ∠*DCP* = 63°57'21"

Az_{AP} = 200°57'37.2"; *P*: (88.531, 88.829); *AC* = −73.853; *BC* = −75.218

11.30 Use WOLFPACK to do Problem 11.9. (See solution to 11.9)

11.31 Use WOLFPACK to do Problem 11.10. (See solution to 11.10)

11.32 Use WOLFPACK to do Problem 11.12. (See solution to 11.12)

11.33 Use WOLFPACK to do Problem 11.13. (See solution to 11.13)

11.34 Use WOLFPACK to do Problem 11.15. (See solution to 11.15)

11.35 Use WOLFPACK to do Problem 11.16. (See solution to 11.16)

11.36 Use WOLFPACK to do Problem 11.17. (See solution to 11.17)

11.37 Write a computational program that solves Example 11.6 using matrices. (Solution will vary.)

11.38 Write a computational program that solves Example 11.8. (Solutions will vary.)

12 AREA

Asterisks (*) indicate problems that have answers given in Appendix G.

***12.1** Compute the area enclosed within polygon *DEFGD* of Figure 12.1 using triangles.

87,970 ft^2; DFG = 61,900 ft^2 and DEF = 26,072 ft^2

12.2 Similar to Problem 12.1, except for polygon *BCDGB* of Figure 12.1.

121,780 ft^2 or 2.7957 ac; BDG = 81,462 ft^2; BCD = 40,320 ft^2

12.3 Compute the area enclosed between line *ABGA* and the shoreline of Figure 12.1 using the offset method.

183,390 ft^2 or 4.2101 ac; *ABG* = 137,481 ft^2; shoreline = 45,911 ft^2

12.4 By rule of thumb, what is the estimated uncertainty in an $430{,}568\ \text{ft}^2$ if the estimated error in the coordinates was ±0.2 ft?

E = **186 ft^2**; By Equation (12.9)

***12.5** Compute the area between a lake and a straight line *AG*, from which offsets are taken at irregular intervals as follows (all distances in feet):

Offset Point	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Stationing	0.00	0 + 54.80	1 + 32.54	2 + 13.02	2 + 98.74	3 + 45.68	4 + 50.17
Offset	2.3	4.2	6.5	5.4	9.1	8.9	3.9

2790 ft^2; 2785.5

12.6 Repeat Problem 12.5 with the following offset in meters.

Offset Point	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Stationing	0.00	20.00	78.94	148.96	163.65	203.69	250.45
Offset	1.15	4.51	6.04	9.57	6.87	3.64	0.65

1350 m^2; 1345.5

12.7 Use the coordinate method to compute the area enclosed by the traverse of Problem 10.8.

3,570,400 ft^2 or 81.965 ac

X	Y	XY (+)	YX (-)
20000.00	15000.00		331186500
22079.10	15575.76	311515200	337919540
21695.22	17255.04	380975754	355862768
20623.70	16487.36	357696902	314717158

19088.39	16535.33	341019685	330706600
20000.00	15000.00	286325850	
		1677533391	1670392566

12.8 Calculate by coordinates the area within the traverse of Problem 10.11.

<u>66,810 m^2 or 6.681 ha</u>

X	Y	XY (+)	YX (-)
630.892	311.411		127535.6
409.541	338.596	213617.5	116133.3
342.985	183.452	75131.12	89327.55
486.926	71.496	24522.06	50824.08
710.866	119.473	58174.51	75374.56
630.892	311.411	221371.5	
		592816.7	459195.1

12.9 Compute the area enclosed in the traverse of Problem 10.8 using DMDs.

<u>3,570,400 ft^2 or 81.965 ac</u>

Dep	Lat	DMD	D-Area
2079.096	575.764	2079.10	1197068.63
-383.872	1679.276	3774.32	6338124.99
-1071.523	-767.684	2318.93	-1780201.6
-1535.310	47.970	-287.91	-13810.947
911.610	-1535.326	-911.61	1399615.46
			7140796.52

***12.10** Determine the area within the traverse of Problem 10.11 using DMDs.

<u>66,810 m^2 or 6.681 ha</u>

Dep	Lat	DMD	D-Area
-221.351	27.185	-221.351	-6017.4
-66.556	-155.144	-509.258	79008.1
143.941	-111.957	-431.873	48351
223.940	47.977	-63.992	-3070.2
-79.974	191.938	79.974	15350.1
			133622

12.11 By the DMD method, find the area enclosed by the traverse of Problem 10.20.

<u>112,710 m^2 or 11.271 ha</u>

Departure	Latitude	DMD	D-Area
-367.856	155.146	-367.856	-57071
-129.557	-558.158	-865.268	482956
497.412	403.012	-497.412	-200463

225422

12.12 Compute the area within the traverse of Problem 10.17 using the coordinate method. Check by DMDs.

182,600 m^2 or 18.260 ha

X	Y	XY (+)	YX(-)	Dep	Lat	DMD	D-Area
6521.951	7037.072		47696240	255.902	-207.923	255.902	-53208
6777.853	6829.149	44539375	46505378	31.982	207.897	543.786	113051
6809.835	7037.045	47696057	49834523	271.905	63.969	847.673	54224.8
7081.740	7101.014	48356734	49719653	-79.972	271.906	1039.606	282675
7001.768	7372.921	52213110	48439663	-431.825	-31.986	527.808	-16883
6569.942	7340.934	51399517	47877212	-47.991	-303.862	47.991	-14583
6521.951	7037.072	46233155					365278
		290437947	290072668				

12.13 Calculate the area inside the traverse of Problem 10.18 by coordinates and check by DMDs.

87,600 ft^2 or 2.011 ac

X	Y	XY (+)	YX(-)	Dep	Lat	DMD	D-Area
7193.66	7308.95		50702552	-256.61	165.874	-256.614	-42566
6937.05	7474.82	53771314	50727492	-150.59	-186.74	-663.821	123965
6786.45	7288.08	50557775	50512735	144.422	-243.84	-669.992	163373
6930.87	7044.24	47805383	50673868	262.785	264.713	-262.785	-69563
7193.66	7308.95	50657382					175209
		202791854	202616646				

12.14 Compute the area enclosed by the traverse of Problem 10.19 using the DMD method. Check by coordinates.

286,080 ft^2 or 6.5676 ac

X	Y	XY (+)	YX(-)	Dep	Lat	DMD	D-Area
10,000.00	5,000.00		52239050	447.809	-303.868	447.809	-136075
10,447.81	4,696.13	46961300	51602720	540.546	287.855	1436.16	413407
10,988.35	4,983.99	52071781	52063806	-542.148	291.078	1434.56	417569
10,446.21	5,275.07	57964315	52750700	-446.207	-275.065	446.207	-122736
10,000.00	5,000.00	52231050					572166

12.15 Find the area of the lot in Problem 10.25.

<u>115,640 ft^2 or 2.6547 ac</u>

X	Y	XY (+)	YX (-)
5000.00	5000.00		26448350
5289.67	5436.12	27180600	26555229
4884.96	5354.54	28323750	25469726
4756.66	5068.37	24758785	25341850
5000.00	5000.00	23783300	
		104046434	103815155

***12.16** Determine the area of the lot in Problem 10.26.

<u>8,868,600 m^2 or 886.86 ha</u>

X	Y	XY (+)	YX (-)
8000.000	5000.000		13250000
2650.000	4702.906	37623248	8239623
1752.028	2015.453	5340950	3854157
1912.303	1511.635	2648427	12093080
8000.000	5000.000	9561515	
		55174140	37436860

12.17 Calculate the area of Lot 16 in Figure 21.2.

<u>10,600 ft^2 or 0.243 ac</u>

12.18 Plot the lot of Problem 10.25 to a scale of 1 in. = 100 ft. Determine its surrounded area using a planimeter.

about **<u>115,640 ft^2 or 2.6547 ac</u>**

12.19 Similar to Problem 12.18, except for the traverse of Problem 10.26.

About **<u>8,868,600 m^2 or 886.86 ha</u>**

12.20 Plot the traverse of Problem 10.19 to a scale of 1 in. = 200 ft, and find its enclosed area using a planimeter.

<u>286,080 ft^2 or 6.5676 ac</u>

12.21 The (*X,Y*) coordinates (in feet) for a closed-polygon traverse *ABCDEFA* follow. *A* (1000.00, 1000.00), *B* (1661.73, 1002.89), *C* (1798.56, 1603.51), *D* (1289.82, 1623.69), *E* (1221.89, 1304.24) and *F* (1048.75, 1301.40). Calculate the area of the traverse by the method of coordinates.

<u>361,010 ft^2 or 8.2877 ac</u>

X	**Y**	**XY (+)**	**YX (-)**

1000.00	1000.00		1661730
1661.73	1002.89	1002890	1803758
1798.56	1603.51	2664600.7	2068239
1289.82	1623.69	2920303.9	1983971
1221.89	1304.24	1682234.8	1367822
1048.75	1301.40	1590167.6	1301400
1000.00	1000.00	1048750	
		10908947	10186919

12.22 Compute by DMDs the area in hectares within a closed-polygon traverse *ABCDEFA* by placing the *X* and *Y* axes through the most southerly and most westerly stations, respectively. Departures and latitudes (in meters) follow. *AB*: E dep. 50, N lat. = 45; *BC*: E dep. = 60, N lat. = 45; *CD* = E dep. =45, S lat = 25; *DE*: W dep = 70, S lat. = 40; *EF*: W dep = 50, S lat = 30; *FA*: W dep. = 35, N lat = 5.

5100 m^2 or 0.51 ha

Dep	Lat	DMD	D-Area
50	45	50	2250
60	45	160	7200
45	-25	265	-6625
-70	-40	240	-9600
-50	-30	120	-3600
-35	5	35	175
			-10200

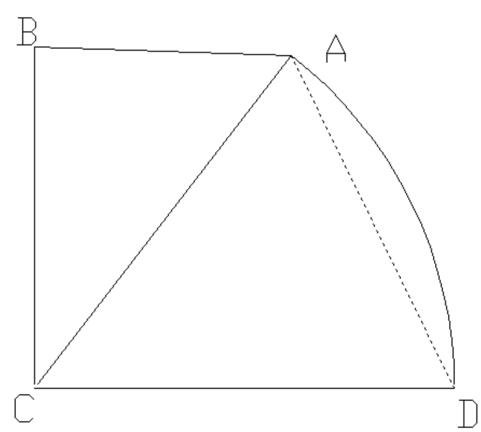
12.23 Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates at angle points: *A* (1275.11,1356.11), *B* (1000.27, 1365.70), *C* (1000.00, 1000.00), *D* (1450.00, 1000.00) with a circular arc of radius *CD* starting at *D* and ending at *A* with the curve outside the course *AD*.

142,700 ft^2 or 3.2759 ac

Area of ABCD is 130,381 ft^2

Angle *ACD* = 90° − 37°41'15" = 52°18'45"

Area of segment is 12,319 ft^2



X	Y	XY (+)	YX (-)
1275.11	1356.11		1356476
1000.27	1365.70	1741417.7	1365700
1000.00	1000.00	1000270	1450000
1450.00	1000.00	1000000	1275110
1275.11	1356.11	1966359.5	
		5708047.2	5447286

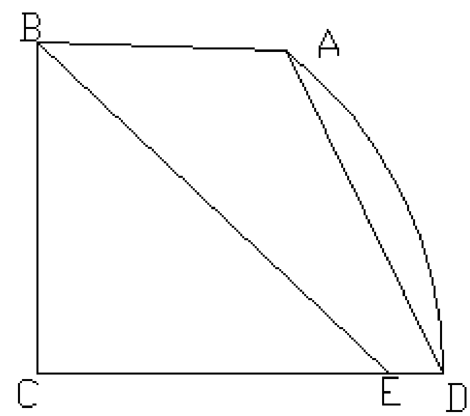
12.24 Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates in feet at angle points: *A* (526.68, 823.98), *B* (535.17, 745.61), *C* (745.17, 745.61), *D* (745.17, 845.61), *E* (546.62, 846.14) with a circular arc of radius 25 ft starting at *E*, tangent to *DE*, and ending at *A*.

<u>21,570 ft^2 or 0.49526 ac</u>

Az_{AB} = 173°49'01.7"
Azimuth to center of circle: 83°49'01.7"
Az_{ED} = 90°09'10.6"
Azimuth to center of circle: 180°09'10.6"
θ = 96°20'08.9"
Area of segment = 214.8 ft^2
Area of remainder = 21,358.6 ft^2

X	Y	XY (+)	YX (-)
526.68	823.98		440969
535.17	745.61	392697.87	555606
745.17	745.61	399028.1	555606
745.17	845.61	630123.2	462227
546.62	846.14	630518.14	445645
526.68	823.98	450403.95	
		2502771.3	2460054

12.25 Divide the area of the lot in Problem 12.23 into two equal parts by a line through point *B*. List in order the lengths and azimuths of all sides for each parcel.



Parcel A

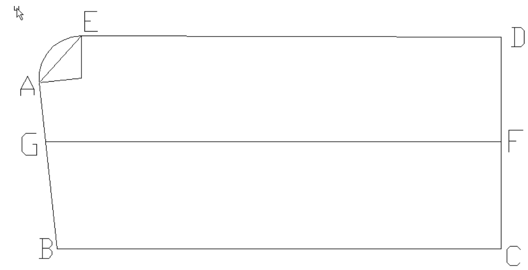
Course	Length	Azimuth
AB	275.007	271°59'54"
BE	534.593	133°09'46"
ED	597.796	90°00'00"
DA	396.738	153°50'38"

Parcel B

Course	Length	Azimuth
BC	365.700	180°02'32"
CE	390.210	270°00'00"
EB	534.593	313°09'46"

1/2 Area = 71,350 ft^2; Area$_{BCD}$ = 82,282.5 ft^2; Area$_{BCE}$ = 10,932.5 ft^2
Az_{CB} = 309°06'59" Az_{DB} = 270°
BD = 579.65 ft; ∠*CDB* = 39°06'59"
1/2(*BD*)(*DE*)sin(*CDB*) = 10932.5 ft^2; *DE* = 59.79 ft

12.26 Partition the lot of Problem 12.24 into two equal areas by means of a line parallel to *BC*. Tabulate in clockwise consecutive order the lengths and azimuths of all sides of each parcel.



Parcel A

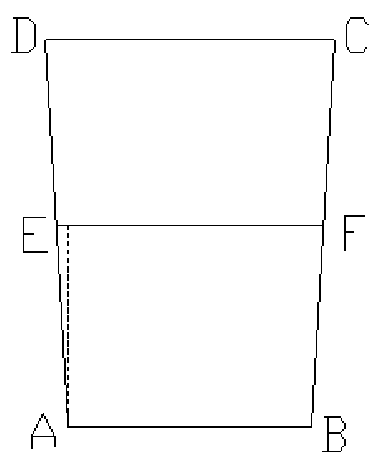
Course	Length	Azimuth
AG	28.02	173°49'02"
GF	215.47	90°00'00"
FD	49.48	0°00'00"
DE	198.55	270°09'11"
EA	29.81	221°58'53"

Parcel B

Course	Length	Azimuth
GB	50.82	173°49'02"
BC	210.00	90°00'00"
CF	50.52	0°00'00"
FG	215.47	180°00'00"

Area = 10,747.5 ft^2; $\angle ABC$ = 96°10'58"; *BC* = 210.00 ft
10,747.5 = h/2[210.00 + (210.00 + $h$tan 6°10'58")]
quadratic equation: 0 = 0.108331h^2 + 420h − 21495; h = 50.52 ft;

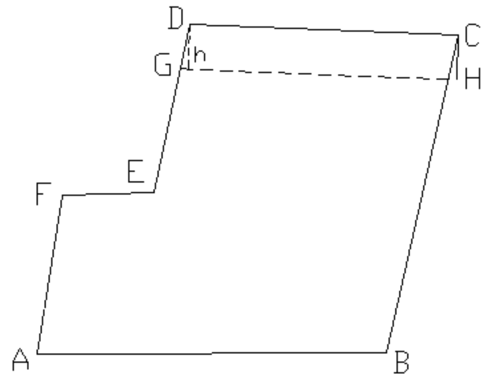
12.27 Lot *ABCD* between two parallel street lines is 350.00 ft deep and has a 220.00 ft frontage (*AB*) on one street and a 260.00 ft frontage (*CD*) on the other. Interior angles at *A* and *B* are equal, as are those at *C* and *D*. What distances *AE* and *BF* should be laid off by a surveyor to divide the lot into two equal areas by means of a line *EF* parallel to *AB*?



***AE* = *BF* = 182.58 ft**

Area = 84,000 ft^2; 1/2 area = 42,000 ft^2
$\angle$@*A* = tan^{-1} (20/350) = 93°16'14"
42000 = h/2[220.00 + (220.00 + 2h tan 3°16'14")]
quadratic equation: 0.057142857h^2 + 220h − 42000 = 0
h = 182.28 ft
EF = 220 + 2(182.28)tan 3°16'14" = 240.83 ft

12.28 Partition 1-acre parcel from the northern part of lot *ABCDEFA* in Problem 12.21 such that its southern line is parallel to the northern line.



85.45 ft

Az_{CB} = 192°50'01.6"; Az_{CD} = 272°16'18"
Az_{DE} = 192°16'17.5"; CD = 509.14 ft
Az_h = 182°16'17.5"
Small angle at C = 10°33'44.1"
Small angle at D = 9°44'00.3"

$$43560 = h/2\{509.14 + [509.14 + h(\tan 9°44'00.3" - \tan 10°33'44.1")]\}$$
$$0 = -0.0149306h^2 + 1018.28h - 87120$$

12.29 Write a computational program for calculating areas within closed polygon traverses by the coordinate method.

Solutions will vary.

12.30 Write a computational program for calculating areas within closed polygon traverses by the DMD method.

Solutions will vary.

13 GLOBAL NAVIGATION SATELLITE SYSTEMS—INTRODUCTION AND PRINCIPLES OF OPERATION

Asterisks $^{(*)}$ indicate problems that have answers given in Appendix G.

13.1 Define the line of apsides.

From Section 13.4.1: The line of apsides joins the point of closest approach (perigee) by a satellite with its farthest point (apogee) and is defined as the *x* axis of the satellite reference coordinate system.

13.2 Briefly describe the orbits of the GLONASS satellites.

See Section 13.10.1: " The ***Glo****bal* ***Na****vigation* ***S****atellite* ***S****ystem* (GLONASS) is the Russian equivalent of GPS. When completed, the GLONASS constellation will contain 24 satellites equally spaced in three orbital planes making a 64.8° nominal inclination angle with the equatorial plane of the Earth. The satellites orbit at a nominal altitude of 19,100 km and have a period of 11.25 hours. When the system has its full compliment of satellites, at least five will always be visible to the user. The system is free from selective availability, but does not permit public access to the P code. Each satellite broadcasts two signals with frequencies that are unique. The frequencies of the satellites are determined as

$$f_{L1}^{j} = 1602.0000 \text{ MHz} + j \times 0.5625 \text{ MHz}$$
$$f_{L2}^{j} = 1246.0000 \text{ MHz} + j \times 0.4375 \text{ MHz}$$

where *j* represents the channel number assigned to the specific satellite,[11] and varies from 1 to 24, and *L*1 and *L*2 represent the broadcast bands."

13.3 Why is a fully operational satellite positioning system designed to have at least four satellites visible at all time?

From Section 13.4.1: To meet the minimum requirements for point positioning.

***13.4** Discuss the purpose of the pseudorandom noise codes.

See Section 13.2, paragraphs 3 thru 8. "The individual satellites are normally identified by their *PseudoRandom Noise* (PRN) number. The receiver simultaneously generates a duplicate PRN code. Matching the incoming satellite signal with the identical receiver-generated signal derives the time it takes for the signal to travel from satellite to receiver. This yields the signal delay that is converted to travel time. From the travel time, and the known signal velocity, the distance to the satellite can be computed."

13.5 What is the purpose of the Consolidated Space Operation Center in GPS?

From Section 13.2, paragraph 4: "The tracking information is relayed to the *master control station* in the Consolidated Space Operations Center (CSOC) located at Schriever Air Force base in Colorado Springs. The master control station uses this data to make precise, near-future predictions of the satellite orbits, and their clock correction parameters. This information is uploaded to the satellites, and, in turn, transmitted by them as part of their *broadcast message* to be used by receivers to predict satellite positions and their clock *biases* (systematic errors)."

13.6 Describe the three segments of GPS.

See Section 13.2: Space, Control, and User segments.

13.7 Describe the content of the GPS broadcast message.

Handover word, Time of Week word, satellite clock bias, satellite positioning data (ephemeris), and ionospheric modeling data. This information is scattered throughout the chapter.

13.8 What is anti-spoofing?

From Section 13.3, paragraph 6: Anti-spoofing encrypts the P code.

13.9 What errors affect the accuracy of satellite positioning?

From Section 13.6, satellite and receiver clock biases, ionospheric and tropospheric refractions, satellite geometry, multipathing, ephemeris errors, instrument setup errors, and measurement of height errors.

13.10 Define the terms "geodetic height," "geoid undulation," and "orthometric height." Include their relationship to each other.

See Section 13.4.3: $h = H + N$ where h is the geodetic height, H is the elevation/orthometric height, or N is the geoid undulation is the vertical distance between the ellipsoid and geoid.

13.11 Define PDOP, HDOP, and VDOP.

From Section 13.6.4:

- Positional dilution of precision: $\sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2}$
- Horizontal dilution of precision: $\sqrt{\sigma_X^2 + \sigma_Y^2}$
- Vertical dilution of precision: $\sqrt{\sigma_Z^2}$

13.12 Define WAAS and EGNOS.

From Section 13.7: " The *Wide Area Augmentation System* (WAAS) developed by the Federal Aviation Administration has a network of ground tracking base stations that collect GPS signals and determine range errors. These errors are transmitted to geosynchronous satellites that relay the corrections to rovers. GPS software typically allows users to access the WAAS system when performing RTK-GPS surveys (see Chapter 15). This option, called *RTK with infill*, accesses the WAAS corrections when base-station radio transmissions are lost. However these corrections will provide significantly less accuracy than relative positioning techniques typically utilized by GPS receivers using carrier phase-shift measurements. In Europe, the *European Geostationary Navigation Overlay Service* (EGNOS) serves a similar role to WAAS."

13.13 How can receiver clock bias error be eliminated from carrier phase measurements?

By either double differencing (Section 13.9.2) or by modeling and solving for directly in the observation equations (Section 13.5.2).

13.14 How can satellite clock bias error be eliminated from carrier phase measurements?

From Section 13.6.1: By either single differencing or by modeling using the satellite clock bias parameters.

13.15 What is single differencing?

From Section 13.5.1: Single differencing is the difference between two observations from one satellite to two receivers.

13.16 What is double differencing?

From Section 13.5.2: Double differencing is the difference between two single difference observations from two satellites.

13.17 List and discuss the ephemerides.

From Section 13.6.3, paragraph 2: "One of three updated post-survey ephemerides are available: (1) ultra-rapid ephemeris, (2) the *rapid ephemeris*, and (3) the *precise ephemeris*. The ultra-rapid ephemeris is available twice a day; the rapid ephemeris is available within two days after the survey; the precise ephemeris (the most accurate of the three) is available two weeks after the survey. The ultra-rapid and rapid ephemerides are sufficient for most surveying applications."

13.18 Describe how the travel time of a GPS signal is measured.

From Section 13.3, paragraph 7 and 8: By matching the offset satellite code (C/A or P) with a receiver generated code.

13.19 If the HDOP during a survey is 1.53 and the UERE is estimated to be 1.65 m, what is the 95 percent horizontal point-positioning error?

<u>**±4.9 m**</u>; 1.53(1.65)1.96

13.20 In Problem 13.19, if the VDOP is 3.3, what is the 95 percent point-positioning error in geodetic height?

<u>**±10.7 m**</u>; 3.3(1.65)1.96

***13.21** What are the geocentric coordinates in meters of a station in meters which has a latitude of $39°27'07.5894''$ N, longitude of $86°16'23.4907''$ W, and height of 203.245 m. (Use the WGS84 ellipsoid parameters.)

From WolfPack: (**320,559.446, −4,921,314.168, 4,031,328.395**)

13.22 Same as Problem 13.21 except with geodetic coordinates of $45°26'32.0489''$ N, longitude of $110°54'39.0646''$ W, and height of 335.204 m?

From WolfPack: **(−1,600,027.879, −4,187,678.858, 4,522,205.930)**

13.23 Same as Problem 13.21 except with geodetic coordinates of $28°47'06.0936''$ N, longitude of $75°52'35.0295''$ W, and height of 845.678 m?

From WolfPack: (**1,365,283.624, −5,425,960.742, 3,053,447.732**)

***13.24** What are the geodetic coordinates in meters of a station with geocentric coordinates of (136,153.995, $-4,859,278.535, 4,115,642.695$)? (Use the WGS84 ellipsoid parameters.)

From WolfPack: (**40°26'29.65168" N, 88°23'42.09876" W, 182.974 m**)

13.25 Same as Problem 13.24, except with geocentric coordinates in meters are (2,451,203.546, $-4,056,568.907$, 4,542,988.809)?

From WolfPack: (**43°58'22.42970" N, 58°51'26.29598" W, 197,408.025 m**)

13.26 Same as Problem 13.24, except with geocentric coordinates in meters are (566,685.776, $-4,911,654.896$, $-4,017,124.050$)?

From WolfPack: (**39°16'54.58310" S, 83°25'06.80760" W, 853.106 m**)

13.27 The GNSS determined height of a station is 588.648 m. The geoid undulation at the point is -28.45 m.

617.098 m by Equation (13.8)

13.28* The GNSS determined height of a station is 284.097 m. The geoid undulation at the point is -30.052 m. What is the elevation of the point?

314.149 m by Equation (13.8)

13.29 Same as Problem 13.28, except the height is 64.684 m and the geoid undulation is -28.968 m.

93.652 m by Equation (13.8)

***13.30** The elevation of a point is 124.886 m. The geoid undulation of the point is −28.998 m. What is the geodetic height of the point?

95.888 m by Equation (13.8)

13.31 Same as Problem 13.30, except the elevation is 686.904 m, and the geoid undulation is

−22.232 m.

664.672 m by Equation (13.8)

13.32 The GNSS observed height of two stations is 124.685 m and 89.969 m, and their orthometric heights are 153.104 m and 118.386 m, respectively. These stations have model-derived geoid undulations of -28.454 m and -28.457 m, respectively. What is the orthometric height of a station with a GNSS measured height of 105.968 m and a model-derived geoid undulation of -28.453 m?

134.384 m

N_{GPS}	ΔN
124.685 − 153.104 = −28.419	−28.419 + 28.454 = 0.035
89.969 − 118.386 = −28.417	−28.417 + 28.457 = 0.040

Avg ΔN = 0.0375 m   H = 105.968 −(−28.453 + 0.0375) = 105.3835

13.33 Why are satellites at an elevation below 10° from the horizon eliminated from the positioning solution?

From Section 13.6.2, last paragraph: To minimize the errors caused by refraction.

13.34 Research the Chinese satellite positioning system known as Compass and prepare a written report on the system.

Responses will vary.

13.35 Create a computational program that converts geocentric coordinates to geodetic coordinates.

Responses will vary.

13.36 Create a computational program that converts geodetic coordinates to geodetic coordinates.

Responses will vary.

13.37 Find at least two Internet sites that describe how GPS works. Summarize the contents of each site.

Responses will vary.

14 GLOBAL NAVIGATION SATELLITE SYSTEMS—STATIC SURVEYS

Asterisks (*) indicate problems that have answers given in Appendix G.

14.1 Explain the differences between a static survey and rapid static survey.

From Section 14.2: A static survey and rapid static survey are essentially the same methods of survey with these differences: **(1)** the rapid static survey should only be performed on lines less than 20 km in length **(2)** in ideal observational conditions. The rapid static survey **(3)** typically uses an epoch rate of 5 sec whereas the static survey typically uses a rate of 15 sec. The **(4)** time of occupation in a rapid static survey is typically about half of what is required for a static survey.

14.2 When using the rapid static surveying method, what is the minimum recommended length of the session required to observe a baseline that is 10 km long for ***(a)** a dual-frequency receiver, **(b)** a single-frequency receiver?

From Table 14.1:

(a)* **<u>20 min</u>**

(b) **<u>40 min</u>**

14.3 What would be the recommended epoch rates for the surveys given in Problem 14.2?

Both (a) and (b) require an epoch rate of 5 sec or less.

14.4 For a 38-km baseline using a dual-frequency receiver, **(a)** what static surveying method should be used, **(b)** for what time period should the baseline be observed, (c) and what epoch rate should be used?

(a) **<u>Static survey</u>**

(b) **<u>96 min</u>**

(c) **<u>15 sec</u>**

14.5 What variables affect the accuracy of a static survey?

Besides what is mentioned in Section 13.6, which includes clock bias, refraction, ephemeral errors, multipathing, centering of receiver over station, measurement of height of receiver above the station, and satellite geometry, the type of receiver used in the survey is important. Highest accuracies will be achieved by GNSS and dual-frequency receivers because of their ability to correct for clock bias and refraction. GNSS receivers have the additional advantage of being able to use more satellites. Sites of stations must be free from overhead obstructions and void of multipathing possibilities for the highest accuracies.

14.6 Why are dual-frequency and GNSS receivers preferred for high-accuracy control stations?

Highest accuracies will be achieved by GNSS and dual-frequency receivers because of their ability to correct for clock bias and refraction. GNSS receivers have the additional advantage of being able to use satellites from 2 or more constellations resulting in increased observational numbers.

14.7 Explain why canopy obstructions are a problem in a static survey.

From Section 14.3.1: Canopy restrictions may possibly block satellite signals, thus reducing observations and possibly adversely affecting satellite geometry.

14.8 Why is it recommended to use a precise ephemeris when processing a static survey?

From 14.5, paragraph 3 and Section 13.6.3: The broadcast ephemeris is a near-future prediction of the location of the satellites whereas a precise ephemeris is their tracked position. Thus orbital errors are removed by processing with a precise ephemeris.

14.9 What are the recommended rates of data collection in a **(a)** static survey, and a **(b)*** rapid static survey?

(a) **15 sec**

(b) **5 sec**

14.10 List the fundamental steps involved in planning a static survey.

From Section 14.3:

1. Obtain the location of the nearest existing control/reference stations
2. Recon the project area and locate suitable locations for new control
3. Select the appropriate survey method
4. Check satellite availability during observation sessions and space weather forecast.
5. Develop a observational scheme.

***14.11** How many nontrivial baselines will be observed in one session with 4 receivers?

3

14.12 What variables should be considered when selecting a site for a static survey?

From Section 14.3.3: "…a reconnaissance trip to the field should be undertaken to check the selected observation sites for (1) overhead obstructions that rise above 10°–15° from the horizon, (2) reflecting surfaces that can cause multipathing, (3) nearby electrical installations that can interfere with the satellite's signal, and (4) other potential problems."

14.13 Why should a control survey using static methods form closed geometric figures?

From 14.3.5: "For control surveys, the baselines should form closed geometric figures since they are necessary to perform closure checks."

14.14 What is the purpose of an obstruction diagram in planning a static survey?

From Section 14.3.1: " The diagram will then show whether crucial satellites are removed by the obstructions and also indicate the best times to occupy the station to avoid the obstructions. "

14.15 Explain why periods of high solar activity should be avoided when collecting satellite observations.

From Section 14.3.1, paragraph 4: "Days of high solar activity, where the k-index is above 4, should be avoided since this can cause high refraction conditions as well as cause loss of lock in extreme conditions."

14.16 Why the survey vehicle should be parked at least 25 m from the observing station in a static survey?

From Section 14.6.2 on multipathing. "Thus, the best approach to reducing this problem is to avoid setups near reflective surfaces. Reflective surfaces include flat surfaces such as the sides of building, vehicles, water, chain link fences, and so on."

14.17 When can a rapid static horizontal control survey with high PDOP continue?

When the HDOP is low, but VDOP is high.

14.18 When using four receivers, how many sessions will it take to independently observe all the baselines of a hexagon?

<u>**5 sessions**</u>; 3 nontrivial lines per session with a total of 15 lines.

14.19 Plot the following ground obstructions on a obstruction diagram.

(a) From an azimuth of 65° to 73° there is a building with an elevation of 20°.

Student graphic

(b) From an azimuth of 355° to 356° there is a pole with an elevation of 35°.

Student graphic

(c) From an azimuth of 125° to 128° there is a tree with an elevation of 26°.

Student graphic

***14.20** In Problem 14.19, which obstruction is unlikely interfere with GPS satellite visibility in the northern hemisphere?

(b) since obstruction is in near pole.

14.21 What items should be considered when deciding which method to use for a static survey?

From Section 14.3.2, paragraph 3: "The selection of the appropriate survey method is dependent on the (1) desired level of accuracy in the final coordinates, (2) intended use of the survey, (3) type of equipment available for the survey, (4) size of the survey, (5) canopy and other local conditions for the survey, and (6) available software for reducing the data."

14.22 What items should be included in a site log sheet?

From Section 14.4, paragraph 2: "Typical ancillary data obtained during a survey includes (1) project and station names, (2) ties to the station, (3) a rubbing of the monument cap, (4) photos of the setup showing identifying background features, (5) potential obstructive or reflective surfaces, (6) date and session number, (7) start and stop times, (8) name of observer, (9) receiver and antenna serial number, (10) meteorological data, (11) PDOP value, (12) antenna height, (13) orientation of antenna, (14) rate of data collection (epoch rate), and (15) notations on any problems experienced."

14.23 What is a satellite availability chart and how is it used?

From Section 14.3.1, paragraph 5: "To aid in selecting suitable observation windows, a satellite availability plot, as shown in Figure 14.7, can be applied." It also shows the PDOP, HDOP, and VDOP, which allows the user to pick the optimal time to collect observations.

***14.24** What order of accuracy does a survey with a standard deviation in the geodetic height difference of 15 mm between two control stations that are 5 km apart meet?

Fourth order, Class II; $\frac{15}{\sqrt{5}} = 6.71$, which is under 4th order, class II of 15.0.

14.25 Do Problem 14.24 when the standard deviation in the geodetic height difference is 5 mm for two control points 15 km apart?

Second order, Class II; $\frac{5}{\sqrt{15}} = 1.29$, which is under 2nd order, class II of 1.3.

14.26 Use the NGS web site to download the station coordinates for the nearest CORS station.

Answers will vary.

14.27 What are CORS and HARN stations?

From Section 14.3.5, paragraph 2: "In recent years, the NGS with cooperation from other public and private agencies has created a national system of ***C****ontinuously* ***O****perating* ***R****eference* ***S****tations*, also called the *National CORS Network*. The location of stations in the CORS network as of 2003 is shown in Figure 14.12. As of January 2007 there are 1221 CORS stations. These stations not only have their positions known to high accuracy,[4] but they also are occupied by a receiver that continuously collects satellite data. The collected data is then downloaded and posted on the NGS Internet site at http://www.ngs.noaa.gov/CORS/."

From Section 14.3.5, paragraph 1: "To meet this need, individual states, in cooperation with the NGS, have developed ***H****igh* ***A****ccuracy* ***R****eference* ***N****etworks* (HARNs). The

HARN is a network of control points that were precisely observed using GPS under the direction of the National Geodetic Survey (NGS)."

14.28 Why should repeat baselines be performed in a static survey?

From Section 14.5.3: "Another procedure employed in evaluating the consistency of the observed data and in weeding out blunders is to make repeat observations of certain baselines. These repeat measurements are taken in different observing sessions and the results compared. Significant differences in repeat baselines indicate problems with field procedures or hardware."

14.29 What is the purpose of developing a site log sheet for each session?

From Section 14.5, paragraph 1: "As the observation files are downloaded, special attention should be given to checking station information that is read directly from the file with the site log sheets. Catching incorrectly entered items such as station identification, antenna heights, and antenna offsets at this point can greatly reduce later problems during processing. Batch processing typically performs the reduction process. "

14.30 Using loop *ACFDEA* from Figure 14.10, and the data from Table 14.6, what is the

(a) Misclosure in the X component? **29.8 mm**

(b) Misclosure in the Y component? **3.9 mm**

(c) Misclosure in the Z component? **−24.2 mm**

(d) Length of the loop misclosure? **48,587.535 m**

(e)* Derived ppm for the loop? **0.79 ppm**

Course	ΔX (m)	ΔY (m)	ΔZ (m)	Length
AC	11644.2232	3601.2165	3399.2550	12,653.52
CF	−10527.7852	994.9377	956.6246	10,617.88
FD	−4600.3787	5291.7785	5414.4311	8859.033
DE	−1837.7459	−6253.8534	−6596.6697	9273.837
EA	5321.7164	−3634.0754	−3173.6652	7183.267
misclosure	0.0298	0.0039	−0.0242	48,587.535

14.31 Do Problem 14.30 with loop *BCFB*.

Note: solutions will vary slightly if students use *BF* in solution.

(a) Misclosure in the X component? **−9.9 mm**

(b) Misclosure in the Y component? **−16.4 mm**

(c) Misclosure in the Z component? **1.5 mm**

(d) Length of the loop misclosure? **32,006.619 m**

(e) Derived ppm for the loop? **0.60 ppm**

Course	ΔX (m)	ΔY (m)	ΔZ (m)	Length
BC	3960.5442	−6681.2467	−7279.0148	10,644.671
CF	−10527.7852	994.9377	956.6246	10,617.876
FB	6567.2311	5686.2926	6322.3917	10,744.072
	−0.0099	−0.0164	0.0015	32,006.619

14.32 Do Problem 14.30 with loop *BFDB*.

Note: Answers may vary slightly if students use baseline *FB*.

(a) Misclosure in the X component? **−2.1 mm**

(b) Misclosure in the Y component? **−4.4 mm**

(c) Misclosure in the Z component? **9.7 mm**

(d) Length of the loop misclosure? **30,814.505 m**

(e) Derived ppm for the loop? **0.35 ppm**

Course	ΔX (m)	ΔY (m)	ΔZ (m)	Length
BF	−6567.2310	−5686.3033	−6322.3807	10,744.071
FD	−4600.3787	5291.7785	5414.4311	8,859.033
DB	11167.6076	394.5204	907.9593	11,211.400
	−0.0021	−0.0044	0.0097	30814.505

14.33 List the contents of a typical survey report.

From Section 14.5.6:

" A final survey report is helpful in documenting the project for future analysis. At a minimum, the report should contain the following items.

1. The location of the survey and a description of the project area. An area map is recommended.
2. The purpose of the survey, and its intended specifications.
3. A description of the monumentation used including the tie sheets, photos and rubbings of the monuments.
4. A thorough description of the equipment used including the serial numbers, antenna offsets, and the date the equipment was last calibrated.
5. A thorough description of the software used including name and version number.
6. The observation scheme used including the itinerary, the names of the field crew personnel and any problems that were experienced during the observation phase.

7. The computation scheme used to analyze the observations and the results of this analysis.
8. A list of the problems encountered in the process of performing the survey, or its analysis including unusual solar activity, potential multipathing problems, or other factors that can affect the results of the survey.
9. An appendix containing all written documentation, original observations, and analysis. Since the computer can produce volumes of printed material in a typical survey, only the most important files should be printed. All computer files should be copied onto some safe backup storage. A CD provides an excellent permanent storage media that can be inserted into the back of the report."

14.34 The observed baseline vector components in meters between two control stations is (3814.244, −470.348, −1593.650). The geocentric coordinates of the control stations are (1,162,247.650, −4,655,656.054, 4,188,020.271) and (1,158,433.403, −4,655,185.709, 4,189,613.926). What are:

(a)* ΔX ppm? **<u>0.76 ppm</u>**

(b) ΔY ppm? **<u>0.85 ppm</u>**

(c) ΔZ ppm? **<u>1.16 ppm</u>**

	dX	*dY*	*dZ*
	1162247.650	-4655656.054	4188020.271
	1158433.403	-4655185.709	4189613.926
Control	3814.247	-470.344	-1593.655
Observed	3814.244	-470.348	-1593.650
Δ	0.003	0.004	0.005
ppm	0.76	0.85	1.16

14.35 Same as Problem 14.34 except the two control station have coordinates in meters of (−1,661,107.767, −4,718,275.246, 3,944,587.541) and (1,691,390.245, −4,712,916.010, 3,938,107.274), and the baseline vector between them was (30282.469, −5359.245, 6480.261).

(a) ΔX ppm? **<u>0.28 ppm</u>**

(b) ΔY ppm? **<u>0.31 ppm</u>**

(c) ΔZ ppm? **<u>0.19 ppm</u>**

	dX	*dY*	*dZ*
	−1,661,107.767	−4,718,275.246	3,944,587.541
	−1,691,390.245	−4,712,916.010	3,938,107.274
Control	30282.478	−5359.235	6480.267
Observed	30282.469	−5359.245	6480.261
Δ	0.009	0.010	0.006
ppm	0.28	0.31	0.19

14.36 List the various survey types that could be performed using static survey.

From Section 14.2: **static, rapid static, and pseudokinematic**

14.37 Employ the user-friendly button in the NGS CORS Internet site at http://www.ngs.noaa.gov/CORS/ to

(a) Download the navigation and observation files for station PSU1 between the hours of 10 and 11 local time for the Monday of the current week using a 5-sec data collection rate.

Solution will be the current file for PSU1

(b) Print the files and comment on the contents of them. (Hint: An explanation of the contents of the RINEX2 data file is contained at http://www.ngs.noaa.gov/CORS/Rinex2.html on the Internet in the CORS site)

Solutions will show portions of files with appropriate comments.

15 GLOBAL NAVIGATION SATELLITE SYSTEMS—KINEMATIC SURVEYS

Asterisks (*) indicate problems that have answers given in Appendix G.

***15.1** What is the typical epoch rate for a kinematic survey?

1 sec

15.2 What are the advantages of a PPK survey over an RTK survey?

From Section 15.4, paragraph 4: "Data latency is not a problem in PPK surveys since the data is post-processed. Other advantages of PPK surveys are (1) that precise ephemeris can be combined with the observational data to remove errors in the broadcast ephemeris and (2) the base station coordinates can be resolved after the fieldwork is complete." Plus also the reduce amount of equipment needed in the field since a radio is not required.

15.3 What are the advantages of an RTK survey over a PPK survey?

From Section 15.4, last paragraph: "The advantages of RTK surveys over PPK surveys are the reduction in office time, and the ability to verify observations in the field. When using RTK, the data can also be downloaded immediately into a GIS (see Chapter 28) or an existing surveying project. This increases the overall productivity of the survey."

15.4 Why are repeater stations used in an RTK survey?

From Section 15.3, paragraph 6: "However, repeater stations can also be used to extend the range of the base station radio in situations where necessary as well as avoid obstructions "

15.5 How can ephemeral errors be eliminated in a kinematic survey?

PPK survey that uses a precise ephemeris From Section 15.5, paragraph 9: "An advantage PPK surveys have over RTK surveys is that precise ephemeris can be used in the processing. As discussed in Section 13.6.3, this will result in a better solution for the positions of the surveyed points since it removes ephemeral errors from the solution."

* **15.6** How much error in horizontal position occurs if the antenna is mounted on a 2.000 m pole that is 10 min out of level?

5.8 mm; (10'*π/180)2.000

15.7 Do Problem 15.6, but this time assume the level is 4 min out of level.

2.3 mm; (4'*π/180)2.000

15.8 How much error in vertical position occurs with the situation described in Problem 15.6?

0.008 mm

*** 15.9** Why should the radio antenna at the base station be mounted as high as possible?

Section 15.3 paragraph 6: "Mounting the radio antenna high can increase the range of the base radio."

15.10 List two reasons why PPK surveys usually provide better results than RTK surveys.

From Section 15.4, paragraph 4: The lack of data latency and the ability to use a precise ephemeris.

15.11 What is OPUS and how can it be used in a PPK survey?

From Section 15.5: OPUS is used to determine the position of the base station that used an autonomous position in a PPK survey.

15.12 What are the available methods for initializing a receiver?

From Section 15.2, (1) occupy two stations with known positions or a previously determined baseline length, (2) antenna swapping, or (3) on-the-fly (OTF) methods.

15.13 Discuss the differences between the stop-an-go and the true kinematic modes of surveying.

From Section 15.4, paragraph 2: "In *true kinematic* mode, data is collected at a specific rate. This method is useful for collecting points along an alignment, or grade elevations for topographic surveys. An alternative to the true kinematic mode is to stop for a few epochs of data at each point of interest. This method, known as *semikinematic* or the *stop-and-go* mode, is useful for mapping and construction surveys where increased accuracy is desired for a specific feature. "

15.14 Discuss the appropriate steps used in processing PPK data.

From Section 15.4, paragraph 4: "In post-processed kinematic (PPK) surveys, the collected data is stored on the survey controller or receiver until the fieldwork is completed. The data is then processed in the office using the same software and processing techniques used in static surveys."

15.15 Why is the use of a real-time network not recommended in machine control?

From Section 15.9, paragraph 3: "Again, real-time networks should be used with caution in machine control since data latency can be large which can lead to significant real-time errors during the finishing process in a construction project. In fact, some manufacturers do not recommend the use of real-time networks at all in machine control projects."

15.16 What is VRS?

From Section 15.7, paragraph 2: "Using the known positions of the base receivers and their observational data, the central processor models errors in the satellite ephemerides, range errors caused by ionospheric and tropospheric refraction, and the geometric integrity of the network stations. *Virtual reference station* (VRS) and *spatial correction parameter* (*FKP*) are examples of two methods used in modeling these errors."

15.17 What limitations occur in an RTK survey?

1. Lock must be maintained on at least 4 satellites at all times.
2. More equipment and heavier battery usage is required.
3. Low-powered radio signals can be blocked by obstructions and have limited range.
4. High-powered radios require an FCC license.
5. Can only use broadcast ephemeris.
6. Positioning has the lowest accuracy of all relative positioning methods.

***15.18** What frequencies found in RTK radios require licensure?

450 - 470 MHz. From Section 15.3: "In North America and in other areas of the world, frequencies in the range of 150–174 MHz in the VHF radio spectrum, and from 450–470 MHz in the UHF radio spectrum can be used for RTK transmissions." and from Section 15.6, paragraph 2: "The Federal Communications Commission (FCC) does not require a license for radios that broadcast in the range from 157 – 174 MHz. However, all other frequencies given in Section 15.4 do require a FCC license"

15.19 What is localization of a survey?

From Section 15.8: It is the process of transforming satellite-derived coordinates into some local reference frame.

15.20 Why is it important to localize a survey?

From Section 15.8, paragraph 2: "The current rendition of the WGS 84 reference frame (used in broadcast ephemeris) closely approximates ITRF 2000. The difference in the origins of the NAD 83 and ITRF 2000 data is about 2.2 m. Thus when performing a stakeout survey, the coordinates for stations produced by receivers can differ significantly from coordinates of the same stations in the regional reference frame that were used to perform the engineering design."

15.21 Discuss how the coordinate differences between a regional datum and satellite-derived coordinates can be resolved.

From Section 15.8; By localization

15.22 What three surveying elements are needed in machine control?

From Section 15.9:

1. DTM
2. Sufficient horizontal and vertical control to support machine control in all areas of project and during all phases of the project.
3. Site calibration parameters for localization of project.

15.23 A 5-mi stretch of road has numerous canopy restrictions. What is the minimum number control stations required to support machine control in this part of the road if a robotic total station is used?

27 control stations. From Section 15.9: Robotic total stations have a working radius of 1000 ft. Thus 5(5280)/1000 = 26.4 stations

15.24 How are robotic total station used in machine control?

From Section 15.9: To fill in areas with canopy restrictions and to provide finish grades.

15.25 How are finished grades determined in machine control?

From Section 15.9: By either robotic total stations, or laser levels.

15.26 What factors may determine the best location for a base station in a RTK survey?

From Section 15.3, paragraph 7: "Several factors may determine the “best” location for the base station in a RTK survey. Since the range of the radio can be increased with increasing height of the radio antenna, it is advantageous to locate the base station on a local high point. Additionally, since the base station in an RTK survey requires the most equipment, it is also preferable to place the base station in an easily accessible location."

15.27 What should be considered in planning a kinematic survey?

Besides conditions mentioned in Chapter 14 such as solar activity, location of control, and so on. Additional considerations must be given to location of the base station, canopy restrictions, and additional equipment that will be required to survey in areas of canopy restrictions. Also the number of satellites must be five or greater if OTF methods of initialization are going to be used and locations to charge batteries.

***15.28** How many total pseudoranges observations will be observed using a 1-sec epoch rate for a total of 10 min with 8 usable satellites?

4800; 60 obs/min(10 min)8 sats

15.29 How many pseudoranges observations will be observed using a 5-sec epoch rate for a total of 30 min with 8 usable satellites?

2880; 12 obs/min(30 min)8 sats

15.30 The baseline vector between the base and roving receivers is 1000 m long. What is the estimated uncertainty in the length of the baseline vectors if a RTK-GPS survey is performed?

1 - 2 cm + 2 ppm yields a range of **1 cm to 2 cm**

15.31 Discuss the importance of knowing the space weather before performing a kinematic GPS survey.

Section 15.4, paragraph 4: "During periods of high solar activity, poor positioning results can be obtained with GPS."

15.32 Why must the antenna be calibrated to the cutting edge of the blade in a machine control system.

From Section 15.9, paragraph 1: "One aspect of this technology is that the antenna must be calibrated with respect to the construction vehicle. For instance, the distance between the antenna reference point and the cutting edge of the blade on a machine must be observed and entered into the machine control system so that the height of the surface under the blade is accurately known."

15.33 Where are the best locations for control used in localizing a project?

From Section 15.8, paragraph 3: "It is important when performing these transformations to have control on the exterior and surrounding the project area to avoid extrapolation errors."

15.34 Why should a localization occur only once on a project?

From Section 15.8, paragraph 4: "However, this procedure should be performed only once for any survey, since errors in the systems can produce significantly different results if the procedure is repeated."

16 ADJUSTMENT BY LEAST SQUARES

Asterisks (*) indicate problems that have answers given in Appendix G.

16.1 What fundamental condition is enforced by the method of weighted least squares?

From Section 16.2, paragraph 5: "If observed values are to be weighted in least squares adjustment, then the fundamental condition to be enforced is that *the sum of the weights times their corresponding squared residuals is minimized* or, in equation form,"

$$\sum_{i}^{m} w_i v_i^2 = w_1 v_1^2 + w_2 v_2^2 + w_3 v_3^2 + \cdots + w_m v_m^2 \longrightarrow \text{minimum}$$

16.2 What are the advantages of adjusting observations by the method of least squares?

From Section 16.2, paragraphs 1: "The method of least-squares adjustment is derived from the equation for the normal distribution curve. *It produces that unique set of residuals for a group of observations that have the highest probability of occurrence.*"

16.3 What are the advantages of the method of least squares over methods of adjustment?

From Section 16.1, paragraph 4: "Least-squares adjustments provide several advantages over other arbitrary methods. First of all, because the method is based upon the mathematical theory of probability, it is the most rigorous of adjustment procedures. It enables all observations to be simultaneously included in an adjustment, and each observation can be weighted according to its estimated precision. Furthermore, the least squares method is applicable to any observational problem regardless of its nature or geometric configuration. In addition to these advantages, the least squares method enables rigorous statistical analyses to be made of the results of the adjustment, i.e., the precisions of all adjusted quantities can be estimated, and other factors investigated. The least squares method even enables presurvey planning to be done so as to ensure that required precisions of adjusted quantities are obtained in the most economical manner."

***16.4** What is the most probable value for the following set of ten distance observations in meters? 532.688, 532.682, 532.682, 532.684, 532.689, 532.686, 532.690, 532.684, 532.686, 532.686

<u>**532.686 m**</u>

16.5 What is the standard deviation of the adjusted value in Problem 16.4?

<u>**±0.0028 m**</u>

16.6 Three horizontal angles were observed around the horizon of station A. Their values are $165°07'54"$, $160°25'36"$ and $34°26'36"$. Assuming equal weighting, what are the most probable values for the three angles?

165°07'52", **160°25'34"**, and **34°26'34"**

Condition $x + y + z = 360°$ or $v_3 = 6" - (v_1 + v_2)$

$$\sum \upsilon_1^2 + \upsilon_2^2 + \left(6" - \upsilon_1^2 - \upsilon_2^2\right) \rightarrow \text{ minimum}$$

Normal equations
$$\begin{aligned} 2\upsilon_1 + \upsilon_2 &= -6" \\ \upsilon_1 + 2\upsilon_2 &= -6" \end{aligned}$$

So, $v_1 = v_2 = v_3 = -2"$

16.7 What are the standard deviations of the adjusted values in Problem 16.6?

±2.8"

$$S_0 = \sqrt{\frac{(-2)^2 + (-2)^2 + (-2)^2}{3-2}} = \pm 3.46; \quad \sigma = 3.46\sqrt{\frac{2}{3}} = \pm 2.8"$$

16.8 In Problem 16.6, the standard deviations of the three angles are $\pm 1.5''$, $\pm 3.0''$, and $\pm 4.9''$, respectively. What are the most probable values for the three angles?

165°07'53.8", **160°25'34.9"**, and **34°26'31.3"**

Normal equations:
$$\begin{aligned} 2w_1v_1 + w_3v_2 &= -w_3 6" \\ w_3v_1 + 2w_2v_2 &= -w_3 6" \end{aligned}$$

Normal Equations		A^TWL	X (")
0.888889	0.041649	−0.2499	−0.23
0.041649	0.222222	−0.2499	−1.08

***16.9** Determine the most probable values for the x and y distances of Figure 16.2, if the observed lengths of AC, AB, and BC (in meters) are 294.081, 135.467, and 158.607, respectively.

$x = 135.469$ and **$y = 158.609$**

Normal equations:
$$\begin{aligned} 2x + y &= 294.081 + 135.467 \\ x + 2y &= 294.081 + 158.607 \end{aligned}$$

Normal Equations		A^TL	X
2	1	429.548	135.469
1	2	452.688	158.609

***16.10** What are the standard deviations of the adjusted values in Problem 16.9?

±0.003 ft for both

$$S_0 = \pm 0.0033;\ \Sigma_{xx} = 0.0033^2\begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

16.11 A network of differential levels is run from existing benchmark Juniper through new stations A and B to existing benchmarks Red and Rock as shown in the accompanying figure. The elevations of Juniper, Red, and Rock are 685.65 ft, 696.75 ft, and 705.27 ft, respectively. Develop the observation equations for adjusting this network by least squares, using the following elevation differences.

From	To	Elev. Diff. (ft)	σ (ft)
Juniper	*A*	−40.58	0.021
A	*B*	8.21	0.010
B	Red	43.44	0.020
B	Rock	51.96	0.026

$A = 685.65 - 40.58 = 645.07$

$-A + B = 8.21$

$-B = -696.75 + 43.44 = -653.31$

$-B = -705.27 + 51.96 = -653.31$

16.12 For Problem 16.11, following steps outlined in Example 16.6 perform a weighted least-squares adjustment of the network. Determine weights based upon the given standard deviations. What are the

(a) Most probable values for the elevations of *A* and *B*? **645.09** and **653.30**

(b) Standard deviations of the adjusted elevations? **±0.01** and **±0.01**

(c) Standard deviation of unit weight? **±0.75**

(d) Adjusted elevation differences and their residuals?

From	To	ΔElev	V	S
Juniper	A	−40.563	0.017	±0.011
A	B	8.214	0.004	±0.007
B	Red	43.450	0.010	±0.010
B	Rock	51.970	0.010	±0.010

(e) Standard deviations of the adjusted elevation differences? (see d)

16.13 Repeat Problem 16.12 using distances for weighting. Assume the following course lengths for the problem.

From	To	Dist (ft)
Juniper	A	1500
A	B	300
B	Red	1200
B	Rock	2300

(a) Most probable values for the elevations of *A* and *B*? **645.09** and **653.30**

(b) Standard deviations of the adjusted elevations? **±0.01** and **±0.01**

(c) Standard deviation of unit weight? **±0.000**

(d) Adjusted elevation differences and their residuals?

From	To	ΔElev	V	S
Juniper	A	−40.563	0.017	±0.011
A	B	8.214	0.004	±0.007
B	Red	43.450	0.010	±0.010
B	Rock	51.970	0.010	±0.010

(e) Standard deviations of the adjusted elevation differences? (see d)

16.14 Use WOLFPACK to do Problem 16.12 and 16.13 and compare the solutions for *A* and *B*.

See solutions in Problem 16.12 and 16.13. The only difference is that the standard deviation in unit weight is different. **Instructor's note:** The solutions are the same since the standard deviations supplied in 11 where based on the distances using the formulas given in Chapter 9.

16.15 Repeat Problem 16.12 using the following data.

From	To	Elev. Diff. (ft)	σ (ft)
Juniper	*A*	−40.62	0.027
A	*B*	8.19	0.015
B	Red	43.48	0.029
B	Rock	51.95	0.027

Instructor's note: You may wish to point out to students how weights control the solution to the problem by having them compare solution in 16.12 to this solution.

(a) Most probable values for the elevations of *A* and *B*? **645.07** and **653.27**

(b) Standard deviations of the adjusted elevations? **±0.032** and **±0.029**

(c) Standard deviation of unit weight? **±1.729**

(d) Adjusted elevation differences and their residuals?

From	To	Elev. Diff.	V	S
Juniper	*A*	-40.58	0.04	±0.032
A	*B*	8.20	0.01	±0.024
B	Red	43.48	0.00	±0.029
B	Rock	52.00	0.05	±0.029

(e) Standard deviations of the adjusted elevation differences? (see d)

16.16 A network of differential levels is shown in the accompanying figure. The elevations of benchmarks *A* and *G* are 435.235 m and 465.643 m, respectively. The observed elevation differences and the distances between stations are shown in the following table. Using WOLFPACK, determine the

(a) Most probable values for the elevations of new benchmarks B, C, D, E, F, and H? See listing below.

(b) Standard deviations of the adjusted elevations? See listing below.

(c) Standard deviation of unit weight? See listing below.

(d) Adjusted elevation differences and their residuals? See listing below.

(e) Standard deviations of the adjusted elevation differences? See listing below.

From	To	Elev. Diff (m)	S (m)
A	*B*	−19.411	0.127
B	*C*	5.007	0.180
C	*D*	24.436	0.154
D	*E*	10.414	0.137
E	*F*	−14.974	0.112
F	*A*	−4.797	0.106
G	*F*	−25.655	0.112
G	*H*	−7.810	0.112
H	*D*	−13.011	0.112
G	*B*	−49.785	0.122
G	*E*	−10.477	0.117

```
Adjusted Elevation Differences
      From        To     Elevation Difference       V       S
=============================================================
         A         B       -19.447               -0.036  0.0856
         B         C         4.782               -0.225  0.1404
         C         D        24.271               -0.165  0.1314
         D         E        10.297               -0.117  0.1070
         E         F       -15.078               -0.104  0.0859
         F         A        -4.825               -0.028  0.0703
         G         F       -25.583                0.072  0.0703
         G         H        -7.801                0.009  0.0971
         H         D       -13.002                0.009  0.0971
         G         B       -49.855               -0.070  0.0856
         G         E       -10.505               -0.028  0.0826

Adjusted Elevations
   Station      Elevation       S
===================================
         B      415.788      0.0856
         C      420.570      0.1420
         D      444.841      0.1064
         E      455.138      0.0826
         F      440.060      0.0703
```

```
          H      457.842      0.0971

    Standard Deviation of Unit Weight: 1.026
```

16.17 Develop the observation equations for Problem 16.16.

$$B = 415.839 + \upsilon_1$$
$$-B + C = 5.007 + \upsilon_2$$
$$-C + D = 24.436 + \upsilon_3$$
$$-D + E = 10.414 + \upsilon_4$$
$$-E + F = -14.974 + \upsilon_5$$
$$-F = -440.032 + \upsilon_6$$
$$F = 439.988 + \upsilon_7$$
$$H = 457.833 + \upsilon_8$$
$$-H + D = -13.011 + \upsilon_9$$
$$B = 415.858 + \upsilon_{10}$$
$$E = 455.166 + \upsilon_{11}$$

16.18 A network of GNSS observations shown in the accompanying figure was made with two receivers using the static method. Known coordinates of the two control stations are in the geocentric system. Develop the observation equations for the following baseline vector components.

Station	X (m)	Y (m)	Z (m)
Jim	34.676	−4,497,514.405	4,507,737.916
Al	−268.920	−4,588,971.649	4,415,199.130

Jim to Troy

76,399.646	3.35E − 03	2.79E − 04	1.55E − 03
−45,109.020		3.18E − 03	7.93E − 05
−45,766.394			3.32E − 03

Al to Troy

76,703.337	3.37E − 03	2.17E − 04	5.24E − 07
46,348.167		3.45E − 03	1.62E − 05
46,772.390			3.38E − 03

$$X_{Troy} = 76,434.322 + \upsilon_1$$
$$Y_{Troy} = -4,542,623.425 + \upsilon_2$$
$$Z_{Troy} = 4,461,971.522 + \upsilon_3$$
$$X_{Troy} = 76,434.417 + \upsilon_4$$
$$Y_{Troy} = -4,542,623.482 + \upsilon_5$$
$$Z_{Troy} = 4,461,971.520 + \upsilon_6$$

16.19 For Problem 16.18, construct the A and L matrices.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 76{,}434.322 \\ -4{,}542{,}623.425 \\ 4{,}461{,}971.522 \\ 76{,}434.417 \\ -4{,}542{,}623.482 \\ 4{,}461{,}971.520 \end{bmatrix}$$

16.20 For Problem 16.18, construct the covariance matrix.

$$\Sigma = \begin{bmatrix} 3.35E-03 & 2.79E-04 & 1.55E-03 & 0 & 0 & 0 \\ 2.79E-04 & 3.18E-03 & 7.93E-05 & 0 & 0 & 0 \\ 1.55E-03 & 7.93E-05 & 3.32E-03 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.37E-03 & 2.17E-04 & 5.24E-07 \\ 0 & 0 & 0 & 2.17E-04 & 3.45E-03 & 1.62E-05 \\ 0 & 0 & 0 & 5.24E-07 & 1.62E-05 & 3.38E-03 \end{bmatrix}$$

16.21 Use WOLFPACK to adjust the baselines of Problem 16.18.

```
                    Degrees of Freedom = 3
                 Reference Variance = 0.6854
          Standard Devaiation of Unit Weight = ±0.83

*************************
Adjusted Distance Vectors
*************************
      From        To          dX           dY           dZ        Vx        Vy        Vz
==========================================================================================
       Jim      Troy    76399.690   -45109.047   -45766.383   0.0441   -0.0268    0.0110
        Al      Troy    76703.286    46348.197    46772.403  -0.0509    0.0302    0.0130

*****************************************************
Advanced Statistical Values
*****************************************************
      From        To      ±S   Vector Length    Prec
======================================================
       Jim      Troy  0.0575      99,831.361 1,737,000
        Al      Troy  0.0575     101,090.094 1,759,000

********************
Adjusted Coordinates
********************
   Station          X                Y                 Z        Sx       Sy       Sz
=====================================================================================
       Jim         34.676   -4,497,514.405    4,507,737.916
        Al       -268.920   -4,588,971.649    4,415,199.130
      Troy     76,434.366   -4,542,623.452    4,461,971.533  0.0330   0.0337   0.0329
```

16.22 Convert the geocentric coordinates obtained for station Troy in Problem 16.21 to geodetic coordinates.

<u>(44°40'30.61060" N, 89°02'09.70749" W, 313.539 m)</u>

16.23 A network of GNSS observations shown in the accompanying figure was made with two receivers using the static method. Use WOLFPACK to adjust the network, given the following data.

Station	X (m)	Y (m)	Z (m)
Bonnie	−1,660,596.783	−4,718,761.893	3,944,402.433
Tom	−1,622,711.656	−4,733,328.952	3,942,760.219

Bonnie to Ray

54,807.272	3.48E − 03	2.89E − 04	1.62E − 03
−65,078.175		3.61E − 03	6.16E − 05
−55,773.186			3.39E − 03

Bonnie to Herb

80,093.477	3.07E − 03	7.87E − 05	5.88E − 05
14,705.261		3.13E − 03	1.36E − 04
49,804.206			3.06E − 03

Tom to Ray

16,922.045	1.93E − 03	4.49E − 05	7.39E − 05
−50,511.098		1.98E − 03	−3.91E − 06
−54,130.990			1.99E − 03

Tom to Herb

42,208.529	1.79E − 03	1.12E − 05	4.48E − 05
29,272.300		1.78E − 03	5.65E − 05
51,446.338			1.80E − 03

Bonnie to Tom (Fixed line-Don't use in adjustment.)

37,885.108	5.55E − 04	1.31E − 05	1.21E − 05
−14,567.048		5.55E − 04	8.04E − 06
−1,642.215			5.54E − 04

Degrees of Freedom = 6
Reference Variance = 1.697
Standard Deviation of Unit Weight = ±1.3

Adjusted Distance Vectors

From	To	dX	dY	dZ	Vx	Vy	Vz
Bonnie	Ray	54807.209	-65078.165	-55773.208	-0.0625	0.0103	-0.0221
Bonnie	Herb	80093.590	14705.249	49804.154	0.1131	-0.0122	-0.0519
Tom	Ray	16922.082	-50511.106	-54130.994	0.0375	-0.0077	-0.0041
Tom	Herb	42208.463	29272.308	51446.368	-0.0659	0.0078	0.0301

Advanced Statistical Values

From	To	±S	Vector Length	Prec
Bonnie	Ray	0.0787	101,733.222	1,292,000
Bonnie	Herb	0.0759	95,455.127	1,257,000
Tom	Ray	0.0787	75,946.647	965,000
Tom	Herb	0.0759	72,699.045	957,000

Adjusted Coordinates

Station	X	Y	Z	Sx	Sy	Sz
Bonnie	-1,660,596.783	-4,718,761.893	3,944,402.433			
Tom	-1,622,711.656	-4,733,328.952	3,942,760.219			
Ray	-1,605,789.574	-4,783,840.058	3,888,629.225	0.0448	0.0466	0.0450
Herb	-1,580,503.193	-4,704,056.644	3,994,206.587	0.0438	0.0439	0.0439

16.24 For Problem 16.23, write the observation equations for the baselines "Bonnie to Ray" and "Tom to Herb."

$$X_{Ray} = -1,605,789.511 + \upsilon_x \qquad X_{Herb} = -1,580,503.127 + \upsilon_x$$
$$Y_{Ray} = -4,783,840.068 + \upsilon_y \qquad Y_{Herb} = -4,704,056.652 + \upsilon_y$$
$$Z_{Ray} = 3,888,629.247 + \upsilon_z \qquad Z_{Herb} = 3,995,848.771 + \upsilon_z$$

16.25 For Problem 16.23, construct the A, X, and L matrices for the observations.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} X_{Ray} \\ Y_{Ray} \\ Z_{Ray} \\ X_{Herb} \\ Y_{Herb} \\ Z_{Herb} \end{bmatrix} \quad L = \begin{bmatrix} -1,605,789.511 \\ -4,783,840.068 \\ 3,888,629.247 \\ -1,580,503.306 \\ -4,704,056.632 \\ 3,994,206.639 \\ -1,605,789.611 \\ -4,783,840.050 \\ 3,888,629.229 \\ -1,580,503.127 \\ -4,704,056.652 \\ 3,995,848.771 \end{bmatrix}$$

16.26 For Problem 16.23, construct the covariance matrix.

$$\Sigma = \begin{bmatrix}
3.48E-03 & 2.89E-04 & 1.62E-03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2.89E-04 & 3.61E-03 & 6.16E-05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.62E-03 & 6.16E-05 & 3.39E-03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.07E-03 & 7.87E-05 & 5.88E-05 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 7.87E-05 & 3.13E-03 & 1.36E-04 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 5.88E-05 & 1.36E-04 & 3.06E-03 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.93E-03 & 4.49E-05 & 7.39E-05 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4.49E-05 & 1.98E-03 & -3.91E-06 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 7.39E-05 & -3.91E-06 & 1.99E-03 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.79E-03 & 1.12E-05 & 4.48E-05 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.12E-05 & 1.78E-03 & 5.65E-05 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.48E-05 & 5.65E-05 & 1.80E-03
\end{bmatrix}$$

***16.27** After completing Problem 16.23, convert the geocentric coordinates for station Ray and Herb to geodetic coordinates. (Hint: See Section 13.4.3)

Station	Latitude	Longitude	h (m)
Ray	37°48'16.20675" N	108°33'19.20792" W	501.250
Herb	39°01'05.04679" N	108°34'18.09694" W	525.589

16.28 Following the procedures discussed in Section 14.5.2, analyze the fixed baseline from station Bonnie to Tom.

		ppm
dX	0.019	0.46
dY	0.011	0.27
dZ	0.000	0.00

16.29 For the horizontal survey of the accompanying figure, determine initial approximations for the unknown stations. The observations for the survey are

Station	**X (ft)**	**Y (ft)**
Dave	2515.62	1941.12

From	**To**	**Azimuth**	**S**
Dave	Wes	23°43'46"	0.001"

From	**To**	**Distance (ft)**	**σ (ft)**
Dave	Steve	2049.59	0.016
Steve	Frank	2089.17	0.016
Frank	Wes	2639.06	0.017
Wes	Dave	3179.50	0.018
Dave	Frank	3181.90	0.018
Steve	Wes	3759.69	0.019

Backsight Station	**Instrument Station**	**Foresight Station**	**Angle**	**σ**
Wes	Dave	Frank	49°01′12″	3.1″
Frank	Dave	Steve	40°12′36″	3.2″
Dave	Steve	Wes	57°44′15″	3.2″
Wes	Steve	Frank	42°45′08″	3.2″
Steve	Frank	Dave	39°17′59″	3.2″
Dave	Frank	Wes	65°26′35″	3.2″
Frank	Wes	Steve	32°30′13″	3.2″
Steve	Wes	Dave	33°01′59″	3.1″

Initial approximations can vary slightly:

Steve: **(4402.83, 1141.59)**

Frank: **(5554.35, 2884.73)**

Wes: **(3795.06, 4851.83)**

***16.30** Using the data in Problem 16.29, write the linearized observation equation for the distance from Steve to Frank.

$$-0.55119dx_{Steve} - 0.83438dy_{Steve} + 0.551191dx_{Frank} + 0.834379dy_{Frank} = 0.023 = 2089 - 17 - 2089.147$$

16.31 Using the data in Problem 16.29, write the linearized observation equation for the angle Wes-Dave-Frank.

$$-59.38918dx_{Wes} + 26.10528dy_{Wes} + 0dx_{Dave} + 0dy_{Dave} + 19.22443dx_{Frank} - 61.90890dy_{Frank} = -0.17''$$

where coefficients are multiplied by 206264.8"/rad.

16.32 Assuming a standard deviation of ±0.001" for the azimuth line Dave-Wes, use WOLFPACK to adjust the data in Problem 16.29.

Adjusted stations

Station	Northing	Easting	Sn	Se	Su	Sv	t
Steve	1,141.58	4,402.84	0.016	0.016	0.019	0.013	45.60°
Frank	2,884.70	5,554.39	0.019	0.013	0.019	0.013	11.05°
Wes	4,851.82	3,795.11	0.014	0.006	0.015	0.000	23.73°

Adjusted Distance Observations

Station Occupied	Station Sighted	Distance	V	S
Dave	Steve	2,049.60	-0.010	0.014
Steve	Frank	2,089.15	0.016	0.014
Frank	Wes	2,639.06	0.002	0.014
Wes	Dave	3,179.51	-0.011	0.015
Dave	Frank	3,181.90	-0.001	0.014
Steve	Wes	3,759.69	0.004	0.014

Adjusted Angle Observations

Station Backsighted	Station Occupied	Station Foresighted	Angle	V	S
Wes	Dave	Frank	49°01'13"	-0.9"	1.1"
Frank	Dave	Steve	40°12'39"	-3.0"	1.4"
Dave	Steve	Wes	57°44'14"	0.9"	1.5"
Wes	Steve	Frank	42°45'08"	0.5"	1.4"
Steve	Frank	Dave	39°17'59"	-0.4"	1.4"
Dave	Frank	Wes	65°26'34"	1.4"	1.4"
Frank	Wes	Steve	32°30'19"	-6.5"	1.1"
Steve	Wes	Dave	33°01'54"	4.9"	0.9"

Adjusted Azimuth Observations

Station Occupied	Station Sighted	Azimuth	V	S
Dave	Wes	23°43'46"	-0.0"	0.0"

-----Standard Deviation of Unit Weight = 1.038578-----

***16.33** Given the following inverse matrix and a standard deviation of unit weight of 1.23, determine the parameters of the error ellipse.

$$(A^TWA)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix} = \begin{bmatrix} 0.00023536 & 0.00010549 \\ 0.00010549 & 0.00033861 \end{bmatrix}$$

$$t = 31°57'43''; \; S_u = \pm 0.025 \text{ ft}; \quad S_v = \pm 0.016 \text{ ft}$$

16.34 Compute S_x and S_y in Problem 16.33.

$$S_x = 1.23\sqrt{0.00023536} = \pm 0.018 \text{ ft}; \; S_y = 1.23\sqrt{0.00033861} = \pm 0.023 \text{ ft}$$

16.35 Given the following inverse matrix and a standard deviation of unit weight of 1.45, determine the parameters of the error ellipse.

$$(A^TWA)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix} = \begin{bmatrix} 0.0004894 & 0.0000890 \\ 0.0000890 & 0.0002457 \end{bmatrix}$$

$t = 71°55'40''$; $S_u = \pm 0.033$ ft; $S_v = \pm 0.021$ ft

16.36 Compute S_x and S_y in Problem 16.35.

$S_x = 1.45\sqrt{0.0004894} = \pm 0.032$ ft; $S_y = 1.45\sqrt{0.0002457} = \pm 0.023$ ft

16.37 The well-known observation equation for a line is $mx + b = y + v_y$. What is the slope and y-intercept of the best fit line for a set of points with coordinates of (478.72, 3517.64), (1446.81, 2950.40), (2329.79, 2432.66), (3345.74, 1837.13), (4382.98, 1229.16)?

m = −0.58617; b = 3798.338

16.38 Use WOLFPACK and the following standard deviations for each observation to do a least squares adjustment of Example 10.4, and describe any differences in the solution. What advantages are there to using the least squares method in adjusting this traverse?

Stations	Angle ± S	Stations	Distance ± S
E-A-B	100°45′37″ ± 16.7″	*AB*	647.25 ± 0.027
A-B-C	231°23′43″ ± 22.1″	*BC*	203.03 ± 0.026
B-C-D	17°12′59″ ± 21.8″	*CD*	720.35 ± 0.027
C-D-E	89°03′28″ ± 10.2″	*DE*	610.24 ± 0.027
D-E-A	101°34′24″ ± 16.9″	*EA*	285.13 ± 0.026
AZIMUTH *AB*	126°55′17″ ± 0.001″		

Adjusted stations

Sta	Northing	Easting	Sn	Se	Su	Sv	t
B	4,611.179	10,517.459	0.0099	0.0132	0.0165	0.0000	126.92°
C	4,408.224	10,523.432	0.0172	0.0178	0.0193	0.0154	130.51°
D	5,102.267	10,716.279	0.0232	0.0192	0.0256	0.0160	147.58°
E	5,255.934	10,125.709	0.0150	0.0149	0.0175	0.0119	44.56°

Adjusted Distance Observations

Station Occupied	Station Sighted	Distance	V	S
A	B	647.26	-0.010	0.016
B	C	203.04	-0.013	0.016
C	D	720.34	0.013	0.017
D	E	610.23	0.005	0.017
E	A	285.14	-0.011	0.017

Adjusted Angle Observations

Station Backsighted	Station Occupied	Station Foresighted	Angle	V	S
E	A	B	100°45'44"	-6.9"	9.1"
A	B	C	231°23'34"	8.8"	12.3"
B	C	D	17°12'51"	7.6"	10.2"
C	D	E	89°03'24"	4.4"	6.1"
D	E	A	101°34'27"	-2.9"	8.4"

Adjusted Azimuth Observations

Station Occupied	Station Sighted	Azimuth	V	S
A	B	126°55'17"	0.0"	0.0"

-----Standard Deviation of Unit Weight = 0.700781

17 MAPPING SURVEYS

Asterisks (*) indicate problems that have answers given in Appendix G.

17.1 Describe what features are located on a topographic map.

From Section 17.1, paragraph 2: "Topographic maps also include planimetric features, but in addition they show the configuration of the Earth's surface."

17.2 What is the purpose of a mapping survey.

From Section 17.1, paragraph 1: "Mapping surveys are made to determine the locations of *natural* and *cultural* features on the Earth's surface and to define the configuration (*relief*) of that surface."

17.3 What factors must be considered when selecting the contour interval to be used for a given topographic map.

From Section 17.5, paragraph 4: "The contour interval selected depends on a map's purpose and scale, and upon the diversity of relief in the area."

17.4 List the different methods that can be used for a ground survey to perform a mapping survey.

From Section 17.9, paragraph 2: "Location of planimetric features and contours can be accomplished by one of the following field procedures: (1) radiation by total station instrument, (2) coordinate squares or "grid" method, (3) offsets from a reference line, (4) use of portable GNSS units, or (5) a combination of these methods."

17.5 Why are spot elevations placed on a map?

From Section 17.5, paragraph 5: " *Spot elevations* are used on maps to mark unique or critical points such as peaks, potholes, valleys, streams, and highway crossings. They may also be used in lieu of contours for defining elevations on relatively flat terrain that extends over a large area."

17.6* On a map sheet having a scale of $^{1\text{ in.} = 360\text{ ft,}}$ what is the smallest distance (in feet) that can be plotted with an engineer's scale? (Minimum scale graduations are 1/60th in.)

$$\underline{\mathbf{6\ ft}} = \frac{1}{60} 360\text{ ft}$$

17.7 What ratio scales are suitable to replace the following equivalent scales: 1:1200, 1:2400, 1:3600, 1:4800, and 1:7200?

<u>1 in. = 100 ft, 1 in. = 200 ft, 1 in. = 300 ft, 1 in. = 400 ft, 1 in. = 600 ft.</u>

17.8 A topographic map has a contour interval of 1 ft and a scale of 1:600. If two adjacent contours are 0.5 in. apart, what is the average slope of the ground between the contours?

4%; H = 50*0.5 = 25 ft; slope = 1/25*100%

17.9* On a map whose scale is 1 in. = 50 ft, how far apart (in inches) would 2-ft contours be on a uniform slope (grade) of 2 percent?

2 in. x = 2/0.02 = 100 ft, map dist = 100 ft/50 ft/in. = 2 in.

17.10* On a map drawn to a scale of 1:1000 contour lines are 16 mm apart at a certain place. The contour interval is 1 m. What is the ground slope, in percent, between adjacent contours?

6% H = 0.016(1000) = 16 m; Slope = 1/16 = 6%

17.11 Similar to Problem 17.10, except for a 5-m interval, 20-mm spacing, and a map scale of 1:5000.

5%, H = 0.020(5000) = 100 m, Slope = 5/100 = 5%

17.12 Sketch at a scale of 10 ft/in., the general shape of contours that cross a 20-ft wide street having a +4.00 percent grade, a 6 in. parabolic crown, and a 6 in. high curb.

Student sketch

17.13 Same as Problem 17.12, except a four-lane divided highway including a 30-ft wide median strip, two 12-ft wide lanes of pavement each side of the median having a +4.00 percent grade with a 1.50 percent side slope; and 10-ft wide shoulders both sides with a 2% side slope. The median slopes 10.0 percent toward the center.

Student sketch

17.14 When should points be located for contours connected by straight lines? When by smooth curves?

Straight line: When slope of ground is uniform as on a highway

Smooth curves: When slope of ground is gradually changing on gently rolling land.

17.15* What conditions in the field need to exist when using kinematic satellite survey?

No overhead obstructions/Canopy restrictions

17.16 What is a digital terrain model?

From Section 17.8, paragraph 1: "Data for use in automated contouring systems is collected in arrays of points whose horizontal positions are given by their *X* and *Y* coordinates and whose elevations are given as *Z* coordinates. Such three-dimensional arrays provide a *digital* representation of the continuous variation of relief over an area and are known as *digital elevation models* (DEMs). Alternatively, the term *digital terrain model* (DTM) is sometimes used."

17.17 Discuss why it is important to locate breaks in grade with "breaklines" in the field if contours will be drawn using a computerized automated contouring system.

From Section 17.8, paragraph 6: "Breaklines are linear topographic features that delineate the intersection of two surfaces that have uniform slopes, and thus define changes in grade. *Automated mapping algorithms use these lines to define sides of the triangles that form the TIN model, and thus elevations are interpolated along them.*"

17.18 What considerations should be given to a mapping survey using GNSS satellites?

From Chapter 15, receivers can not locate points close to buildings due to multipathing or under overhead obstructions. Additionally, in topographic surveys, the geodetic heights must be converted to orthometric heights (elevations).

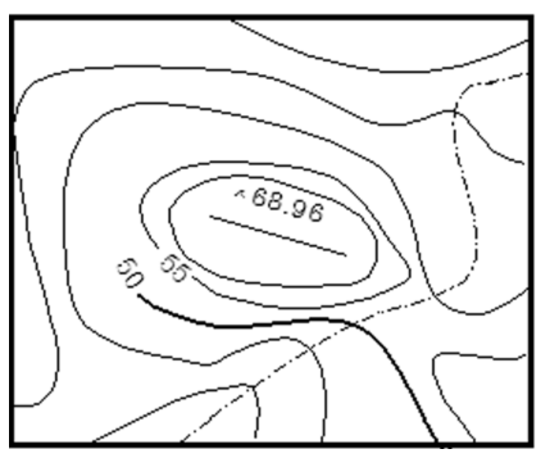
17.19 Using the labels given in parentheses in the legend of the accompanying figure create a set of notes using the drawing designators listed in Table 17.2.

Point	Drawing Designator	Point	Drawing Designator
1	.IP	13	.IP
2	CTREE	14	DTREE
3	CTREE	15	DTREE
4	DTREE	16	.IP
5	.BDG	17	.BDG
6	.BDG	18	.BDG.DECK
7	.BDG	19	.DECK
8	.BDG	20	.DECK
9	.ROAD	21	.BDG.DECK!
10	.IP.ROAD!	22	.BDG+
11	.ROAD	23	CTREE
12	.IP.ROAD!	24	.IP+

17.20 How could GNSS survey methods be used where the area of interest has some overhead obstructions?

The surveyor could locate all possible features and grade points using satellite surveying methods followed by establishing a minimum of 2 intervisible control points that can be occupied by a total station to fill in areas inaccessible with GNSS antennas.

17.21 Using the rules of contours, list the contouring mistakes that are shown in the accompanying figure and list the contouring rule it violates.



1. 65-ft contour violates item 10, which states that contours go in pairs along sides of ridges and item 11.
2. 55-ft contour should not be labeled.
3. 50-ft contour is not continuous which violates item 1.
4. 45-ft contour violates item 9 since it

points both up and down the stream

17.22 Discuss how a data collector with a total station instrument can be combined with satellite surveying methods to collect data for a topographic map.

A data collector can often interface with both a total station and a GNSS receiver. Thus the GNSS receiver can establish the coordinate system and necessary control to support the survey. It can also be used to collect data on features and grade points where overhead obstructions and multipathing are not a problem. The total station can then be interfaced with the same data collector in the same project file and be used to complete the survey in areas where the GNSS receiver could not be used.

17.23 Prepare a set of field notes to locate the topographic details in Figure 17.5. Scale additional distances and angles if necessary.

Point	Hor. Angle	Zen. Angle	Distance	Notes
3	16°37'44"	90°25'50"	565.855	DTREE 24-in. Maple
4	70°35'24"	91°15'48"	436.472	MH Sanitary manhole
5	225°14'22"	88°30'36"	265.934	.BDG SE Corner of Building

17.24 Why is it dangerous to run the control traverse at the same time as planimetric data is being collected?

From Section 17.4, paragraph 4: "Regardless of the methods used in conducting the control surveys for mapping projects, specified maximum allowable closure errors for both horizontal and vertical control should be determined in advance of the fieldwork, then used to guide it. The locations of the features, which comprise the map (often also called *map details*), are based upon the framework of control points whose positions and elevations are established. Thus, any errors in the surveyed positions or elevations of the control points will result in erroneous locations of the details on the map. Therefore, it is advisable to run, check, and adjust the horizontal and vertical control surveys before beginning to locate map details, rather than carry on both processes simultaneously."

17.25 Cite the advantages of locating topographic details by radial methods using a total station instrument with a data collector.

From Section 17.9.1, paragraph 1: "This method is especially efficient if a data collector (see Sections 2.12 through 2.15) is used to record the point identities and their associated descriptions, vertical distances, horizontal distances, and directions. The data collector permits downloading the observations directly into a computer for processing through an automated mapping system. The field procedure of radiation with a total station can be made most efficient if the instrument is placed at a good vantage point (on a hill or ridge) that overlooks a large part or all of the area to be surveyed. This permits more and longer radial lines and reduces the number of setups required."

17.26 What does the term “point cloud” describe in laser-scanning?

From Section 17.9.5, paragraph 2: "The resulting grid of scanned, three-dimensional points can be so dense that a visual image of the scene is formed. This so-called “point-cloud” image differs from a photographic image in that every point has a three-dimensional coordinate assigned to it. These coordinates can be used to obtain dimensions between any two observed points in the scene."

17.27 List the methods for establishing control to support a topographic mapping project.

From Section 17.4, paragraph 1: "*Horizontal control* for a mapping survey is provided by two or more points on the ground, permanently or semi-permanently monumented, and precisely fixed in position horizontally by distance and direction or coordinates. It is the basis for locating map features. Horizontal control can be established by the traditional ground surveying methods of *traversing, triangulation,* or *trilateration* (see Section 19.12), or by using *GNSS* surveys (see Chapters 14 and 15)."

From Section 17.4, paragraph 3: "*Vertical control* is provided by benchmarks in or near the tract to be surveyed and becomes the foundation for correctly portraying relief on a topographic map. Vertical control is usually established by running lines of differential levels starting from and closing on established benchmarks (see Chapters 4 and 5). Project benchmarks are established throughout the mapping area in strategic locations and their elevations determined by including them as turning points in the differential leveling lines. In rugged areas, trigonometric leveling with total station instruments is practical and frequently used to establish vertical control for mapping. GNSS surveys are also suitable for establishing vertical control for topographic mapping but the ellipsoidal heights derived from GNSS surveys must first be converted to orthometric heights Using Equation (13.8). The latter two methods are of sufficient accuracy to support most mapping surveys."

For Problems 17.28 through 17.31, calculate the *X, Y* and *Z* coordinates of point *B* for radial readings taken to *B* from occupied station *A*, if the backsight azimuth at *A* is 25°32'48", the elevation of *A* = 610.098 m, and *hi* = 1.45 m. Assume the *XY* coordinates of *A* are (10,000.000, 5000.000).

17.28* Clockwise horizontal angle = 55°37'42", zenith angle = 92°34'18", slope distance = 435.098 m, *hr* = 2.000 m.

<u>**(10429.514, 5066.684, 590.026)**</u>

17.29 Clockwise horizontal angle = 272°42'22", zenith angle = 92°28'16", slope distance = 58.905 m, *hr* = 1.500 m.

<u>**(9948.161, 5027.857, 607.508)**</u>

17.30 Clockwise horizontal angle = 55°15'06", zenith angle = 88°35'24", slope distance = 103.023 m, *hr* = 1.500 m.

<u>**(10101.666, 5016.469, 612.583)**</u>

17.31 Clockwise horizontal angle = 307°56'52", zenith angle = 87°17'40", slope distance = 304.902 m, *hr* = 1.500 m.

(9867.079, 5272.550, 624.440)

17.32 Describe how the arbitrary coordinates of a point cloud are transformed into a conventional coordinate system.

From Section 17.9.5, paragraph 3: "The original point cloud has coordinates in an arbitrary three-dimensional coordinate system. If it is necessary to have coordinates in a project-based coordinate system, a traditional survey can be used to establish coordinates on targets in the scene. Control must be strategically located at the edges of each scene. A minimum of three control points per scene is required. However, additional control is often used to provide redundancy. Multiple scenes can be connected using common control targets. After determining the project coordinates of the control, a three-dimensional conformal coordinate transformation discussed in Section 17.10, is performed to transform points from the arbitrary coordinate system to the project coordinate system."

17.33 List various equipment used for making hydrographic depth soundings, and discuss the limitations, advantages, and disadvantages of each.

From Section 17.13: Sounding pole, lead lines, or echo (depth) sounder.

17.34* On a map having a scale of 200 ft/in. the distance between plotted fixes 49 and 50 of Figure 17.14 is 3.15 in. From measurements on the profile of Figure 17.13, determine how far from fix 50 the 20-ft contour (existing between fixes 49 and 50) should be plotted on the map.

0.47 in. Ratio between fixes 49 and 50 = 0.15. Solution = 0.15*3.15 = 0.47 in.

17.35 Similar to Problem 17.34, except locate the 16-ft contour between fixes 50 and 51 if the corresponding map distance is 2.98 in.

2.12 in. Ratio = 0.71 in. Solution = 0.71(2.98) = 2.12 in.

17.36 Why is it important to show the shoreline and some planimetric features for navigation hydrographic maps?

Navigators, fisherman, and others can visually locate their position using references to shoreline features.

17.37 Create a computational program that solves performs a three-dimensional conformal coordinate transformation as described in Section 17.10.

Solutions will vary.

18 MAPPING

Asterisks (*) indicate problems that have answers given in Appendix G.

18.1 Give the terms to which the acronyms TIN, DTM, and DEM apply.

TIN - Triangulated Irregular Network

DTM - Digital Terrain Model

DEM - Digital Elevation Model

***18.2** On a map drawn to a scale of 1:6000, a point has a plotting error of 1/30-in. What is the equivalent ground error in units of feet?

200 in. = 6000(1/30)

18.3 What is the scale of the USGS 7-1/2 min quadrangle map sheet?

1:24,000

18.4 What design elements must be considered when laying out a map?

From Section 18.6, paragraph 2: "To achieve maximum effectiveness in map design, the following elements or factors should be considered: (1) clarity, (2) order, (3) balance, (4) contrast, (5) unity, and (6) harmony."

18.5 Why is scale depicted graphically on a map?

From Section 18.12, paragraph 4: "If a map sheet is enlarged or reduced in a reproduction process, the graphical scale will change accordingly, and thus the original scale of the map will be preserved on the reproduction."

18.6 Why should contours be broken where labeling occurs?

From Section 18.11, paragraph 3: "Text should take precedence over line work. If necessary, lines should be broken where text is placed, as this improves clarity. An example of this is in the labeling of contours, where the lines are preferably broken and the contour elevation inserted in the break. It is best to select straight, or nearly straight, sections of contours for labeling."

18.7 Discuss why a map designed for a planning board hearing may not be the same as a map designed for an engineer?

From Section 18.6, paragraph 9: "In designing maps, it is important to remember that different audiences may require different maps. For example, it would be difficult for a layperson to read and understand a map produced for an engineering project. Accordingly, maps that are developed for design professionals are not generally suitable for public hearings. In fact because laypeople often have no training in map reading, it may be best to develop specialized three-dimensional maps or models that depict relief, boundaries, proposed buildings, landscaping, and so on."

18.8 What is the content of DTMs?

From Section 18.14, paragraph 10: "A DTM shows only topographic features of the earth and is devoid of any vegetation, or structures that lie on the surface. A DTM can be created with a TIN or by locating spot elevations in a grid, although, the former is more prevalent in practice."

18.9 List the advantages of compiling maps using field-to-finish software?

From Section 18.8.2, paragraph 1: "Fundamentally, CADD systems plot points and lines in a manner similar to manual drafting techniques. However compared to manual map drafting, computer-assisted mapping offers advantages of increased accuracy, speed, flexibility, and reduced cost. Computers are capable of quickly performing many drafting chores that are tedious and time consuming if done by manual methods, e.g., drawing complicated line types and symbols, and performing lettering. With CADD systems, lettering reduces to simply choosing letter sizes and styles and selecting and monitoring placement. Since these systems can often read files of coordinates, such as those from data collectors, the plotting process can become almost totally automated (see Section 17.11). For example, many common features of a map such as bar scale, north arrow, legend, and title block can be created as *blocks* and imported into any map with varied scales. This process simplifies the entire map production process and creates a standardized look for a mapping agency or company. Additionally, the digital environment of a CADD system allows for the easy arrangement of the mapping elements, which simplifies the process of map design and enables colors to be readily selected and changed."

***18.10** For a 20-ft contour interval, what is the greatest error in elevation expected of any definite point read from a map if it complies with National Map Accuracy Standards?

<u>±20 ft</u>;

From Section 18.4, paragraph 3: "The NMAS vertical accuracy requirements specify that not more than 10 percent of elevations tested shall be in error by more than one-half the contour interval, and **none can exceed the interval**."

18.11 An area that varies in elevation from 463–634 ft is being mapped. What contour intervals will be drawn if a 20-ft interval is used? Which lines are emphasized?

<u>480, 500, 520, 540, 560, 580, 600, and 620</u>

18.12 Similar to Problem 18.11, except elevations vary from 37–165 m and a 10-m interval is used.

<u>40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, and 160</u>

18.13 What two questions must be answered before designing a map?

From Section 18.6, paragraph 1: "Before beginning the design of a map, the following two basic questions should be answered: (1) What is the purpose of the map? and (2) Who is the map intended to serve? All maps have a purpose, which in turn dictates the information that the map must convey. Once the purpose of the map is fixed, emphasis should be placed on achieving the design that best meets its objectives and conveys the necessary information clearly to its users."

18.14 Describe how clarity can be enhanced through the use of fonts.

From Section 18.11, paragraphs 1, 2 & 3: "To produce a professional looking drawing and one that clearly conveys the intended information, a suitable style of lettering must be selected. That style should be used consistently throughout the map, but the size varied in accordance with the importance of each particular item identified. Lettering that is too big or bold should not be used, but the letters must be large enough to be readable without difficulty.

Lettering should be carefully placed so that it is clearly associated with the item it identifies and so that letters do not interfere with other features being portrayed. Typically, the best balance results if names are centered in the objects being identified. Also both appearance and clarity are generally improved by aligning letters parallel with linear objects that run obliquely, as has been done with the traverse lengths and bearings of Figure 18.2. For ease in map reading, letters should be placed so that the map can be read from either the bottom or its right side.

Text should take precedence over line work. If necessary, lines should be broken where text is placed, as this improves clarity. "

***18.15** What is the largest acceptable error in position for 90% of the well-defined points on a map with a 1:24,000 scale that meets national map accuracy standards.

From Section 18.4, paragraph 2: "To meet the NMAS horizontal position specification, for maps produced at scales larger than 1:20,000, not more than 10 percent of well-defined points tested shall be in error by more than 1/30 in. (0.8 mm)."

18.16 Discuss how balance is achieved on a map.

From Section 18.6, paragraph 5: "All elements on a map have weight, and they should be distributed uniformly around the “visual center” of the map to create good overall *balance.* The visual center is slightly above the geometrical center of the map sheet. In general, the weight of an element is affected by factors such as size, color, font, position, and line width. Map elements that appear at the center have less weight than those on the edges. Elements in the top or right half of the map will appear to have more weight than those in the bottom or left half of the map. Also map elements identified with thicker line widths will appear to have heavier weights than their slimmer counterparts. Colors such as red appear heavier than blue or yellow. ... The use of thumbnail sketches can often help to achieve a balanced layout for a map. It is important to place highest weights on those elements that enhance the purpose of the map."

18.17 Discuss why insets are sometimes used on maps.

From Section 18.6, paragraph 3: **To enhance clarity**: "If considerable detail must be included on a map, the information could be placed in a table. Other alternatives consist of preparing larger-scale *inset maps* of areas that contain dense detail, or creating an overlay to display some of the detail."

18.18* If a map is to have a 1-in. border, what is the largest nominal scale that may be used for a subject area with dimensions of 604 ft and 980 ft on a paper of dimensions 24 by 36 in?

1 in./30 ft, or 1:360

Usable height = 22 in. Usable width = 34 in.

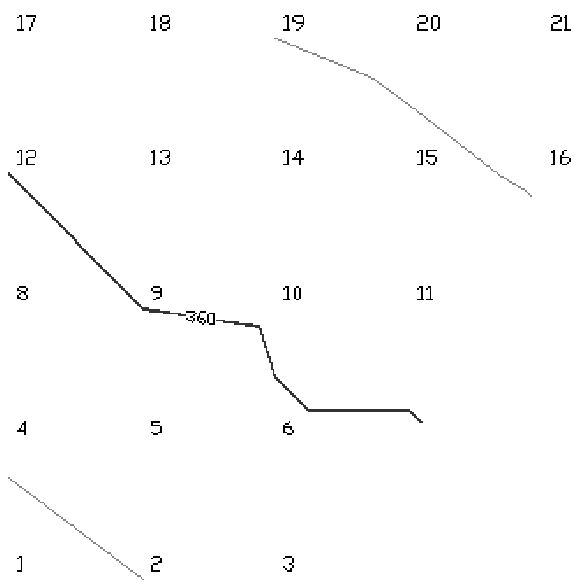
Scale for height = 604/22 = 27.5 ft/in. Scale for width = 980/34 = 28.8 ft/in.

18.19 Similar to Problem 18.18, except the dimensions of the subject area are 653 ft and 475 ft.

30 ft/in. or 1:360

Scale for height= 475/22 = 21.6 ft/in. Scale for width = 653/34 = 19.4 ft/in.

18.20 Draw 2-ft contours for the data in Plate B.2 of Appendix B.



18.21 If 90 percent of all elevations on a map must be interpolated to the nearest ±2 ft, what contour interval is necessary according to the National Map Accuracy Standards? Explain.

4 ft

18.22 If an area having an average slope of 4% is mapped using a scale of 1:1000 and contour interval of 0.5 m, how far apart will contours be on the map?

12.5 mm; run = 0.5m/0.04 = 12.5 m; x = 12,500 mm/1000 = 12.5 mm

18.23 Similar to Problem 18.22, except average slope is 6%, map scale is 300 ft/in., and contour interval is 10 ft.

0.56 in.; run = 10 ft/0.06 = 166.67 ft; x = 166.67/100 = 0.56 in.

***18.24** Similar to Problem 18.22, except average slope is 5%, map scale is 1:500, and contour interval is 0.5 m.

2 cm; run = 0.5 m/0.05 = 10 m; x = 10,000 mm/500 = 20 mm = 2 cm

***18.25** The three-dimensional (X, Y, Z) coordinates in meters of vertexes A, B, and C in Figure 18.14 are (5412.456, 4480.621, 248.147), (5463.427, 4459.660, 253.121) and (5456.081, 4514.382, 236.193), respectively. What are the coordinates of the intersection of the 250-m contour with side AB? With side BC?

AB: (5431.445, 4472.812); BC = (5462.073, 4469.745)

AB = 55.113 m; Az_{AB} = 112°21'14.8"; BC = 55.213 m; Az_{BC} = 352°21'15.1"

Interpolations:

$$AB : x = 55.113\frac{1.853}{4.974} = 20.532 \text{ m}; \;\; BC : x = 55.213\frac{-3.121}{-16.928} = 10.180 \text{ m}$$

18.26 The three-dimensional (X,Y,Z) coordinates in feet for vertices A, B, and C in Figure 18.14 are (8649.22, 6703.67, 143.86), (8762.04, 6649.77, 165.88) and (8752.64, 6770.20, 146.84), respectively. What are the coordinates of the intersections of the 150-ft contour as it passes through the sides of the triangle?

AB: (8680.68, 6688.64); BC: (8754.20, 6750.21)

AB = 125.034 ft; Az_{AB} = 115°32'10.5"; BC = 120.796 ft; Az_{BC} = 355°32'12.9"

Interpolations:

$$AB : x = 125.034\frac{6.14}{22.02} = 34.86 \text{ ft}; \;\; BC : x = 122.97\frac{15.88}{19.04} = 100.748 \text{ ft}$$

18.27 Similar to Problem 18.26, except compute the coordinates of the intersection of the 152-ft contour.

AB: (8690.92, 6683.74); BC = (8755.19, 6737.56)

AB = 125.034 ft; Az_{AB} = 115°32'10.5"; BC = 120.796 ft; Az_{BC} = 355°32'12.9"

Interpolations:

$$AB : x = 125.03\frac{8.14}{22.02} = 46.22 \text{ ft}; \;\; BC : x = 122.97\frac{15.88}{19.04} = 88.06 \text{ ft}$$

18.28 Discuss how contrast can be improved on a map.

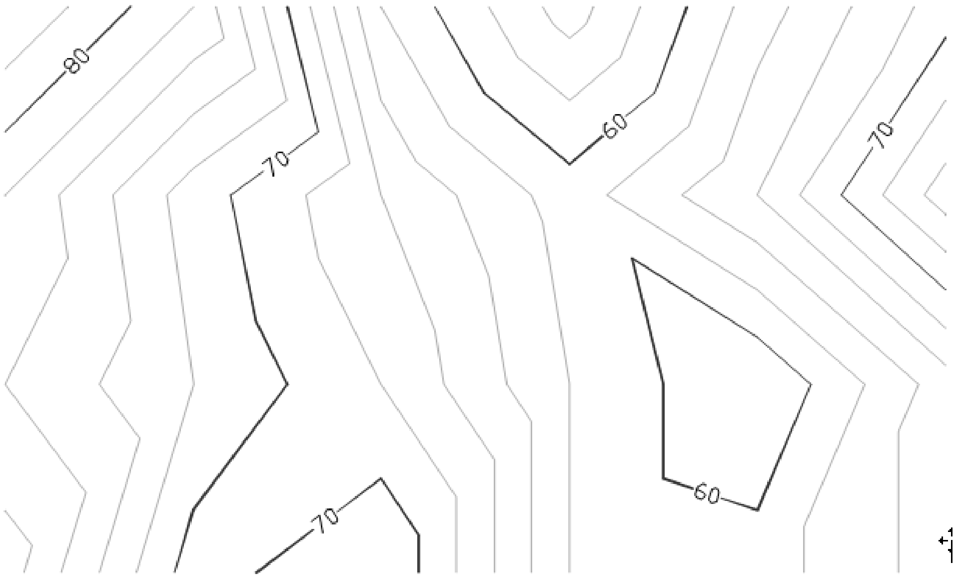
From Section 18.6, paragraph 6: "*Contrast* relates primarily to the use of different line weights, and fonts of varying sizes. Contrast can be used to enhance balance, order, and clarity. For example, the title of the map should be displayed in a larger font than the other textual elements. This will attract the viewer's attention, thereby enhancing the order and clarity of the map. Various fonts can also be used to provide balance with other elements on the map. Another example where contrast supports the clarity of a map is in contouring. Here *index contours* (every fifth contour) should be drawn with a heavier line than the other contours. This enhances the map's clarity and facilitates the determination of elevations."

84	78	62	55	63	69
78	71	66	61	66	75
76	72	68	62	58	65

The following table gives elevations at the corners of 50-ft coordinate squares, and they apply to Problems 18.29 and 18.31.

18.29 At a horizontal scale of 1 in. = 50 ft draw 2-ft contours for the area.

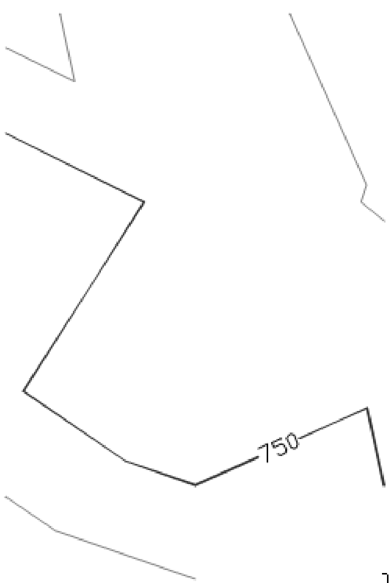
18.30 Similar to Problem 18.29, except at the bottom of the table add a fourth line of elevations: 79, 69, 72, 62, 61, and 65 (from left to right).



18.31 In problem 18.30, about which number in the table can a 5-ft closed contour be drawn?

58 ft; 60-ft contour is closed as shown in solution to 18.30 about 58-ft grid point.

18.32 A rectangular lot running N-S and E-W is 150 by 100 ft. To locate contours it is divided into 50-ft square blocks, and the following horizontal rod readings are taken at the corners successively along the E-W lines, proceeding from west to east, and beginning with the northernmost line: 6.8, 5.6, 3.6; 6.3, 6.9, 5.1; 5.9, 4.7, 2.6; and 3.4, 4.9, 6.3. The HI is 323.9 ft. Sketch 2-ft contours.



18.33 If a map is drawn with 10-ft contour intervals, what contours between 70 ft and 330 ft are drawn with heavier line weight?

100, 200, 300

18.34 Find the web site at which USGS quadrangle sheets may be purchased.

www.usgs.gov or **store.usgs.gov**

18.35 Download and prepare a report on Part 3 of the FGDC Geospatial Positioning Accuracy Standards.

Independent project.

19 CONTROL SURVEYS AND GEODETIC REDUCTIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

19.1 Define the *geoid* and *ellipsoid*.

From Section 19.2, paragraph 1: "The geoid is an equipotential gravitational surface located approximately at mean sea level, which is everywhere perpendicular to the direction of gravity."

From Section 19.2, paragraph 2: "The ellipsoid is a mathematical surface obtained by revolving an ellipse about the Earth's polar axis. The dimensions of the ellipse are selected to give a good fit of the ellipsoid to the geoid over a large area, and are based upon surveys made in the area."

19.2 What are the possible monumentation types for a control station with a quality code of *A*.

From Section 19.11, paragraph 3: "Qualty code A monuments are the most reliable and are expected to hold a precise elevation. These monuments are typically rock outcrops, bedrock, and similar features plus massive structures with deep foundations; large structures with foundations on bedrock; or sleeved deep settings (10 ft or more) with galvanized steel pipe or galvanized steel, stainless steel, or aluminum rods."

19.3 What is *nutation*?

From Section 19.3, paragraph 1: "Additionally, the pole wanders in much smaller radial arcs that are superimposed upon precession. These smaller circles are known as a nutation, and are completed about once every 18.6 years."

***19.4** What is the difference between the equatorial circumference of the Clarke 1866 ellipsoid and that of the GRS80 ellipsoid?

$\underline{\mathbf{\Delta C = 436.0\ m}}$ $= 2\pi(6378206.4 - 6378137.0)$

19.5 Determine the first and second eccentricities for the GRS80 ellipsoid.

By Equation (19.2): $\underline{\mathbf{e = 0.081819191;}}$ $\underline{\mathbf{e' = 0.08209448}}$

19.6 Why shouldn't points defined in different versions of the NAD83 datum be mixed in a survey?

Points in different reference frames come from different adjustments and are, thus, part of different coordinate systems. Thus, points in NAD83 (legacy) will not agree with NAD83(2007).

19.7 What are the radii in the meridian and prime vertical for a station with latitude 29°06'58.29740" using the GRS80 ellipsoid?

By Equation 19.5: **R_M = 6,350,531.514 m**

By Equation 19.4: **R_N = 6,383,197.617 m**

19.8 For the station listed in Problem 19.7, what is the radius of the great circle at the station that is at an azimuth of 302°49'21" using the GRS80 ellipsoid?

By Equation 19.6: **R_α = 6,373,565.327 m**

***19.9** What are the radii in the meridian and prime vertical for a station with latitude 42°37'26.34584" using the GRS80 ellipsoid?

By Equation 19.5: **R_M = 6,364,725.399 m**

By Equation 19.4: **R_N = 6,387,949.711 m**

19.10 For the station listed in Problem 19.9, what is the radius of the great circle at the station that is at an azimuth of 153°29'32" using the GRS80 ellipsoid?

By Equation 19.6: **R_α = 6,369,338.224 m**

***19.11** The orthometric height at Station Y 927 is 304.517 m, and the geoidal undulation at that station −31.893 m. What is its geodetic height?

By Equation (19.7): h = **272.624 m**

19.12 The geodetic height at Station Z104 is 102.054 m. Its geoidal undulation is −22.101 m. What is its orthometric height?

By Equation (19.7): H = **124.155 m**

19.13 The orthometric height of a particular benchmark is 87.95 ft. The geoidal height at the station is –30.66 m. Is the station above or below the ellipsoid? Draw a sketch depicting the geoid, ellipsoid, and benchmark.

Below ; By Equation (19.7): h = 87.95(12/39.37) − 30.66 = −3.805 m. So it is below.

19.14 The instantaneous position of the pole at the time of an azimuth observation is x −0.55" and y = −0.83". The position of the station is (42°37'23.0823" N, 128°56'01.0089" W) and the observed azimuth of a line is 18°52'37". What is the astronomic azimuth of the line corrected for polar motion?

By Equation (19.3): **18°52'35.7"** $Az_A = Az_{obs} - (x \sin \lambda + y \cos \lambda) \sec \phi$

***19.15** The deflection of the vertical components ξ and η are −2.85" and −5.94", respectively. The observed zenith angle is 42°36'58.8". What is the geodetic zenith angle and for the observations in Problem 19.14?

z = 42°36'54.2"; α = 18°52'46.3" by Equations (19.34) and (19.32), respectively.

19.16 To within what tolerance should the elevations of two benchmarks 15-km apart be established if first-order class II standards were used to set them? What should it be if second-order class I standards were used?

First order, Class III: ±2.7 mm; By equations in Table 19.5

19.17 Name the orders and classes of accuracy of both horizontal and vertical control surveys, and give their relative accuracy requirements.

See Table 19.4 and 19.5

***19.18** Given the following information for stations *JG00050* and *KG0089*, what should be the leveled height difference between them?

Station	Height (m)	Gravity (mgal)
JG0050	474.442	979,911.9
KG0089	440.552	979,936.2

−33.880 m; By Equations (19.42) and (19.43).

19.19 Similar to Problem 19.18 except that the station data for EY5664 and EY1587 is

Station	Height (m)	Gravity (mgal)
EY5664	2.960	979,811.6
EY1587	3.679	979,791.2

0.719 m by Equations (19.42) and (19.43)

19.20 Similar to Problem 19.18 except that the station data for CV0178 and DQ0080 is

Station	Height (m)	Gravity (mgal)
CV0178	1028.652	979,279.4
DQ0080	1013.487	979,269.9

−15.176 m by Equations (19.42) and (19.43)

***19.21** A slope distance of 2458.663 m is observed between two points *Gregg* and *Brian* whose orthometric heights are 458.966 m and 566.302 m, respectively. The geoidal undulations are −25.66 m and −25.06 m at *Gregg* and *Brian*, respectively. The height of the instrument at station *Gregg* at the time of the observation was 1.525 m and the height of the reflector at station *Brian* was 1.603 m. What are the geodetic and mark-to-mark distances for this observation? (Use an average radius for the Earth of 6,371,000 m for R_α)

D_2 (geodetic) = **2456.310 m**; D_3 (mark-to-mark) = **2458.868 m** by Equations (19.21) through (19.23)

19.22 If the latitude of station *Gregg* in Problem 19.21 was 45°22'58.6430" and the azimuth of the line was 110°33'03.8" what are the geodetic, and mark-to-mark distances for this observation? (Use the GRS80 ellipsoid).

D_2 (geodetic) = **<u>2456.310 m</u>**; D_3 (mark-to-mark) = **<u>2458.868 m</u>** by Equations (19.21) through (19.23)

where R_M = 6,367,810.618 m; R_N = 6,388,981.703; and R_α = 6,386,365.179 m

19.23 A slope distance of 6704.511 m is observed between two stations *A* and *B* whose geodetic heights are 916.963 m and 928.578 m, respectively. The height of the instrument at the time of the observation was 1.500 m, and the height of the reflector was 1.825 m. The latitude of Station *A* is 33°08'36.2947" and the azimuth of *AB* is 202°28'21.9". What are the geodetic, and mark-to-mark distances for this observation?

D_2 (geodetic) = **<u>6704.507 m</u>**; D_3 (mark-to-mark) = **<u>6705.490 m</u>** by Equations (19.21) through (19.23)

where R_M = 6,354,503.682 m; R_N = 6,384,528.207; and R_α = 6,358,872.952 m

19.24 Discuss the advantages of combined networks for establishing control.

From Section 19.12.4: "These surveys provide the greatest geometric strength and the highest coordinate accuracies for traditional survey techniques."

***19.25** Compute the back azimuth of a line 5863 m long in the east-west direction at a mean latitude of 45°01'32.0654" whose forward azimuth is 88°16'33.2" from north. (Use an average radius for the Earth of 6,371,000 m.)

<u>268°19'43.2"</u> = 88°16'33.2 − 190" + 180° and θ = 190" from Equation (19.15).

19.26 Compute the back azimuth of a line 6505 m long in the east-west direction at a mean latitude of 38°52'02" whose forward azimuth is 83°24'37.5" from north. (Use an average radius for the Earth of 6,371,000 m.)

<u>263°27'27.2"</u> = 83°24'37.5 − 169.7" + 180° and θ = 169.7" from Equation (19.15).

19.27 In Figure 19.14 azimuth of *AB* is 102°36'20" and the angles to the right observed at *B, C, D, E*, and *F* are 132°01'05", 241°45'12", 141°15'01", 162°09'24", and 202°33'19", respectively. An astronomic observation yielded an azimuth of 82°24'03" for line *FG*. The mean latitude of the traverse is 42°16'00", and the total departure between points *A* and *F* was 24,986.26 ft. Compute the angular misclosure and the adjusted angles. (Assume the angles and distances have already been corrected to the ellipsoid.)

Misclosure = −15.3" Correction +3"/angle.

Adjusted Angles: 132°01'08", 241°45'15", 141°15'04", 162°09'27", and 202°33'22"

θ = 206.7" by Equation (19.15)

19.28 In Figure 19.20 slope distance *S* and vertical angles α and β were observed as 76,953.82 ft, +4°18'42", and −4°26'28", respectively. Ellipsoid height of point *A* is 1672.21 ft. What is length *A'B'* on the ellipsoid? (Use an average radius for the Earth of 6,371,000 m.)

76,687.950 m δ = 4°22'35.0"; AB_1 = 76,729.444; BB_1 = 5872.203; ψ = 0°41'24.0";

B_1B_2 = 35.359; AB_2 = 76,694.086

19.29 In Figure 19.19 slope distance *S* was observed as 19,875.28 m. The orthometric elevations of points *A* and *B* were 657.73 m and 1805.54 m, respectively, and the geoidal undulation at both stations was −30.5 m. The instrument and reflector heights were both set at 1.50 m. Calculate geodetic distance *A'B'* (Use an average radius for the Earth of 6,371,000 m.)

19,875.275 m D_3 = 19,838.364 m; D_2 = 19,838.372 m

19.30 In Figure 19.20, slope distance *S* and zenith angle α at station *A* were observed as 2072.33 m and 82°17'18", respectively to station *B*. If the elevation of station *A* is 435.967 m and the geoidal undulations at stations *A* and *B* are both −28.04m, what is ellipsoid length *A'B'*? (Use an average radius for the Earth of 6,371,000 m.)

2053.556 m δ = 7°43'05.7"; AB_1 = 2053.556; BB_1 = 278.318; ψ = 0°01'06.5";

B_1B_2 = 0.0448; AB_2 = 2053.511 m

***19.31** Components of deflection of the vertical at an observing station of latitude 43°15'47.5864" are ξ = −6.87" and η = −3.24" If the observed zenith angle on a course with an astronomic azimuth of 204°32'44" is 85°56'07", what are the azimuth and zenith angles corrected for deviation of the vertical?

z = 85°56'00.1"; α = 204°32'47.3" by Equations (19.34) and (19.32), respectively.

19.32 At the same observation station as for Problem 19.31, the observed zenith angle on a course with an azimuth of 12°08'07" is 80°15'54", what are the azimuth and zenith angles corrected for deviation of the vertical?

z = 80°15'47.1"; α = 12°08'10.6" by Equations (19.34) and (19.32), respectively.

19.33 Using the reduced azimuths of Problems 19.31 and 19.32, what is the reduced geodetic angle that is less than 180°?

Angle: **167°35'23.3"**

19.34 What is a dynamic height of a point?

From Section 19.14.4, paragraph 3: "Dynamic heights are the geopotential number divided by a *reference gravity* ..."

19.35 Compute the collimation correction factor C for the following field data, taken in accordance with the example and sketch in the field notes of Figure 19.18. With the instrument at station 1, high, middle, and low cross-hair readings were 5.512, 5.401, and 5.290 ft on station A and 4.978, 3.728, and 2.476 ft on station B. With the instrument at station 2, high, middle, and low readings were 7.211, 6.053, and 4.894 ft on A and 4.561, 4.358, and 4.155 ft on B.

0.0022 ft/ft

	r1	i1	R1	I1		
1	5.512		4.978			
	5.401	0.111	3.728	1.250	C	0.0022
	5.290	0.111	2.476	1.252		
	5.401	0.222	3.727	2.502		
	R2	I2	r2	i2		
2	7.211		4.651			
	6.053	1.158	4.358	0.293		
	4.894	1.159	4.155	0.203		
	6.053	2.317	4.388	0.496		

19.36 A leveling instrument having a collimation factor of −0.007 m/m of interval was used to run a section of three-wire differential levels from BM A to BM B. Sums of backsights and foresights for the section were 1320.892 m and 1333.695 m, respectively. Backsight stadia intervals totaled 1557.48, while the sum of foresight intervals was 805.67. What is the corrected elevation difference from BM A to BM B?

−18.066 m by Equation (19.17)

19.37 The relative error of the difference in elevation between two benchmarks directly connected in a level circuit and located 90 km apart is ±0.009 m. What order and class of leveling does this represent?

Second order, Class I; c = 0.95 mm

19.38 Similar to Problem 19.37, except the relative error is 0.025 ft for benchmarks located 35 km apart.

Second Order, Class II c = 1.28 mm

19.39 The baseline components of a GPS baseline vector observed at a station A in meters are (1204.869, 798.046, −666.157). The geodetic coordinates of the first base station are 44°27'36.0894" N latitude and 74°44'09.4895" W longitude. What are the changes in the local geodetic coordinate system of (Δn, Δe, Δu)?

(−158.393, 1372.464, −759.213)

R				neu
-0.1844	0.675703	0.713739		-158.393
0.964723	0.263267	0		1372.464

0.187904 -0.68856 0.654665 -759.213

19.40 In Problem 19.39, what are the slant distance, zenith angle, and azimuth for the baseline vector?

S = 1591.337, Az = 119°45'05.9", z 96°34'59.7" by Equation (19.50)

19.41 If the slant distance between two stations is 843.273 m, the zenith angle between them is 85°58'44" and the azimuth of the line is 312°23'59", what are the changes in the local geodetic coordinates?

(−43.668, −621.189, 59.134) by Equation (19.51)

19.42 Create a computational program to solve Problem 19.40.

19.43 Create a computational program to solve Problem 19.41.

20 STATE PLANE COORDINATES AND OTHER MAP PROJECTIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

20.1 Name three developable surfaces that are used in map projections.

Plane, Cone, Cylinder

20.2 Name the two basic projections used in state plane coordinate systems. What are their fundamental differences? Which one is preferred for states whose long dimensions are north-south? East–west?

From Section 20.2, paragraph 2: Lambert conformal conic (LCC) and transverse Mercator (TM).

Fundamental differences are that LCC uses the cone as a developable surface and the transverse Mercator uses a cylinder. Additionally, LCC has constant scale going east-west whereas TM as constant scale going north-south. Thus the TM is preferred for states long in the North–south direction and LCC is preferred for states that are long in the east-west direction.

20.3 What does it mean to be a conformal projection?

From Section 20.2, paragraph 4: " *conformal projection* (one that preserves true angular relationships around points in a small region)"

20.4 How is a geodetic distance reduced to the map projection surface in state plane coordinate system map projections?

From Section 20.8.1, paragraph 2: " This is accomplished by multiplying the ellipsoidal length of the line by an appropriate *scale factor*."

20.5 What corrections must be made to measured slope distances prior to computing state plane coordinates?

From Section 20.8.1, paragraph 1: "The reduction of distances is normally done in two steps: (1) reduce the observations from their ground lengths to ellipsoid lengths (geodetic distance) and (2) reduce the ellipsoid lengths to their grid equivalents."

20.6 What is the primary difference between grid and geodetic azimuths?

From Section 20.8.2, paragraph 1: " The primary difference between these directions is the convergence angle γ."

20.7 Develop a table of SPCS83 elevation factors for geodetic heights ranging from 0 ft to 3000 ft. Use increments of 500 ft and an average radius for the Earth of 6,371,000 m.

Elevation (ft)	**Scale**
0	##########
500	##########
1000	##########
1500	##########
2000	##########
2500	##########

20.8 Similar to Problem 20.7, except for geodetic heights from 0 m to 1200 m using 100 m increments.

Height (m)	**Scale**
0	##########
100	##########
200	##########
300	##########
400	##########
500	##########
600	##########
700	##########
800	##########
900	##########
1000	##########
1100	##########
1200	##########

20.9 Explain how surveys can be extended from one state plane coordinate zone to another or from one state to another.

From Section 20.10, paragraph 1: "The general procedure for extending surveys from one zone to another requires that the survey proceed from the first zone into the overlap area with the second. Then the geodetic latitudes and longitudes are computed for two intervisible stations using their grid coordinates in the first zone. (Recall that this conversion is called the *inverse problem.*) Using the geodetic positions of the two points, their state plane coordinates in the new zone are then computed. (This is the *direct problem.*) Finally the grid azimuth for the line in the new zone can be obtained from the new coordinates of the two points."

20.10 Develop a table similar to Table 20.1 for a range of latitudes from 40°30' N to 40°39' N in the Pennsylvania North Zone with standard parallels of 40°53' N and 41°57' N, and a grid origin at (40°10' N, 77°45' W).

Latitude	R (m)	Tab. Diff.	k
40°30'	7342329.667	30.84819	1.000083949
40°31'	7340478.776	30.84814	1.000079382
40°32'	7338627.887	30.84809	1.000074899
40°33'	7336777.002	30.84805	1.000070499
40°34'	7334926.119	30.84800	1.000066182
40°35'	7333075.239	30.84796	1.000061949
40°36'	7331224.361	30.84793	1.000057798
40°37'	7329373.485	30.84789	1.000053731
40°38'	7327522.612	30.84786	1.000049747
40°39'	7325671.740	30.84783	1.000045847

***20.11** The Pennsylvania North Zone SPCS83 state plane coordinates of points *A* and *B* are as follows:

Point	E(m)	N(m)
A	541,983.399	115,702.804
B	541,457.526	115,430.257

Calculate the grid length and grid azimuth of line *AB*.

592.304 m, 242°36'12"

20.12 Similar to Problem 20.11, except points *A* and *B* have the following New Jersey SPCS83 state plane coordinates:

Point	E(m)	N(m)
A	135,995.711	19,969.603
B	136,459.204	20,302.524

570.668 m, 54°31'08"

20.13 What are the SPCS83 coordinates (in ft) and convergence angle for a station in the North zone of Pennsylvania with geodetic coordinates of 41°12'33.0745" N and 76°23'48.9765" W?

From WolfPack: **X = 2,340,786.03 ft Y = 382,748.62 ft**

Scale = 0.9999634113 Radius : 7,263,572.7586

Convergence angle = **0°53'42.4417"**

***20.14** Similar to Problem 20.13 except that the station's geodetic coordinates are 41°14'20.03582" N and 80°58'46.28764" W. Give coordinates in meters.

From WolfPack:

X = 329,339.9360 m   Y = 124,121.9922 m

Scale = 0.9999616706   Radius : 7,260,273.1778

Convergence angle = **−2°08'11.2512"**

20.15 What is the scale factor for the station in Problem 20.13? **k = 0.9999634113**

20.16 What is the scale factor for the station in Problem 20.14? **k = 0.9999616706**

***20.17** What are the SPCS83 coordinates in meters for a station in New Jersey with geodetic coordinates of 40°50'23.2038" N and 74°15'36.4908" W?

(170227.750, 222784.094)

20.18 Similar to Problem 20.17 except that the geodetic coordinates of the station are 41°11'25.0486" N and 74°29'36.9641"W.

(150536.764, 261678.411)

20.19 What are the convergence angle and scale factor at the station in Problem 20.17?

0°09'24.6903" and 0.9999050344

20.20 What are the convergence angle and scale factor at the station in Problem 20.18?

0°00'15.1706" and 0.9999000035

***20.21** What are the geodetic coordinates for a point *A* in Problem 20.11?

(41°12'23.2037"N, 78°26'30.3340"W)

20.22 Similar to Problem 20.21 except for point B in Problem 20.11?

(41°12'14.2321"N, 78°26'52.8116"W)

***20.23** What are the geodetic coordinates for a point *A* in Problem 20.12?

X = 135,995.7110 m   Y = 19,969.6030 m

Latitude = 39°00'47.2423"N, Longitude = 74°39'42.1526"W

Scale = 0.9999024141

Convergence angle = -0°06'06.4647"

20.24 Similar to Problem 20.23 except for point *B* in Problem 20.12.

X = 136,459.2040 m   Y = 20,302.5240 m

Latitude = 39°00'58.0655"N, Longitude = 74°39'22.9092"W

Scale = 0.9999022570

Convergence angle = -0°05'54.3739"

20.25 In computing state plane coordinates for a project area whose mean orthometric height is 305 m, an average scale factor of 0.99992381 was used. The average geoidal separation for the area is −23.83 m. The given distances between points in this project area were computed from SPCS83 state plane coordinates. What horizontal length would have to be observed to lay off these lines on the ground? (Use 6,371,000 m for an average radius for the Earth.)

Elevation factor = 0.999955869; Combined factor = 0.999858130

(a)* 2834.79 ft **2834.449 ft**

(b) 230.204 m **230.1763 m**

(c) 823.208 ft **823.109 ft**

20.26 Similar to Problem 20.25, except that the mean project area elevation was 38 m, the geoidal separation −31.37 m, the scale factor 0.99997053, and the computed lengths of lines from SPCS83 were:

Elevation factor = 0.999998959; Combined factor = 0.999969489

(a) 132.028 m **132.0240 m**

(b) 502.39 ft **502.375 ft**

(c) 910.028 m **910.0002 m**

20.27 The horizontal ground lengths of a three-sided closed polygon traverse were measured in feet as follows: *AB* = 501.92, *BC* = 336.03, and *CA* = 317.88 ft. If the average orthometric height of the area is 4156.08 ft and the average geoid separation is −23.05 m, calculate ellipsoid lengths of the lines suitable for use in computing SPCS83 coordinates. (Use 6,371,000 m for an average radius for the Earth.)

Elevation factor = 0.999804821

AB = **501.822 ft**

BC = **335.964 ft**

CA = **317.818 ft**

20.28 Assuming a scale factor for the traverse of Problem 20.27 to be 0.99996294, calculate grid lengths for the traverse lines.

AB = **501.803 ft**

BC = **335.952 ft**

CA = **317.806 ft**

20.29 For the traverse of Problem 20.27, the grid azimuth of a line from *A* to a nearby azimuth mark was 10°07'59" and the clockwise angle measured at *A* from the azimuth mark to *B*, 213°32'06". The measured interior angles were *A*= 41°12'26", *B* = 38°32'50", and *C*= 100°14'53". Balance the angles and compute grid azimuths for the traverse lines. (Note: Line BC bears easterly.)

Angle Summary

Station	Obs. Ang.	Adj. Angle
A	41°12'26.0"	**41°12'23.0"**
B	38°32'50.0"	**38°32'47.0"**
C	100°14'53.0"	**100°14'50.0"**
misclosure	9"	

Course	Azimuth
AB	**223° 40' 05"**
BC	**82° 12' 52"**
CA	**2° 27' 42"**

20.30 Using grid lengths of Problem 20.28 and grid azimuths from Problem 20.29, calculate departures and latitudes, linear misclosure and relative precision for the traverse.

From WolfPack

```
                                               Unbalanced
Course      Length       Azimuth          Dep          Lat
-------------------------------------------------------
 AB          501.80    223°40'05.0"  -346.485   -362.980
 BC          335.95     82°12'52.0"   332.855     45.510
 CA          317.81      2°27'42.0"    13.650    317.513
          ----------                ---------  ---------
  Sum =   1,155.56                      0.021      0.042

            Balanced                             Coordinates
        Dep          Lat      Point          X              Y
-----------------------------------------------------------
     -346.494   -362.999     A   2,082,437.05   1,260,646.78
      332.849     45.498     B   2,082,090.56   1,260,283.78
       13.644    317.501     C   2,082,423.41   1,260,329.28

     Linear misclosure  = 0.047
     Relative Precision = 1 in 24,500
```

20.31 If station *A* has SPCS83 state plane coordinates *E* = 634,728.082 m and *N* = 384,245.908 m, balance the departures and latitudes computed in Problem 20.30 using the compass rule, and determine SPCS83 coordinates of stations *B* and *C*.

Station	N (m)	E (m)
B	634,622.472	384,135.264
C	634,723.925	384,149.133

***20.32** What is the combined factor for the traverse of Problems 20.27 and 20.28?

<u>0.99976777</u>

20.33 The horizontal ground lengths of a four-sided closed polygon traverse were measured as follows: *AB* = 479.549 m, *BC* = 830.616 m, *CD* = 685.983 m and *DA* = 859.689 m. If the average orthometric height of the area is 1250 m, the geoidal separation is −30.0 m, and the scale factor for the traverse 0.9999574, calculate grid lengths of the lines for use in computing SPCS83 coordinates. (Use 6,371,000 m for an average radius of the Earth.)

k 0.9999574
EF 0.99980854
CF 0.9997660

Course	Obs. Dist.	Grid Dist.
AB	479.549	**479.4368**
BC	830.616	**830.4216**
CD	685.983	**685.8224**
DA	859.689	**850.4878**

20.34 For the traverse of Problem 20.33, the grid bearing of line BC is N57°39'48"W. Interior angles were measured as follows: *A* = 120°26'28", *B* = 73°48'58", *C*= 101°27'00", and *D* 64°17'26". Balance the angles and compute grid bearings for the traverse lines. (Note: Line CD bears southerly.)

misclosure = −8"

Station	Adj. Angle
A	120°26'30"
B	73°49'00"
C	101°27'02"
D	64°17'28"

Course	Azimuth
BC	S43°47'14"W
CD	S72°4'42"E
DA	S13°38'12"E
AB	N61°27'12"E

20.35 Using grid lengths from Problem 20.33 and grid bearings from Problem 20.34, calculate departures and latitudes, linear misclosure, and relative precision for the traverse. Balance the departures and latitudes by the compass rule. If the SPCS83 state plane coordinates of point *B* are *E*= 255,086.288 m and *N* = 280,654.342 m, calculate SPCS83 coordinates for points *C*, *D*, and *A*.

From WolfPack:

```
                                                Unbalanced
Course       Length       Bearing          Dep         Lat
```

```
-------------------------------------------------------
 BC        830.422   N57°39'48.0"W -701.6396   444.1868
 CD        685.822   S43°47'14.0"W -474.5769  -495.1052
 DA        859.488   S71°55'18.0"E  817.0576  -266.7136
 AB        479.437   N48°31'12.0"E  359.1878   317.5591
         ----------                 ---------  ---------
  Sum =  2,855.169                    0.0289    -0.0729

            Balanced                        Coordinates
        Dep        Lat      Point         X              Y
-----------------------------------------------------------
    -701.6480   444.2080     1    255,086.288    280,654.342
    -474.5838  -495.0877     2    254,384.640    281,098.550
     817.0489  -266.6917     3    253,910.056    280,603.462
     359.1829   317.5713     4    254,727.105    280,336.771

     Linear misclosure  = 0.0784
     Relative Precision = 1 in 36,400
```

20.36 The traverse in Problems 10.9 through 10.11 was performed in the Pennsylvania North Zone of SPCS83. The average elevation for the area was 505.87 m and the average geoidal separation was −28.25 m. Using the data in Table 20.1 and a mean radius for the earth, reduce the observations to grid and adjust the traverse. Compare this solution with that obtained in Chapter 10. (Use 6,371,000 m for an average radius of the earth.)

Using the initial coordinates (in meters) from Chapter 10, the scale factor at each station is (from WolfPack):

A	310,630.890 m	121,311.410 m	0.9999635428
B	310,409.540 m	121,338.600 m	0.9999635323
C	310,342.990 m	121,183.450 m	0.9999636230
D	310,486.930 m	121,071.500 m	0.9999636844
E	310,710.870 m	121,119.470 m	0.9999636515

Reduced distances are:

Course	Obs. Dist.	k_{avg}	C.F.	Grid. Dist.
AB	223.011	0.999963538	0.9998886	222.9862
BC	168.818	0.999963578	0.9998886	168.7992
CD	182.358	0.999963654	0.9998887	182.3377
DE	229.024	0.999963668	0.9998887	228.9985
EA	207.930	0.999963597	0.9998886	207.9068

Traverse Adjustment from WolfPack:

Angle Summary

Station	Unadj. Angle	Adj. Angle
1	119°37'10.0"	119°37'14.0"
2	106°12'58.0"	106°13'02.0"
3	104°39'22.0"	104°39'26.0"
4	130° 1'54.0"	130°01'58.0"
5	79°28'16.0"	79°28'20.0"

Angular misclosure (sec): -20"

Course	Length	Azimuth	Unbalanced Dep	Unbalanced Lat
1-2	222.986	277°00'04.0"	-221.3236	27.1795
2-3	168.799	203°13'06.0"	-66.5467	-155.1280
3-4	182.338	127°52'32.0"	143.9276	-111.9460
4-5	228.999	77°54'30.0"	223.9179	47.9698
5-1	207.907	337°22'50.0"	-79.9627	191.9146
Sum =	1,011.028		0.0124	-0.0102

Balanced Dep	Balanced Lat	Point	Coordinates X	Coordinates Y
-221.3263	27.1817	1	310,630.892	121,311.411
-66.5488	-155.1263	2	310,409.566	121,338.593
143.9253	-111.9441	3	310,343.017	121,183.466
223.9151	47.9721	4	310,486.942	121,071.522

```
    -79.9653   191.9166     5    310,710.857    121,119.494

    Linear misclosure  = 0.0160
    Relative Precision = 1 in 63,000

    Area: 66,800 sq. ft.
           1.533 acres {if distance units are feet}

Adjusted Observations
---------------------
    Course    Distance   Azimuth     Point      Angle
-----------------------------------------------------
       1-2     222.989  277°00'06"      1  119°37'17"
       2-3     168.798  203°13'09"      2  106°13'03"
       3-4     182.335  127°52'32"      3  104°39'23"
       4-5     228.996   77°54'27"      4  130°01'56"
       5-6     207.910  337°22'48"      5   79°28'21"
```

Note: That solutions from Chapter 10 and here provide same linear misclosure and relative precision by the coordinates vary as shown below since the traverse was scaled incorrectly in Chapter 10.

	Initial Coordinates		Grid Coordinates	
Sta	E (m)	N (m)	E (m)	N (m)
A	310,630.892	121,311.411	310,630.892	121,311.411
B	310,409.541	121,338.596	310,409.566	121,338.593
C	310,342.985	121,183.452	310,343.017	121,183.466
D	310,486.926	121,071.496	310,486.942	121,071.522
E	310,710.866	121,119.473	310,710.857	121,119.494

20.37 The traverse in Problems 10.12 through 10.14 was performed in the New Jersey zone of SPCS83. The average elevation for the area was 234.93 m and the average geoidal separation was −32.86 m. Using the data in Table 20.3 and 20.4, and a mean radius for the earth, reduce the observations to grid and adjust the traverse. Compare this solution with that obtained in Chapter 10.

Using the initial coordinates from Chapter 10, the scale factor at each station is:

Sta	E (m)	N (m)	k
A	243,605.596	25,393.201	1.00000786
B	243,725.068	25,204.889	1.00000813
C	243,887.203	25,251.916	1.00000851
D	243,882.370	25,434.207	1.00000849
E	243,703.170	25,576.819	1.00000808

The reduced distances are:

EF 0.9999683

Course	kavg	C.F.	Obs. Dist.	Grid Dist.
AB	1.00000799	0.9999763	223.011	223.0057
BC	1.00000832	0.9999766	168.818	168.8140
CD	1.00000850	0.9999768	182.358	182.3538
DE	1.00000829	0.9999766	229.024	229.0186
EA	1.00000797	0.9999762	207.930	207.9251

The adjusted traverse from WolfPack:

```
Angle Summary
Station   Unadj. Angle     Adj. Angle
--------------------------------------
    1     119°37'10.0"     119°37'14.0"
    2     106°12'58.0"     106°13'02.0"
    3     104°39'22.0"     104°39'26.0"
    4     130° 1'54.0"     130°01'58.0"
    5      79°28'16.0"      79°28'20.0"

Angular misclosure (sec): -20"
```

```
                                               Unbalanced
Course      Length       Azimuth         Dep         Lat
-------------------------------------------------------
 1-2       223.006    147°36'25.0"  119.4696  -188.3044
 2-3       168.814     73°49'27.0"  162.1309    47.0292
 3-4       182.354    358°28'53.0"   -4.8327   182.2898
 4-5       229.019    308°30'51.0" -179.1966   142.6117
 5-1       207.925    207°59'11.0"  -97.5713  -183.6102
        ----------                 ---------  ---------
  Sum =  1,011.117                   -0.0001     0.0161

           Balanced                          Coordinates
       Dep         Lat     Point         X              Y
-----------------------------------------------------------
     119.4696  -188.3080     1    243,605.596     25,393.201
     162.1309    47.0265     2    243,725.066     25,204.893
      -4.8327   182.2868     3    243,887.197     25,251.920
    -179.1966   142.6081     4    243,882.364     25,434.206
     -97.5713  -183.6135     5    243,703.167     25,576.814

     Linear misclosure  = 0.0161
     Relative Precision = 1 in 62,600
```

Note: That solutions from Chapter 10 and here provide same linear misclosure and relative precision by the coordinates vary as shown below since the traverse was scaled incorrectly in Chapter 10.

Sta	Initial Coordinates E (m)	N (m)	Grid Coordinates E (m)	N (m)
A	243,605.596	25,393.201	243,605.596	25,393.201
B	243,725.068	25,204.889	243,725.066	25,204.893

C	243,887.203	25,251.916	243,887.197	25,251.920
D	243,882.370	25,434.207	243,882.364	25,434.206
E	243,703.170	25,576.819	243,703.167	25,576.814

20.38 The traverse in Problem 10.22 was performed in the New Jersey SPCS 1983. The average elevation of the area was 67.2 m and the average geoidal separation was −28.5m. Using 6,371,000 m for the mean radius of the earth, reduction the observations to grid and adjust the traverse using the compass rule. Compare this solution with that obtained in Problem 10.22.

The initial coordinates, k, and convergence angles are

Sta	X	Y	K	γ
A	185,435.381	24,947.460	0.999915456	0°15'28.743"
B	185,205.079	24,777.913	0.999915256	0°15'22.657"
C	185,009.961	24,973.057	0.999915087	0°15'17.601"
D	184,701.298	25,053.025	0.999914823	0°15'09.535"
E	184,539.770	24,880.286	0.999914685	0°15'05.251"

The elevation factor is 0.999993926
The reduced distances are:

Course	Obs. Dist.	kavg	CF	Grid Dist.
AB	285.993	0.999915356	0.9999093	285.9671
BC	275.968	0.999915172	0.9999091	275.9429
CD	318.871	0.999914955	0.9999089	318.8419
DE	236.504	0.999914754	0.9999087	236.4824

The traverse computations are:

```
Angle Summary
Station   Unadj. Angle     Adj. Angle
-------------------------------------
    1       82°57'54.0"      82°57'48.0"
    2      261°21'42.0"     261°21'36.0"
    3      149°31'27.0"     149°31'21.0"
    4      118°33'32.0"     118°33'26.0"
    5      215° 0'51.0"     215°00'45.0"

Angular misclosure (sec): 30"
```

```
                                                Unbalanced
Course      Length       Azimuth          Dep         Lat
-------------------------------------------------------------
 1-2        285.967    233°38'30.0" -230.2965  -169.5308
 2-3        275.943    315°00'06.0" -195.1154   195.1268
 3-4        318.842    284°31'27.0" -308.6523    79.9618
 4-5        236.482    223°04'53.0" -161.5261  -172.7230
          ----------                ---------  ---------
  Sum =   1,117.234                 -895.5904   -67.1653
```

```
Misclosure in Departure = -895.5904 - -895.6110 = 0.0206
Misclosure in Latitude  = -67.1653 - -67.1740 = 0.0087

           Balanced                              Coordinates
        Dep         Lat      Point         X              Y
------------------------------------------------------------
    -230.3018  -169.5331      1    185,435.381     24,947.460
    -195.1205   195.1246      2    185,205.079     24,777.927
    -308.6582    79.9593      3    185,009.959     24,973.052
    -161.5305  -172.7249      4    184,701.300     25,053.011
                              5    184,539.770     24,880.286

     Linear misclosure  = 0.0224
     Relative Precision = 1 in 49,900
```

Note: That solution from Chapter 10 and here are different since this is a link traverse. The relative precision went from 1:18,300 to 1:49,900. The coordinates vary as shown below since the traverse was scaled incorrectly in Chapter 10.

	Initial coordinates		Final coordinates	
Sta	E (m)	N (m)	E (m)	N (m)
A	185,435.381	24,947.460	185,435.381	24,947.460
B	185,205.079	24,777.913	185,205.079	24,777.927
C	185,009.961	24,973.057	185,009.959	24,973.052
D	184,701.298	25,053.025	184,701.300	25,053.011
E	184,539.770	24,880.286	184,539.770	24,880.286

20.39 The traverse in Problem 10.21 was performed in the Pennsylvania North Zone of SPCS83. The average elevation for the area was 367.89 m and the average geoidal separation was −30.23 m. Using the mean radius of the earth of 6,371,000 m, reduce the observations to grid and adjust the traverse using the compass rule. Compare this solution with that obtained in Problem 10.21.

The initial coordinates, scale factors, and convergence angles are:

Sta	X (ft)	Y (ft)	k	γ
A	2,521,005.86	379,490.84	0.99996463	1°19'41.271"
B	2,521,180.16	379,589.99	0.99996461	1°19'42.799"
C	2,521,432.84	379,566.02	0.99996462	1°19'44.980"
D	2,521,575.16	379,714.76	0.99996459	1°19'46.241"

The elevation factor is 0.99994700.

The reduced distances are:

Course	Obs. Dist.	k_{avg}	C.F.	Grid Dist.
AB	200.55	0.9999646	0.99991163	200.532
BC	253.84	0.9999646	0.99991162	253.818
CD	205.89	0.9999646	0.99991161	205.872

The recomputed traverse using grid distances is:

```
Angle Summary
Station   Unadj. Angle     Adj. Angle
---------------------------------------
    1     258°12'18.0"     258°12'27.0"
    2     215° 2'53.0"     215°03'02.0"
    3     128°19'11.0"     128°19'20.0"
    4     237°34' 5.0"     237°34'14.0"

Angular misclosure (sec): -36"
```

```
                                              Unbalanced
Course      Length       Azimuth         Dep          Lat
-------------------------------------------------------
 1-2        200.53     60°21'55.0"   174.301      99.157
 2-3        253.82     95°24'57.0"   252.685     -23.956
 3-4        205.87     43°44'17.0"   142.332     148.744
         ----------                 ---------  ---------
  Sum =     660.22                   569.319     223.945

Misclosure in Departure = 569.319 - 569.300 = 0.019
Misclosure in Latitude  = 223.945 - 223.920 = 0.025
```

```
           Balanced                          Coordinates
        Dep         Lat      Point          X              Y
------------------------------------------------------------
      174.296      99.149      1   2,521,005.86     379,490.84
      252.678     -23.966      2   2,521,180.16     379,589.99
      142.326     148.736      3   2,521,432.83     379,566.02
                               4   2,521,575.16     379,714.76

     Linear misclosure  = 0.031
     Relative Precision = 1 in 21,300
```

Note: That solutions from Chapter 10 and here are different since this is a link traverse. The relative precision went from 1:8100 to 1:21,300. The coordinates vary slightly as shown below since the traverse was scaled incorrectly in Chapter 10.

	Initial coordinates		Final coordinates	
Sta	X (ft)	Y (ft)	X (ft)	Y (ft)
A	2,521,005.86	379,490.84	2,521,005.86	379,490.84
B	2,521,180.16	379,589.99	2,521,180.16	379,589.99
C	2,521,432.84	379,566.02	2,521,432.83	379,566.02
D	2,521,575.16	379,714.76	2,521,575.16	379,714.76

***20.40** If the geodetic azimuth of a line is 205°06'36.2" the convergence angle is −0°42'26.1" and the arc-to-chord correction is +0.8" what is the equivalent grid azimuth for the line?

205°39'03.1" = 205°06'36.1" + 0°42'36.2" + 0.8"

20.41 If the geodetic azimuth of a line is 18°47'20.1" the convergence angle is −1°08'06.8" and the arc-to-chord correction is −1.5", what is the equivalent grid azimuth for the line?

19°55'25.4" = 18°47'20.1" + 1°08'06.8" − 1.5"

20.42 Using the values given in Problems 20.40 and 20.41, what is the obtuse grid angle between the two azimuths?

174°06'22.3"

20.43 The grid azimuth of a line is 102°37'08". If the convergence angle at the endpoint of the azimuth is 2°05'52.9" and the arc-to-chord correction is 0.7", what is the geodetic azimuth of the line?

104°43'00.2" = 102°37'08" + 2°05'52.9" − 0.7"

20.44 Similar to Problem 20.43, except the convergence angle is −1°02'20.7" and the arc-to-chord correction is −0.6".

101°34'47.9" = 102°37'08" − 1°02'20.7" + 0.6"

20.45 What is the arc-to-chord correction for the line from A to B in Problems 20.21 and 20.22?

0.03"; φ_3 = 41°12'20.21317"

20.46 What is the geodetic azimuth of the line from A to B in Problem 20.45?

242°08'44.8"; γ = −0°27'27.4549" t = 242°36'12.3"

20.47 Using the defining parameters given in Example 20.10, compute oblique stereographic map projection coordinates for Station B.

Sta	χ	m	A	E	N	k
B	0.71759585	0.75229451	6368873.37	**−868.958**	**188.599**	1.000000
C	0.71758724	0.75230019	8114471	581.130	170.326	1.274083
D	0.71754043	0.75233104	9260051.2	534.544	−239.047	1.453954

20.48 Similar to Problem 20.47 except for Station C.

NE = (**170.326, 581.130)** See Problem 20.47 for intermediate values.

20.49 Similar to Problem 20.47 except for Station D.

NE = (**−239.047, 534.544**) See Problem 20.47 for intermediate values.

20.50 Create a computational program that reduces distances from the ground to a mapping grid.

Solutions will vary.

21 BOUNDARY SURVEYS

21.1 Define the following terms:

(a) Easement
From Section 21.11, paragraph 2:

"An *easement* is a right, by grant or agreement, which allows a person or persons to use the land of another for a specific purpose."

(b) Subdivision surveys
From Section 21.2, paragraph 1:

Subdivision surveys establish new smaller parcels of land within lands already surveyed.

(c) Color of title

From Section 21.11, paragraph 1: " a claim to a parcel of real property based on some written instrument, though a defective one."

(d) Adverse possession
From Section 21.11, paragraph 1:

"Adverse rights can generally be applied to gain ownership of property by occupying a parcel of land for a period of years specified by state law, and performing certain acts. To claim land or rights to it by *adverse possession,* its occupation or use must be (1) actual, (2) exclusive, (3) open and notorious, (4) hostile, and (5) continuous. It may also be necessary for the property to be held under *color of title* (a claim to a parcel of real property based on some written instrument, though a defective one). In some states all taxes must be paid. The time required to establish a claim of adverse possession varies from a minimum of 5 years in California to a maximum of 60 years for urban property in New York. The customary period is 20 years."

21.2 What is the responsibility of a surveyor in a retracement survey?

From Section 21.2, paragraph 4:

"The responsibility of a professional surveyor is to weigh all evidence and try to establish the originally intended boundary between the parties involved in any property-line dispute, although without legal authority to force a compromise or settlement."

21.3 Visit your county courthouse and obtain a copy of a metes-and-bounds property description. Write a critique of the description, with suggestions on how the description could have been improved.

Student critique required.

21.4 In a description by metes and bounds, what purpose may be served by the phrase "more or less" following the acreage?

From Section 21.4, paragraph 5:

"The expression 'more or less,' which may follow a computed area, allows for minor errors, and avoids nuisance suits for insignificant variations."

21.5 Write a metes-and-bounds description for the house and lot where you live. Draw a map of the property.

Student metes-and-bounds description of personal lot.

21.6 Write a metes-and-bounds description for the exterior boundary of lot 16 in Figure 21.2.

Metes-and bounds description of boundary in Figure 21.2.

21.7 What are the advantages of the block and lot system of describing property?

From Section 21.5, paragraph 2:

The block and lot system is a short and unique method of describing property for transfer of title.

21.8 From the property description of the parcel of tide and submerged land described in Section 21.6, compute the parcel's area.

26,297 ft^2

Station	X (+2,106,000)	Y (+164,000)	Plus (ft^2)	Minus (ft^2)
POB	973.68	301.93		361,120.357
2	1196.04	285.08	277,576.694	343,681.045
3	1205.56	410.72	491,237.549	814,539.904
4	1983.20	427.57	515,461.289	416,316.358
POB	973.68	301.93	598,787.576	
			1,883,063.1 1	1,935,657.66

Area = |−52,594.56/2| = 26,297 ft^2

21.9 What is the difference between the point of commencement and point of beginning in a property description?

From Section 21.2, paragraphs 6 & 7:

"1. *Point of commencement (POC)*. This is an established reference point such as a corner of the PLSS or NSRS monument to which the property description is tied or referenced. It serves as the starting point for the description.

2. *Point of beginning (POB)*. This point must be identifiable, permanent, well referenced, and one of the property corners. Coordinates, preferably state plane, should be given if known or computable. Note that a POB is no more important than

others and a called for monument in place at the next corner establishes its position, even though bearing and distance calls to it may not agree."

21.10 What major advantage does the coordinate method of property description have over other methods?

From Section 21-6, paragraph 1:

One of the most significant is that they greatly facilitate the relocation of lost and obliterated corners. Every monument which has known state plane coordinates becomes a "witness" to other corner markers whose positions are given in the same system. State plane coordinates also enable the evaluation of adjoiners with less fieldwork.

21.11 What are the advantages of the block and lot system of describing property?

From Section 21.57, paragraph 2:

"The block-and-lot system is a short and unique method of describing property for transfer of title. Standard practice calls for a map or plat of each subdivision to be filed with the proper office. The plat must show the types and locations of monuments, dimensions of all blocks and lots, and other pertinent information such as the locations and dimensions of streets and easements, if any."

21.12 List in their order of importance the following types of evidence when conducting retracement surveys: (a) measurements, (b) call for a survey, (c) intent of the parties, (d) monuments, and (e) senior rights.

From Section 21.7, paragraph 3:

(e) Senior rights, (c) intent of parties, (b) call for survey, (d) monuments, (a) measurements

21.13 In performing retracement surveys, list in their order of importance, the four different types of measurements called for in a description.

From Section 21.7, paragraph 3: In most states, it is (**1**) distance, (**2**) direction, (**3**) area, and (**4**) coordinates. However in some states, such as Pennsylvania it is (**1**) direction, (**2**) distance, (**3**) area, and (**4**) coordinates.

21.14 Why do rules governing retracement surveys vary by state?
From Section 21.7, paragraph 1:
"The rules used in retracement surveys are guided by case law and as such can vary from state to state and with time."

21.15 Identify all types of pertinent information or data that should appear on the plat of a completed property survey.

From Section 21-5, paragraph 2:

The plat must show the types and locations of monuments, dimensions of all blocks and lots, and other pertinent information such as the locations and dimensions of streets and easements, if any.

21.16 What are the first steps in performing a subdivision survey?

From Section 21.8, paragraph 8:
Perform an exterior survey

(a) Obtain recorded deed descriptions of the parent tract of land to be subdivided, and of all adjoiners, from the Register of Deeds office. Note any discrepancies between the parent tract and its adjoiners.

(b) Search for monuments marking corners of the parent tract, those of its adjoiner's where necessary, and, where appropriate, for U.S. Public Land Survey monuments to which the survey may be referred or tied. Resolve any discrepancies with adjoiners.

(c) Make a closed survey of the parent tract and adequately reference it to existing monuments.

(d) Compute departures, latitudes, and misclosures to see whether the survey meets requirements. Balance the survey if the misclosure is within allowable limits.

(e) Resolve, if possible, any encroachments on the property, or differences between occupation lines and title lines, so there will be no problems later with the final subdivision boundary

21.17 Two disputing neighbors employ a surveyor to check their boundary line. Discuss the surveyor's authority if (a) the line established is agreeable to both clients, and (b) the line is not accepted by one or both of them.

(a) Obtain written agreement, record the accepted lines on the original plat, and have corrected deeds recorded.

(b) From Section 21.4, paragraph 4: Marks can be set on the ground, but their acceptance cannot be forced by a surveyor – it must be resolved in the courts.

21.18 What is required to adversely possess land?
From Section 21.11, paragraph 1.
"To claim land or rights to it by *adverse possession,* its occupation or use must be (1) actual, (2) exclusive, (3) open and notorious, (4) hostile, and (5) continuous. It may also be necessary for the property to be held under *color of title* (a claim to a parcel of real property based on some written instrument, though a defective one). In some states all taxes must be paid. The time required to establish a claim of adverse possession varies from a minimum of 5 years in California to a maximum of 60 years for urban property in New York. The customary period is 20 years."

21.19 Compute the misclosures of lots 18 and 19 in Figure 21.2. On the basis of your findings, would this plat be acceptable for recording? Explain.

Lot 18

Course	Length	Azimuth	Departure	Latitude
AB	57.22	163°09’10”	16.58	–54.76

BC	27.83	129°19'35"	21.53	–17.64
CD	88.55	95°30'	88.14	–8.49
DE	123.27	5°30'	11.81	122.70
EA	144.20	253°09'10"	–138.01	–41.79
	Σ = 441.07		Σ = 0.05	Σ = 0.02

Linear misclosure: 0.05
Relative precision: 1:8000

Lot 19

Course	Distance	Azimuth	Departure	Latitude
AB	56.76	95°30'	56.50	–5.44
BC	113.30	156°17'40"	45.55	–103.74
CD	10.00	246°17'40"	–9.16	–4.02
DE	70.41	260°53'50"	–69.52	–11.14
EF	36.16	275°30'	–35.99	3.47
FA	123.27	5°30'	11.82	122.70
	Σ = 409.90		Σ = –0.80	Σ = 1.83

Linear misclosure: 2.00
Relative precision: 1:200

Neither lot closes mathematically. However, lot 19 has a relative precision of only 1:200, which is not acceptable for a property survey.

21.20 Compute the areas of lots 18 and 19 of Figure 21-2.

Lot 18		**Double area**		**Lot 19**		**Double area**	
X	**Y**	↘	↗	**X**	**Y**	↘	↗
16.58	–54.76		–1,179.0	56.50	–5.44		–247.8
21.53	–17.64	–292.5	–1,554.8	45.55	–103.74	–5,861.3	950.3
88.14	–8.49	–182.8	–100.3	–9.16	–4.02	–183.1	279.5
11.81	122.7	10,814.8	–16,933.8	–69.52	-11.14	102.0	400.9
–138.01	–41.79	–493.5	–692.9	–35.99	3.47	–241.2	41.0
16.58	–54.76	7,557.4		11.82	122.70	–4,416.0	6,932.6
		17,403.4	–20,460.7	56.50	–5.44	–64.3	
						–10,663.9	8,356.4

Lot 18:

Area of segment = ½ × 25^2[67°39'10"×π/180 – sin(67°39'10")] = 80.0 sq. ft.
Area: ½|17,403.4+20,460.7| + 80.0 = **<u>19,010 sq. ft.</u>**

Lot 19:

Area of segment = ½ × 139.62^2[29°12'20"×π/180° – sin(29°12'20")] = 212 sq. ft.

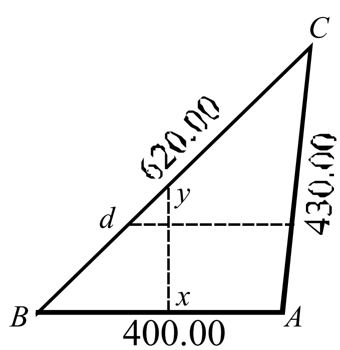
Area: ½|10,663.9 + 8356.4| + 212 = **9720 sq. ft.**

21.21 Determine the misclosure of lot 50 of Figure 21.2, and compute its area.

Computer solution:

```
~~~~~~~~~~~~~~~~~~~~~~~ Traverse Computation ~~~~~~~~~~~~~~~~~~~~~~~

Title: Problem 21-21 Type: Polygon traverse
                                                  Unbalanced
Course      Length       Azimuth         Dep         Lat
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
 1-2         30.38      8°30'00.0"     4.490     30.046
 2-3        123.00     35°00'00.0"    70.550    100.756
 3-4         90.77      9°30'00.0"    14.981     89.525
 4-5         90.11    143°00'00.0"    54.230    -71.965
 5-6         95.50    148°42'00.0"    49.614    -81.601
 6-7         19.67    238°42'00.0"   -16.807    -10.219
 7-8        133.19    245°00'30.0"  -120.719    -56.271
 8-1         62.59    251°19'00.0"   -59.292    -20.050
         ----------                 ---------  ---------
  Sum =     645.21                    -2.953    -19.779

            Balanced                          Coordinates
        Dep         Lat     Point          X              Y
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
        4.629     30.978     1       1,000.00       1,000.00
       71.113    104.526     2       1,004.63       1,030.98
       15.397     92.308     3       1,075.74       1,135.50
       54.642    -69.203     4       1,091.14       1,227.81
       50.051    -78.673     5       1,145.78       1,158.61
      -16.717     -9.616     6       1,195.83       1,079.94
     -120.110    -52.188     7       1,179.12       1,070.32
      -59.005    -18.131     8       1,059.01       1,018.13

     Linear misclosure  = 19.998
     Relative Precision = 1 in 0

     Area: 18,200 sq. ft.
            0.418 acres {if distance units are feet}
```

21.22 For the accompanying figure; using a line perpendicular to *AB* through *x*, divide the parcel into two equal parts, and determine lengths *xy* and *By*.



a = 620 s – a = 105

b = 430 s – b = 295
c = <u>400</u> s – c = 325
Σ = 1450 $Area = \sqrt{725(105)(295)(325)} = 85{,}431\ ft^2$

2/5 Area = 34,172 ft^2

$$B = \cos^{-1}\left(\frac{620^2 + 400^2 - 430^2}{2 \times 620 \times 400}\right) = 43°32'53"$$

Area BXY = ½(BX)(XY) = 34,172
XY = BX tan B = 0.950555 BX
34,172 = ½×0.950555 BX^2; so

BX = 268.14 ft
XY = 268.14 (0.950555) = 254.88 ft
$BY = \sqrt{266.14^2 + 254.88^2} = 368.50ft$

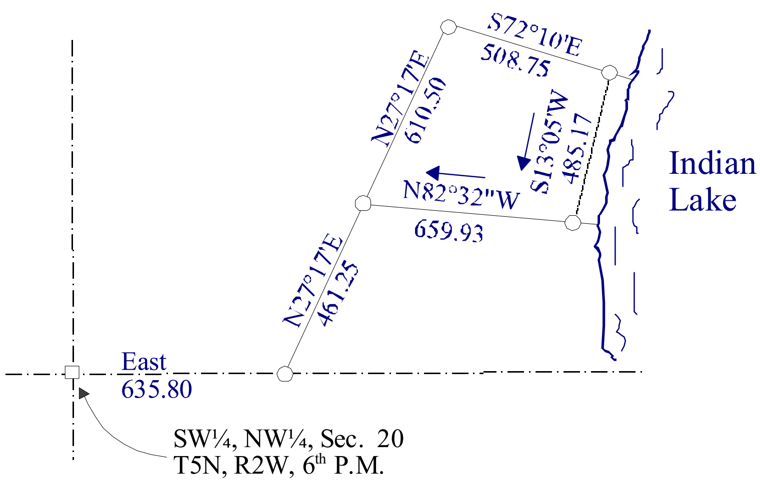
21.23 For the figure of Problem 21.22, calculate the length of line *de*, parallel to *BA*, which will divide the tract into two equal parts. Give lengths *Bd* , *de* and *eA.*

From Problem 22.22, Area = 85,431 sq. ft.
Required Area = 85,431 – 34,172 = 51,260 sq. ft.

51,260 = [400 + *de*/2]*h*; *de* = 400 – *h*/tan *B*;
where from Problem 22-22, *B* = 43°32’53”
$C = \sin^{-1}$[2(85,431)/(620.0×43.0)] = 39°51’31”

A = 180° – 43°32’53” – 39°51’31” = 96°35’36”
51,260 = [400/2 + 400/2 – *h*/(2 tan *B*) + (*h*/2) tan (*A*+90°)]*h*
51,260 = 400 *h* – 0.4682 h^2
h = 157.00 ft

de = 400 – 157.00/tan(43°32’53”) + 157.00 tan(6°35’36”) = 252.98 ft
Bd = 157.00/sin(43°32’53”) = 227.88 ft
Departure of *Ae* = 227.88 sin 43°32’53” + 252.98 – 400 = 9.98 ft
$Ae = \sqrt{9.98^2 + 157.00^2} = 1576.31\ ft$



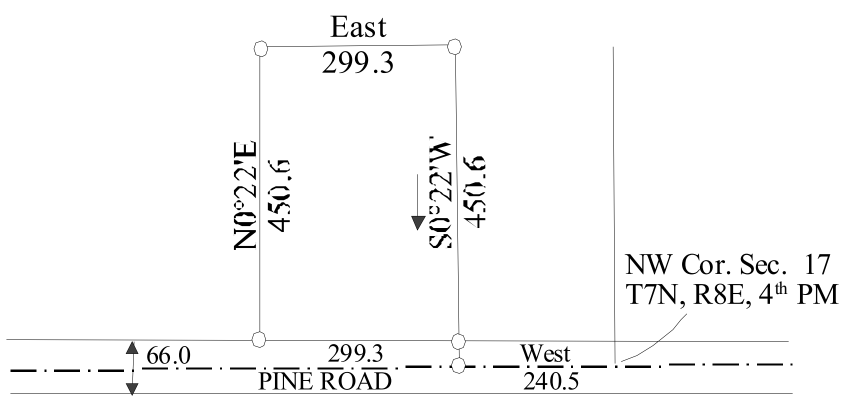
21.24 Prepare a metes-and-bound description for the parcel shown. Assume all corners are marked with 1-in. diameter steel rods, and a 20 ft meander line setback from Indian Lake.

Commencing at a 1-in diameter steel rod at the SW¼ , NW¼ , Sec. 20, T5N, R2W, 6th P.M.; thence east, 635.80 ft to a 1-in diameter steel rod; then N27°17'E, 461.25 ft to a 1-in diameter steel rod which is the point of beginning of this parcel; thence N27°17'E, 610.50 ft to a 1-in diameter steel rod; thence S72°10'E, 508.75 ft to a 1-in diameter steel rod, located N72°10'E, 20 ft more or less from Indian Lake; said pipe being the beginning of a meander line along said Indian Lake; thence S13°05'W, 485.17 ft along the meander line to a 1-in diameter steel rod at the end of the meander line, said pipe being N82°32'W, 659.93 ft to a 1-in diameter steel rod at the point of beginning, including all lands lying between the meander line and the westerly shore of Indian Lake which lie between the extensions of the northerly and southerly boundaries of the parcel herein described; said parcel containing 7.39 acres, more or less.

21.25 Draw a plat map of the parcel in Problem 21.24 at a convenient scale. Label all monuments and the lengths and directions of each boundary line on the drawing. Include a title, scale, North arrow, and legend.

Scaled drawing required.

21.26 Prepare a metes-and-bounds description for the property shown. Assume all corners



are marked with 2-in. diameter iron pipes.

Commencing at a 2-in diameter iron pipe at the NW corner, Sec 17, T7N, R8E, 4th PM; thence West, 240.5 ft along the centerline of Pine Road to a 2-in diameter iron pipe; thence North, 33.0 ft to a 2-in diameter iron pipe which is the point of beginning for this parcel; thence West, 299.3 ft. along the northerly right of way line of Pine Road to a 2-in diameter iron pipe; thence N0°22'E, 450.6 ft to a 2-in diameter iron pipe; thence East, 299.3 ft to a 2-in diameter iron pipe; thence S0°22'W, 450.6 ft to a 2-in diameter iron pipe at the point of beginning, said parcel containing 3.10 acres, more or less.

21.27 Create a 1.25-acre tract on the westerly side of the parcel in Problem 21-26 with a line parallel to the westerly property line. Give the lengths and bearings of all lines for both new parcels.

Required area = 1.25(43,560) = 54,450 sq. ft. = 450.6 x where x = 120.84 ft

West Lot

Course	Length	Bearing
BC	450.60	N0°22'E
CF	145.01	East
FE	450.06	S0°22'W
EB	145.01	West

East Lot: AE = 299.13 – 120.84 = 178.46 ft

Course	Length	Bearing
EF	450.60	N0°22'E
FD	120.84	East
DA	450.06	S0°22'W
AE	120.84	West

21.28 Discuss the ownership limits of a condominium unit.

Generally, to the center of the walls, floors and ceilings of the dwelling.

21.29 Define *common elements* and *limited common elements* in relation to condominiums. Given examples of each.

From Section 21.12, paragraph 2:

Common elements are elements jointly owned and used by all units such as sidewalks, stairways, swimming pool, tennis courts, etc.

Limited common elements are elements reserved for the exclusive use of a particular unit such as a designated parking space.

21.30 What types of measurements are typically made by surveyors in performing work for condominium developments?

As-built surveys.

22 PUBLIC LAND SURVEYS

***22.1** Convert 65.44 chains to feet.

<u>4319 ft</u>

22.2 What two factor originally governed the survey and disposition of the public lands in the U.S.?

From Section 22.1, paragraph 5:

1. A recognition of the value of grid-system subdivision based on experience in the colonies and another large-scale systematic boundary survey—the 1656 Down Survey in Ireland.
2. The need of the colonies for revenue from the sale of public land.

22.3 Who was Rufus Putnam?

From Section 22.2, paragraph 2: "In 1796 General Rufus Putnam was appointed as the first U.S. *Surveyor General*"

22.4 Describe the tangent method of establishing a standard parallel.

A 90° angle is turned to the east or west, as required from a true meridian, and corners are set every 40 ch. At the same time, proper offsets, which increase with increasing latitudes, are taken from *Standard Field Tables* issued by the BLM, and measured north from the tangent to the parallel.

22.5 Why are the boundaries of the public lands established by duly appointed surveyors unchangeable, even though incorrectly set in the original surveys?

From Section 22.2, paragraph 8: "Correcting mistakes or errors now would disrupt too many accepted property lines and result in an unmanageable number of lawsuits."

22.6 What is the convergence in feet of meridians for the following conditions:

(a)* 24 mi apart, extended 24 mi, at mean latitude $45°20'$ N.

<u>776.99 ft</u> By Equation 22.2: $c = \frac{4}{3}24(24)\tan(45°20') = 776.99 \text{ ft}$

(b) 24 mi apart, extended 24 mi, at mean latitude $34°25'$ N.

<u>526.19 ft</u> By Equation 22.2: $c = \frac{4}{3}24(24)\tan(34°25') = 526.19 \text{ ft}$

22.7 What is the angular convergence, in seconds, for the two meridians defining a township exterior at a mean latitude of:

(a) 42°00′ N? **4'42"** By Equation (22.1): $\theta = 52.13(6)\tan(42°00') = 281.6''$

(b) 34°00′ N? **3'31"** By Equation (22.1): $\theta = 52.13(6)\tan(34°00') = 211''$

22.8 What is the nominal distance in miles between the following?

(a)* First Guide Meridian East, and the west Range Line of R8E.

30 mi $9(6) - 24 = 30$

(b) SE corner of Sec. 14, T 6 S, R 5 E, Indian PM, and the NW corner of Sec. 23, T 6 S, R 3 E, Indian PM.

13 mi $2 + 6 + 5 = 13$

22.9 Discuss when meander corners are to be set in a public land survey.

From Section 22.18, paragraph 1: "A *meander corner* (MC) was established on survey lines intersecting the bank of a stream having a width greater than 3 ch, or a lake, bayou, or other body of water of 25 acres or more."

Sketch and label pertinent lines and legal distances, and compute nominal areas of the parcels described in Problems 22.10 through 22.12.

22.10 E 1/2, SE 1/4, Sec. 6, T 2 S, R 3 E, Salt River PM. **80 ac with sketch**

22.11 SW 1/4, NW 1/4, Sec. 15, T 1 N, R 2 E, Fairbanks PM. **40 ac with sketch**

22.12 NE 1/4, SE 1/4, SE 1/4, Sec. 30, T 1 S, R 4 E, 6th PM. **10 ac with sketch**

22.13 What are the nominal dimensions and acreages of the following parcels:

(a) NW 1/4, NE 1/4, Sec. 28. **20 ch by 20 ch; 40 ac**

(b) S 1/2, Sec. 3. **40 ch by 80 ch; 320 ac**

(c) SE 1/4, NE 1/4, SW 1/4, Sec. 16. **10 ch by 10 ch; 10 ac**

22.14 How many rods of fence are required to enclose the following:

(a)* A parcel including the NE 1/4, NE 1/4, Sec. 32, and the NW 1/4, NW 1/4, Sec. 33, T 2 N, R 3 E?

240 rods

(b) A parcel consisting of Secs. 8, 9, and 16 of T 2 N, R 1 W?

2650 rods

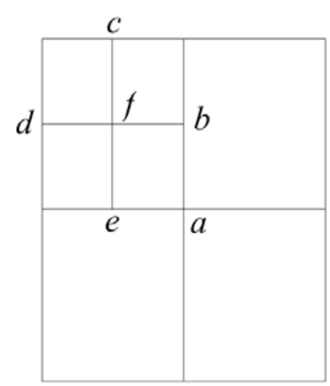
22.15 What lines of the U.S. public-land system were run using astronomic observations?

initial point, principal meridians, guide meridians, range lines, and possibly baselines and township lines

22.16 In subdividing a township, which section line is run first? Which last?

From Section 22.11: The first is the north-south line between section 35 and 36. The last is the north-south line between sections 5 and 6.

22.17 Corners of the SE 1/4 of the NW 1/4 of Sec. 22 are to be monumented. If all section and quarter-section corners originally set are in place, explain the procedure to follow, and sketch all lines to be run and corners set.



Check the section and quarter corners that are in place. Connect opposite quarter corners and set the center corner *a*. Split the distances between the four corners of the NW 1/4, and set the quarter-quarter corners *b, c, d,* and *e*. Connect opposite quarter-quarter corners and set *f* the center of the NW 1/4. The SE 1/4 is shown as a solid area.

22.18 The quarter-section corner between Secs. 15 and 16 is found to be 40.28 ch from the corner common to Secs. 9, 10, 15, and 16. Where should the quarter-quarter-section corner be set along this line in subdividing Sec. 15?

Set the quarter-quarter corner at 40.28 ch/2, which is **20.14** ch on the section line from the NW corner of Section 15.

22.19 As shown in the figure, in a normal township the exterior dimensions of Sec. 6 on the west, north, east, and south sides are 80, 78, 81, and 79 ch, respectively. Explain with a sketch how to divide the section into quarter sections. (See the following figure.)

Assuming all the section corners are in place, set the center of section at the intersection of EG and HF. Set the quarter-quarter corners 20 ch north from E, B, F, I, C, and G, and 20 ch west from A, E, B, F, I, and H. Connect the quarter-quarter corners thus throwing all the discrepancies to the north and west. Thus all quarter-quarter parcels are 20 ch by 20 ch, except for those along the upper tier of the section.

22.20 The problem figure shows original record distances. Corners A, B, C, and D are found, but corner E is lost. Measured distances are $AB = 10{,}602.97$ ft and $CD = 10{,}718.03$ ft. Explain how to establish corner E. (See the following figure.)

AE = **5340.95 ft**; DK = **5345.58 ft**

$$AE = \frac{81.82}{161.82} 10{,}602.97 = 5340.95 \text{ and } DK = \frac{79.6}{159.6} 10{,}718.03 = 5345.58$$

To restore the corners in Problems 22.21 through 22.24, which method is used, single proportion or double proportion?

***22.21** Township corners on guide meridians; section corners on range lines

single; single

22.22 Section corners on section lines; township corners on township lines.

double; double

22.23 Quarter-section corners on range lines.

single

22.24 Quarter-quarter-section corners on section lines.

single

22.25 Why are meander lines not accepted as the boundaries defining ownership of lands adjacent to a stream or lake?

From Section 22.18, paragraph 3: "Meander lines follow the mean high water mark and are used only for plotting and protraction of the area. They are not boundaries defining the limits of the property adjacent to the water."

22.26 What is a meander corner?

From Section 22.18, paragraph 1: "A *meander corner* (MC) was established on survey lines intersecting the bank of a stream having a width greater than 3 ch, or a lake, bayou, or other body of water of 25 acres or more."

22.27 Explain the difference between “obliterated corner” and “lost corner.”

From Section 22.18, paragraphs 2 – 4: "An *obliterated corner* is one for which there are no remaining traces of the monument or its accessories, but whose location has been perpetuated or can be recovered beyond reasonable doubt. The corner may be restored from the acts or testimony of interested landowners, surveyors, qualified local authorities, witnesses, or from written evidence. Satisfactory evidence has value in the following order:

1. Evidence of the corner itself.
2. Bearing trees or other witness marks.

3. Fences, walls, or other evidence showing occupation of the property to the lines or corners.
4. Testimony of living persons.

A *lost corner* is one whose position cannot be determined beyond reasonable doubt, either from traces of the original marks or from acceptable evidence or testimony that bears on the original position. It can be restored only by rerunning lines from one or more independent corners (existing corners that were established at the same time and with the same care as the lost corner)."

22.28 Explain the value of using proportionate measurements from topographic calls in relocating obliterated or lost corners?

From Section 22.19, last paragraph: "When the original surveys were run, *topographic calls* (distances along each line from the starting corner to natural features such as streams, swamps, and ridges) were recorded. Using the recorded distances to any of these features found today and applying single- or double-proportionate measurements to them, may help locate an obliterated corner or produce a more reliable reestablished lost corner."

22.29 Why are the areas of many public-lands sections smaller than the nominal size?

Convergence of the meridians and errors and mistakes in the original surveys that allowed low precisions.

22.30 Visit the NILS web site and briefly describe the four components of NILS.

The four major components are survey management, measurement management, parcel management, and the Geocommunicator. The students should briefly describe each of these.

22.31 Visit the BLM website at http://www.blm.gov/wo/st/en/prog/more/nils.html, and prepare a paper on the NILS project.

Individual report.

23 CONSTRUCTION SURVEYING

Asterisks (*) indicate problems that have answers given in Appendix G.

23.1 Discuss how machine control has changed the construction project site.

From Section 23.11, paragraph 2: "Using machine control, the surveyor's role in construction surveying shifts to tasks such as establishing the project reference coordinate systems, creating a DTM of the existing surface for the design and grading work, managing the electronic design on the job site, calibrating the surveying equipment with respect to the construction site, and developing the necessary data for system operation."

23.2 Discuss how line and grade can be set with a total station instrument.

From Section 23.9: At an occupied station, a backsight is taken with the instrument on a line of known azimuth and the direct to a required stake turned. The instrument is operated in tracking mode and a prism is placed on-line and adjusted in position to read the required horizontal distance, where the stake is driven. The stake can be set at its required elevation or marked as appropriate for the job.

23.3 How are rotating beam lasers used?

From Section 23.2.1, paragraph 4: "They expedite the placement of grade stakes over large areas such as airports, parking lots, and subdivisions, and are also useful for topographic mapping."

23.4 In what types of construction is a reflectorless EDM most advantageous?

From Section 23.2.2, paragraph 2: "Both devices are useful in observing distances to inaccessible locations, a feature that is particularly useful in assembling and checking the placement of structural members in bridges, buildings, and other large fabricated objects."

23.5 Discuss how a laser is used in pipeline layout.

From Section 23.4, paragraph 7: "If laser devices are used for laying pipes, the beam is oriented along the pipe's planned horizontal alignment and grade, and the trench opened. Then with the beam set at some even number of feet above the pipe's planned invert, measurements can be made using a story pole to set the pipe segments. Thus, the laser beam is equivalent to a batter board string line. On some jobs that have a deep wide cut, the laser instrument is set up in the trench to give line and grade for laying pipes. And, if the pipe is large enough, the laser beam can be oriented inside it."

23.6 What is a story pole and how is it used in pipeline layout?

From Section 23.4, paragraph 5: "A graduated pole or special rod, often called a *story pole,* is used to measure the required distance from the string to the pipe invert."

23.7 What is the purpose of localization when performing a GNSS stakeout survey?

From Section 23.10, paragraphs 2 and 3: "The localization process transforms this set of low accuracy GNSS coordinates into the project control reference frame eliminating the inaccuracies of the autonomous base station coordinates.

As discussed in Sections 15.8 and 19.6.6, care must be taken to ensure that points located using GNSS are placed in the same reference frame as the project coordinates."

23.8 What information is typically conveyed to the contractor on stakes for laying a pipeline?

From Section 23.5, paragraph 2: "Information conveyed to the contractor on stakes for laying pipelines usually consists of two parts: (1) giving the depth of cut (or fill), normally only to the nearest 0.1 ft, to enable a rough trench to be excavated; and (2) providing precise grade information, generally to the nearest 0.01 ft, to guide in the actual placement of the pipe invert at its planned elevation."

23.9 A sewer pipe is to be laid from station 10+00 to station 12+50 on a 0.50% grade, starting with invert elevation 83.64 ft at 10+00 Calculate invert elevations at each 50-ft station along the line.

Station	Elevation (ft)
1000	83.64
1050	83.89
1100	84.14
1150	84.39
1200	84.64
1250	84.89

***23.10** A sewer pipe must be laid from a starting invert elevation of 650.73 ft at station 9+25 to an ending invert elevation 653.81 ft at station 12+75. Determine the uniform grade needed, and calculate invert elevations at each 50-ft station.

Station	Elevation (ft)	grade
925	650.73	0.88%
950	650.95	
1000	651.39	
1050	651.83	
1100	652.27	
1150	652.71	
1200	653.15	
1250	653.59	
1275	653.81	

23.11 Grade stakes for a pipeline running between stations 0+00 and 6+37 are to be set at each full station. Elevations of the pipe invert must be 843.95 ft at station 0+00 and 847.22 ft at 6+37, with a uniform grade between. After staking an offset centerline, an instrument is set up nearby, and a plus sight of 5.32 taken on BM *A* (elevation 853.63 ft). The following minus sights are taken with the rod held on ground at each stake: (0 + 00, 5.36); (1 + 00, 5.86); (2 + 00, 5.88); (3 + 00, 6.47); (4 + 00, 7.53); (5 + 00, 8.42); (6 + 00, 8.89); (6+37, 9.12) and (*A*, 5.36). Prepare a set of suitable field notes for this project (see Plate B.6 in Appendix B) and compute the cut required at each stake. Close the level circuit on the benchmark.

					Grade:	0.51%
Station	+Sight	HI	−Sight	Ground Elevation	Pipe Invert	Cut/Fill
A	5.32	858.95		853.63		
0+00			5.36	853.59	843.95	C9.6
1+00			5.86	853.09	844.46	C8.6
2+00			5.88	853.07	844.98	C8.1
3+00			6.47	852.48	845.49	C7.0
4+00			7.53	851.42	846.00	C5.4
5+00			8.42	850.53	846.52	C4.0
6+00			8.89	850.06	847.03	C3.0
6+37			9.12	849.83	847.22	C2.6
A			5.32	853.63		

23.12 If batter boards are to be set exactly 6.00 ft above the pipe invert at each station on the project of Problem 23.11, calculate the necessary rod readings for placing the batter boards. Assume the instrument has the same HI as in Problem 23.11.

Station	+Sight	HI	Rod Reading
A	5.32	858.95	
0+00			9.00
1+00			8.49
2+00			7.97
3+00			7.46
4+00			6.95
5+00			6.43
6+00			5.92
6+37			5.73

23.13 What are the requirements for the placement of horizontal and vertical control in a project?

From Section 23.3, paragraph 3: "The control points must be:

1. Convenient for use, that is, located sufficiently close to the item being built so that work is minimized and accuracy enhanced in transferring alignment and grade.

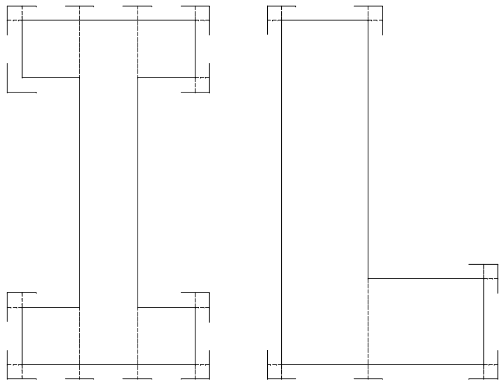
2. Far enough from the actual construction to ensure working room for the contractor and to avoid possible destruction of stakes.

3. Clearly marked and understood by the contractor in the absence of a surveyor.

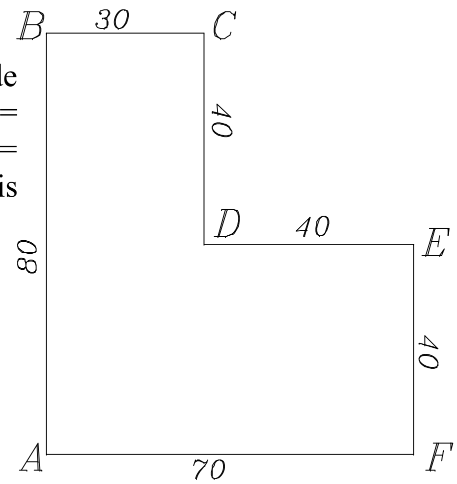
4. Supplemented by guard stakes to deter removal, and referenced to facilitate restoring them. Contracts usually require the owner to pay the cost of setting initial control points and the contractor to replace damaged or removed ones.

5. Suitable for securing the accuracy agreed on for construction layout (which may be to only the nearest foot for a manhole, 0.01 ft for an anchor bolt, or 0.001 ft for a critical feature)."

23.14 By means of a sketch, show how and where batter boards should be located: (**a**) for an I-shaped building (**b**) For an L-shaped structure.



23.15 A building in the shape of an L must be staked. Corners *ABCDEF* all have right angles. Proceeding clockwise around the building, the required outside dimensions are AB = 80.00 ft, BC = 30.00 ft, CD = 40.00 ft, DE = 40.00 ft, EF = 40.00 ft, and FA = 70.00 ft. After staking the batter boards for this building and stretching string lines taut, check measurements of the diagonals should be made. What should be the values of *AC, AD, AE, FB, FC, FD,* and *BD?*



$$AC = \sqrt{80^2 + 30^2} = 85.44\,\text{ft}$$

$$AD = \sqrt{40^2 + 30^2} = 50.00\,\text{ft}$$

$$AE = \sqrt{40^2 + 70^2} = 80.62\,\text{ft}$$

$$FB = \sqrt{80^2 + 70^2} = 106.30\,\text{ft}$$

$$FC = \sqrt{80^2 + 40^2} = 89.44\,\text{ft}$$

$$FD = \sqrt{40^2 + 40^2} = 56.57\,\text{ft}$$

$$BD = \sqrt{40^2 + 30^2} = 50.00\,\text{ft}$$

***23.16** Compute the floor area of the building in Problem 23.15.

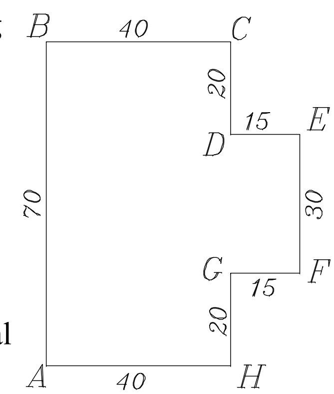
Area: = 30(80) + 40(40) = <u>**4000 ft²**</u>

***23.17** The design floor elevation for a building to be constructed is 332.56 ft. An instrument is set up nearby, leveled, and a plus sight of 6.37 ft taken on BM *A* whose elevation is 330.05 ft. If batter boards are placed exactly 1.00 ft above floor elevation, what rod readings are necessary on the batter board tops to set them properly?

2.86 ft

Station	+Sight	HI	−Sight	Elev
A	6.37	336.42		330.05
			2.86	332.56

23.18 Compute the diagonals necessary to check the stakeout of the building in Figure 23.8.



$AC = BH = \sqrt{40^2 + 70^2} = 80.62\text{ ft}$

$BE = AF = \sqrt{55^2 + 20^2} = 58.52\text{ ft}$

$BF = AE = \sqrt{50^2 + 55^2} = 74.33\text{ ft}$

$BD = AG = \sqrt{40^2 + 20^2} = 44.72\text{ ft}$

23.19 Explain how the corner of a building can be plumbed using a total station?
From Section 23.6, paragraph 7: "One method of checking plumbness of constructed members is to carefully aim a total station's line of sight on a reference mark at the base of the member. The line of sight is then plunged to its top. For an instrument that has been carefully leveled and that is in proper adjustment, the line of sight will define a vertical plane as it is plunged. It should not be assumed that the instrument is in good adjustment, however, and the line should be plunged in both the face left and face right positions. It is necessary to check plumbness in two perpendicular directions when using this procedure. To guide construction of vertical members in real-time, two instruments can be set up with their lines of sight oriented perpendicular to each other, and verticality monitored as construction progresses."

23.20 Where is the invert of a pipe measured?

From Section 23.4, paragraph 5: "the *invert* (flow line or lower inside surface) of the pipe "

23.21 Discuss the importance of localizing a GNSS survey.
From Section 23.10, paragraph 3: "As discussed in Sections 15.8 and 19.6.6, care must be taken to ensure that points located using GNSS are placed in the same reference frame as the project coordinates."

23.22 Explain why slope intercepts are placed an offset distance from the actual slope intercept.

From Section 23-7, paragraph 7: "They are usually offset 4 ft from the construction limits to protect them from destruction during the construction process."

23.23 What information is normally written on a slope stake?

From Section 23.7: The minimum information is the amount of cut (C) or fill (F) from the ground at the stake to the centerline subgrade elevation, distance from the stake to the centerline, and the value of the cut or fill slope.

23.24 What are grade stakes?

From Section 23.7, paragraph 8: "*Grade stakes* are set at points that have the same ground and grade elevation. This happens when a grade line changes from cut to fill, or vice-versa."

23.25 Discuss the procedure and advantages of using total station instruments with data collectors for slope staking.

From Section 23.7, paragraph 13 and Section 23.9, paragraph 12:

Coordinates and elevations of slope stake positions are determined, and they are laid out by measuring differences in horizontal distances, directions, and elevations from an occupied reference station.

23.26 What are spads and how are they used in mine surveys?

From Section 23.8, paragraph 3: "Later setups are made beneath spads (surveying nails with hooks) anchored in the ceiling."

23.27 A highway centerline subgrade elevation is 985.20 ft at station 12+00 and 993.70 ft at 17+00 with a smooth grade in between. To set blue tops for this portion of the centerline, a level is setup in the area and a plus sight of 4.19 ft taken on a benchmark whose elevation is 992.05 ft. From that HI, what rod readings will be necessary to set the blue tops for the full stations from 12+00 through 17+00?

HI = 996.24 g = 1.70%

Station	Subgrade Elevation	Rod Reading
12+00	985.20	11.04
13+00	986.90	9.34
14+00	988.60	7.64
15+00	990.30	5.94
16+00	992.00	4.24
17+00	993.70	2.54

23.28 Similar to Problem 23.27, except the elevations at stations 12+00 and 17+00 are 1713.35 and 1707.10 ft, respectively, the BM elevation is 1710.84 ft, and the backsight is 5.28 ft.

HI = 1716.12 g = –1.25%

Station	Subgrade Elevation	Rod Reading
12+00	1713.35	2.77
13+00	1712.10	4.02
14+00	1710.85	5.27
15+00	1709.60	6.52
16+00	1708.35	7.77
17+00	1707.10	9.02

23.29 Discuss the checks that should be made when laying out a building using coordinates.

From Section 23.6, paragraph 3: "Measuring the distances between adjacent points, and also the diagonals checks the layout."

23.30 What are the jobs of a surveyor in a project using machine control?

From Section 23.11, paragraph 2: "Using machine control, the surveyor's role in construction surveying shifts to tasks such as establishing the project reference coordinate systems, creating a DTM (see Section 17.14) of the existing surface for the design and grading work, managing the electronic design on the job site, calibrating the surveying equipment with respect to the construction site, and developing the necessary data for system operation."

23.31 Describe the procedure for localization of a GNSS survey.

From Section 23.10, paragraph 3: "As discussed in Section 15.9, sufficient project control known in the local reference frame must be established at the perimeter of the construction project. Then prior to staking any points, the GNSS receiver must occupy this control and determine their coordinates in the WGS 84 reference frame; these are GNSS coordinates. Using the project coordinates and the GNSS coordinates, transformation parameters (see Section 19.6.6) are computed so that the GNSS-derived coordinates can be transformed into the local project reference frame. It is important that this transformation occur only once in a project and include important control in the transformation. That is, if a benchmark on a bridge abutment was used to design a replacement structure, then this benchmark should be included in the localization process regardless of its location in the project. The localization process should only occur once during a construction project to avoid the introduction of varying orientation parameters caused by random errors. Once the localization is accepted, the transformation parameters should be distributed amongst the various GNSS receivers involved in the project."

23.32 Why is localization important in a GNSS survey?

From Section 23.10, paragraph 2: "The localization process, discussed next, transforms this set of low accuracy GNSS coordinates into the project control reference frame eliminating the inaccuracies of the autonomous base station coordinates."

23.33 How can finished grades be established in machine control projects?

From Section 23.11, paragraph 4: "However in finished grading, a robotic total station or laser level is required. As previously mentioned, one manufacturer has combined a laser level with the GNSS receiver to provide millimeter accuracies in both horizontal and vertical location."

23.34 What is the minimum number of horizontal control points needed to establish finish-grades using a robotic total station on a machine-controlled project that is 8 mi in length?

<u>43 control stations</u>; $\frac{8(5280)}{1000} = 42.24$

23.35 What advantages does GNSS-supported machine control have over robotic total station methods? What are the disadvantages?

From Section 23.11, the advantage is that fewer control are required to support a project and that one base station can support a project over a 20 km radius whereas a robotic total station must be dedicated to each piece of equipment and many more control stations are required along the entire project. The main disadvantage is that GNSS-supported machine control can not be used for finished grade elevations and can not be used where overhead obstructions will interfere with signal reception.

23.36 Discuss the advantages of using laser-scanning technology when planning for a new pipeline in a refinery.

From Section 23.12: "However in projects that involve extensive detail, danger to instrument-operator, or interruption of daily-commerce, laser scanning can provide superior results in a fraction of the time. Figure 23.14 shows a rendered image of a bridge that was surveyed for renovations. In the bridge survey, enormous quantities of data were collected from an on-shore location. The digital image of the bridge is shown in the lower-right inset. The rendered, three-dimensional image of the bridge allows designers to obtain accurate measurements between points in the image. Figure 17.12 depicts the point-cloud image of a refinery with the path of a new pipe shown in white. This three-dimensional image allowed engineers to design the new pipe alignment so that existing obstructions were cleared. A traditional survey would have either lacked the detail provided by the three-dimensional laser-scanned image or cost considerably more to locate all the existing elements. Using laser-scanning technology in these projects saved thousands of dollars and provided safe conditions for the field crews."

23.37 Do an article review on an application of machine control.

Independent study.

24 HORIZONTAL CURVES

Asterisks $^{(*)}$ indicate problems that have answers given in Appendix *G*.

24.1 Why is a reverse curve objectionable for transportation alignments?

From Section 24.17, paragraph 1: "They should be used only for low-speed traffic routes, and in terrain where simple curves cannot be fitted to the ground without excessive construction costs since the rapid change in curvature causes unsafe driving conditions."

24.2 For the following circular curves having a radius *R*, what is their degree of curvature by (1) arc definition and (2) chord definition?

(a)* 500.00 ft (1) **11°27′33″** (2) **11°28′42″**

(b) 750.00 ft (1) **7°38′22″** (2) **7°38′42″**

(c) 2000.00 ft (1) **2°51′53″** (2) **2°51′54″**

Compute *L, T, E, M, LC, R*, and stations of the PC and PT for the circular curves in Problems 24.3 through 24.6. Use the chord definition for the railroad curve and the arc definition for the highway curves.

***24.3** Railroad curve with D_c = 4°00′, I = 24°00′, and PI station = 36 + 45.00 ft.

24.4 Highway curve with D_a = 2°40′, I = 14°20′, and PI station = 24 + 65.00 ft.

24.5 Highway curve with R = 500.000 m, I = 18°30′, and PI station = 6+517.500 m.

24.6 Highway curve with R = 750.000 m, I = 18°30′, and PI station = 12+324.800 m.

	24.3	**24.4**	**24.5**	**24.6**
PI	36+45.00	24+65.00	6+517.500	12+324.800
D_c	4			
D_a		2.666666667		
R	1432.68542	2148.5925	500	750
I	24	14.33333333	18.5	18.5
L	600	537.5	161.443	242.164
T	304.53	270.16	81.430	122.145
E	32.01	16.92	6.587	9.881
M	31.31	16.79	6.502	9.753
LC	595.74	536.10	160.743	241.114
PC	33+40.47	21+94.84	6+436.070	12+202.655

PT_{Back}	42+45.00	30+02.50	6+678.943	12+566.964
$PT_{Forward}$	39+49.53	27+35.16	6+598.930	12+446.945

Tabulate *R* or *D, T, L, E, M,* PC, PT, deflection angles, and incremental chords to lay out the circular curves at full stations (100 ft or 30 m) in Problems 24.7 through 24.14.

24.7 Highway curve with $D_a = 3°46'$, $I = 16°30'$, and PI station = 29 + 64.20 ft.

```
           Intersection Angle = 16°30'00"
          Degree of Curvature = 3°46'00"
                       Radius = 1,521.13
         Circular Curve Length = 438.05
             Tangent Distance = 220.55
     Circular Curve Long Chord = 436.54
              Middle Ordinate = 15.74
                     External = 15.91

               PI Stationing = 29+64.20
               31+81.70 Back = 31+84.75 Ahead
============================================================
```

Station	Chord	Defl. Increment	Defl. Angle
31+81.70	81.69	1°32'19"	8°15'00"
31+00.00	99.98	1°53'00"	6°42'41"
30+00.00	99.98	1°53'00"	4°49'41"
29+00.00	99.98	1°53'00"	2°56'41"
28+00.00	56.35	1°03'41"	1°03'41"
27+43.65			

24.8 Railroad curve with $D_c = 2°30'$, $I = 15°00'$, and PI station = 58 + 65.42 ft.

```
           Intersection Angle = 15°00'00"
          Degree of Curvature = 2°30'00"
                       Radius = 2,292.01
         Circular Curve Length = 600.00
             Tangent Distance = 301.75
     Circular Curve Long Chord = 598.34
              Middle Ordinate = 19.61
                     External = 19.78

               PI Stationing = 58+65.42
               61+63.67 Back = 61+67.17 Ahead
============================================================
```

Station	Chord	Defl. Increment	Defl. Angle
61+63.67	63.67	0°47'45"	7°30'00"
61+00.00	100.00	1°15'00"	6°42'15"
60+00.00	100.00	1°15'00"	5°27'15"
59+00.00	100.00	1°15'00"	4°12'15"
58+00.00	100.00	1°15'00"	2°57'15"
57+00.00	100.00	1°15'00"	1°42'15"
56+00.00	36.33	0°27'15"	0°27'15"
55+63.67			

24.9 Highway curve with $R = 800$ m, $I = 12°00'$, and PI station = 3 + 281.615 m.

```
           Intersection Angle = 12°00'00"
          Degree of Curvature = 2°10'59"
                       Radius = 800.000
        Circular Curve Length = 167.552
             Tangent Distance = 84.083
    Circular Curve Long Chord = 167.246
              Middle Ordinate = 4.382
                     External = 4.407

               PI Stationing = 3+281.615
              3+365.083 Back = 3+365.698 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
3+365.083	5.083	0°10'55"	6°00'00"
3+360.000	29.998	1°04'27"	5°49'05"
3+330.000	29.998	1°04'27"	4°44'37"
3+300.000	29.998	1°04'27"	3°40'10"
3+270.000	29.998	1°04'27"	2°35'42"
3+240.000	29.998	1°04'27"	1°31'15"
3+210.000	12.468	0°26'47"	0°26'47"
3+197.532			

24.10 Highway curve with $R = 700$ m, $I = 14°30'$, and PI station = 1 + 632.723 m.

```
           Intersection Angle = 14°30'00"
          Degree of Curvature = 2°29'41"
                       Radius = 700.000
        Circular Curve Length = 177.151
             Tangent Distance = 89.051
    Circular Curve Long Chord = 176.679
              Middle Ordinate = 5.597
                     External = 5.642

               PI Stationing = 1+632.723
              1+720.823 Back = 1+721.774 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
1+720.823	10.823	0°26'35"	7°15'00"
1+710.000	29.998	1°13'40"	6°48'25"
1+680.000	29.998	1°13'40"	5°34'46"
1+650.000	29.998	1°13'40"	4°21'06"
1+620.000	29.998	1°13'40"	3°07'26"
1+590.000	29.998	1°13'40"	1°53'46"
1+560.000	16.328	0°40'06"	0°40'06"
1+543.672			

24.11 Highway curve with $R = 850$ ft, $I = 40°00'$, and PI station = 85 + 40.00 ft.

```
         Intersection Angle = 40°00'00"
       Degree of Curvature = 6°44'26"
                    Radius = 850.00
      Circular Curve Length = 593.41
           Tangent Distance = 309.37
  Circular Curve Long Chord = 581.43
            Middle Ordinate = 51.26
                   External = 54.55

              PI Stationing = 85+40.00
              88+24.04 Back = 88+49.37 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
88+24.04	24.04	0°48'36"	20°00'00"
88+00.00	99.94	3°22'13"	19°11'24"
87+00.00	99.94	3°22'13"	15°49'10"
86+00.00	99.94	3°22'13"	12°26'57"
85+00.00	99.94	3°22'13"	9°04'44"
84+00.00	99.94	3°22'13"	5°42'31"
83+00.00	69.36	2°20'17"	2°20'17"
82+30.63			

24.12 Highway curve with $L = 350$ m, $R = 400$ m, and PI station = 4 + 332.690 m.

```
         Intersection Angle = 50°08'02"
       Degree of Curvature = 4°21'57"
                    Radius = 400.000
      Circular Curve Length = 350.000
           Tangent Distance = 187.092
  Circular Curve Long Chord = 338.941
            Middle Ordinate = 37.675
                   External = 41.592
              PI Stationing = 4+332.690
             4+495.598 Back = 4+519.782 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
4+495.598	25.594	1°50'00"	25°04'01"
4+470.000	29.993	2°08'55"	23°14'01"
4+440.000	29.993	2°08'55"	21°05'06"
4+410.000	29.993	2°08'55"	18°56'11"
4+380.000	29.993	2°08'55"	16°47'16"
4+350.000	29.993	2°08'55"	14°38'21"
4+320.000	29.993	2°08'55"	12°29'26"
4+290.000	29.993	2°08'55"	10°20'31"
4+260.000	29.993	2°08'55"	8°11'36"
4+230.000	29.993	2°08'55"	6°02'41"
4+200.000	29.993	2°08'55"	3°53'47"
4+170.000	24.398	1°44'52"	1°44'52"
4+145.598			

24.13 Highway curve with $T = 265.00$ ft, $R = 1250$ ft, and PI station = 87 + 33.55 ft.

```
       Intersection Angle = 23°56'20"
     Degree of Curvature = 4°35'01"
                   Radius = 1,250.00
    Circular Curve Length = 522.27
         Tangent Distance = 265.00
Circular Curve Long Chord = 518.48
          Middle Ordinate = 27.18
                 External = 27.78

            PI Stationing = 87+33.55
           89+90.82 Back = 89+98.55 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
89+90.82	90.80	2°04'53"	11°58'10"
89+00.00	99.97	2°17'31"	9°53'17"
88+00.00	99.97	2°17'31"	7°35'47"
87+00.00	99.97	2°17'31"	5°18'16"
86+00.00	99.97	2°17'31"	3°00'45"
85+00.00	31.45	0°43'15"	0°43'15"
84+68.55			

24.14 Railroad curve with $T = 155.00$ ft, $D_C = 2°35'$, and PI station = 48 + 10.00 ft.

```
       Intersection Angle = 7°59'41"
     Degree of Curvature = 2°35'00"
                   Radius = 2,218.09
    Circular Curve Length = 309.47
         Tangent Distance = 155.00
Circular Curve Long Chord = 309.25
          Middle Ordinate = 5.40
                 External = 5.41

            PI Stationing = 48+10.00
           49+64.47 Back = 49+65.00 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
49+64.47	64.47	0°49'58"	3°59'50"
49+00.00	100.00	1°17'30"	3°09'52"
48+00.00	100.00	1°17'30"	1°52'22"
47+00.00	45.00	0°34'52"	0°34'52"
46+55.00			

In Problems 24.15 through 24.18 tabulate the curve data, deflection angles, and total chords needed to lay out the following circular curves at full-station increments using a total station instrument set up at the PC.

24.15 The curve of Problem 24.7.

```
                Intersection Angle = 16°30'00"
              Degree of Curvature = 3°46'00"
                           Radius = 1,521.13
             Circular Curve Length = 438.05
                  Tangent Distance = 220.55
        Circular Curve Long Chord = 436.54
                   Middle Ordinate = 15.74
                          External = 15.91

                    PI Stationing = 29+64.20
                    31+81.70 Back = 31+84.75 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
31+81.70	436.54	1°32'19"	8°15'00"
31+00.00	355.54	1°53'00"	6°42'41"
30+00.00	256.05	1°53'00"	4°49'41"
29+00.00	156.28	1°53'00"	2°56'41"
28+00.00	56.35	1°03'41"	1°03'41"
27+43.65			

24.16 The curve of Problem 24.8

```
                Intersection Angle = 15°00'00"
              Degree of Curvature = 2°30'00"
                           Radius = 2,292.01
             Circular Curve Length = 600.00
                  Tangent Distance = 301.75
        Circular Curve Long Chord = 598.34
                   Middle Ordinate = 19.61
                          External = 19.78

                    PI Stationing = 58+65.42
                    61+63.67 Back = 61+67.17 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
61+63.67	598.34	0°47'45"	7°30'00"
61+00.00	535.15	1°15'00"	6°42'15"
60+00.00	435.71	1°15'00"	5°27'15"
59+00.00	336.05	1°15'00"	4°12'15"
58+00.00	236.24	1°15'00"	2°57'15"
57+00.00	136.32	1°15'00"	1°42'15"
56+00.00	36.33	0°27'15"	0°27'15"
55+63.67			

24.17 The curve of Problem 24.9

```
          Intersection Angle = 12°00'00"
        Degree of Curvature = 2°10'59"
                     Radius = 800.000
       Circular Curve Length = 167.552
           Tangent Distance = 84.083
   Circular Curve Long Chord = 167.246
            Middle Ordinate = 4.382
                   External = 4.407

              PI Stationing = 3+281.615
             3+365.083 Back = 3+365.698 Ahead
```

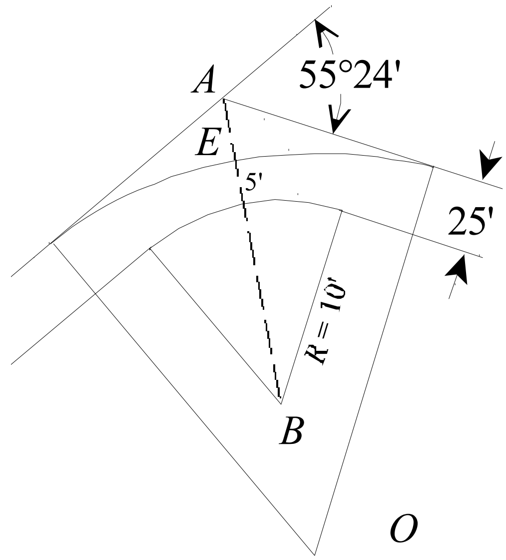
Station	Chord	Defl. Increment	Defl. Angle
3+365.083	167.246	0°10'55"	6°00'00"
3+360.000	162.189	1°04'27"	5°49'05"
3+330.000	132.317	1°04'27"	4°44'37"
3+300.000	102.398	1°04'27"	3°40'10"
3+270.000	72.444	1°04'27"	2°35'42"
3+240.000	42.463	1°04'27"	1°31'15"
3+210.000	12.468	0°26'47"	0°26'47"
3+197.532			

24.18 The curve of Problem 24.10

```
          Intersection Angle = 14°30'00"
        Degree of Curvature = 2°29'41"
                     Radius = 700.000
       Circular Curve Length = 177.151
           Tangent Distance = 89.051
   Circular Curve Long Chord = 176.679
            Middle Ordinate = 5.597
                   External = 5.642

              PI Stationing = 1+632.723
             1+720.823 Back = 1+721.774 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
1+720.823	176.679	0°26'35"	7°15'00"
1+710.000	165.937	1°13'40"	6°48'25"
1+680.000	136.113	1°13'40"	5°34'46"
1+650.000	106.226	1°13'40"	4°21'06"
1+620.000	76.290	1°13'40"	3°07'26"
1+590.000	46.320	1°13'40"	1°53'46"
1+560.000	16.328	0°40'06"	0°40'06"
1+543.672			

24.19 A rail line on the center of a 50-ft street makes a 55°24′ turn into another street of equal width. The corner curb line has R = 10 ft. What is the largest R that can be given a circular curve for the track centerline if the law requires it to be at least 5 ft from the curb?



$AB = (25+10)\sin(55°24') = 24.694$ ft

$AE = 24.694 - (10+5) = 9.694$ ft

$R = 9.694/[1/\cos(55°24'/2) - 1) = 74.89$ ft

Tabulate all data required to lay out by deflection angles and incremental chords, at the indicated stationing, for the circular curves of Problems 24.20 and 24.21.

24.20 The R for a highway curve (arc definition) will be rounded off to the nearest larger multiple of 100 ft. Field conditions require M to be approximately 30 ft to avoid an embankment. The PI = 94 + 18.70 and I = 23°00′ with stationing at 100 ft.

$$R = \frac{20}{1-\cos\left(23°00'/2\right)} = 996.25$$, so round R to 1000 ft.

```
        Intersection Angle = 23°00'00"
      Degree of Curvature = 5°43'46"
                    Radius = 1,000.00
     Circular Curve Length = 401.43
          Tangent Distance = 203.45
 Circular Curve Long Chord = 398.74
           Middle Ordinate = 20.08
                  External = 20.49

            PI Stationing = 94+18.70
           96+16.67 Back = 96+22.15 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
96+16.67	16.67	0°28'40"	11°30'00"
96+00.00	99.96	2°51'53"	11°01'20"
95+00.00	99.96	2°51'53"	8°09'27"
94+00.00	99.96	2°51'53"	5°17'34"
93+00.00	84.73	2°25'41"	2°25'41"
92+15.25			

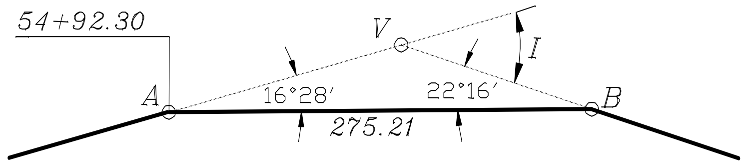
24.21 For a highway curve *R* will be rounded off to the nearest multiple of 10 m. Field measurements show that *T* should be approximately 80 m to avoid an overpass. The PI = 6 + 356.400 m and *I* = 13°20′ with stationing at 30 m.

$$R = \frac{80}{\tan\left(13°20'/2\right)} = 684.444 \text{ m}$$, so round off to 680 m.

```
          Intersection Angle = 13°20'00"
        Degree of Curvature = 8°25'33"
                     Radius = 680.0
       Circular Curve Length = 158.2
            Tangent Distance = 79.5
   Circular Curve Long Chord = 157.9
             Middle Ordinate = 4.6
                    External = 4.6

              PI Stationing = 63+56.4
               64+35.2 Back = 64+35.9 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
64+35.2	15.2	0°38'20"	6°40'00"
64+20.0	30.0	1°15'50"	6°01'40"
63+90.0	30.0	1°15'50"	4°45'50"
63+60.0	30.0	1°15'50"	3°30'00"
63+30.0	30.0	1°15'50"	2°14'10"
63+00.0	23.1	0°58'21"	0°58'21"
62+76.9			

24.22 A highway survey PI falls in a pond, so a cut off line AB = 275.12 ft is run between the tangents. In the triangle formed by points A, B, and PI, the angle at A = 16°28′ and at B = 22°16′. The station of A is 54+92.30 ft. Calculate and tabulate curve notes to run, by deflection angles and incremental chords, a 4°30′ (arc definition) circular curve at full-station increments to connect the tangents.



$I = 16°28' + 22°16' = 38°44'$

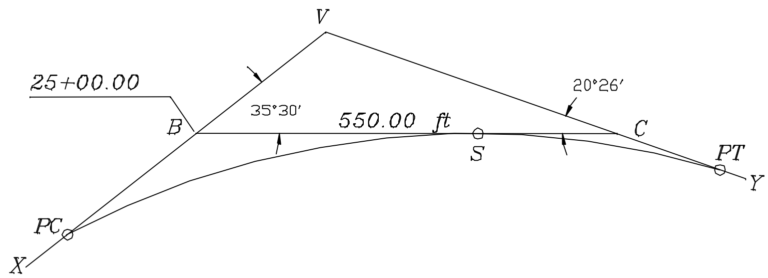
$$AV = \frac{275.21\sin(22°16')}{\sin(180° - 38°44')} = 166.665 \text{ ft}$$

$\text{PI} = 54 + 92.30 + 166.66 = 56 + 58.96$

```
         Intersection Angle = 38°44'00"
        Degree of Curvature = 4°30'00"
                     Radius = 1,273.24
      Circular Curve Length = 860.74
           Tangent Distance = 447.55
  Circular Curve Long Chord = 844.44
            Middle Ordinate = 72.05
                   External = 76.37

               PI Stationing = 56+58.96
             60+72.15 Back = 61+06.51 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
60+72.15	72.15	1°37'25"	19°22'00"
60+00.00	99.97	2°15'00"	17°44'35"
59+00.00	99.97	2°15'00"	15°29'35"
58+00.00	99.97	2°15'00"	13°14'35"
57+00.00	99.97	2°15'00"	10°59'35"
56+00.00	99.97	2°15'00"	8°44'35"
55+00.00	99.97	2°15'00"	6°29'35"
54+00.00	99.97	2°15'00"	4°14'35"
53+00.00	88.57	1°59'35"	1°59'35"
52+11.41			

24.23 In the figure, a single circular highway curve (arc definition) will join tangents *XV* and *VY* and also be tangent to *BC*. Calculate *R, L*, and the stations of the PC and PT.



$$I = 35°30' + 20°26' = 55°56'$$

$$R = \frac{550}{\tan(35°30'/2) + \tan(20°20'/2)} = 1099.27 \text{ ft}$$

$$L = 1099.27I = 1073.13 \text{ ft}$$

$$T = 1099.27\tan(55°56'/2) = 583.67 \text{ ft}$$

$$BV = \frac{550.00\sin(20°26')}{\sin(55°56')} = 231.79 \text{ ft}$$

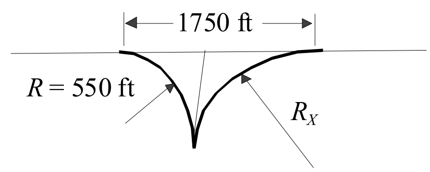
PI = 2500 + 231.79 = 27 + 31.79

PC = 2731.79 − 583.67 = 21 + 48.12

PT_{Back} = 2148.12 + 1073.13 = 32 + 21.25

$PT_{Forward}$ = 2731.79 + 583.67 = 33 + 15.46

***24.24** Compute R_x to fit requirements of the figure and make the tangent distances of the two curves equal.



<u>1392.04 ft</u>

$$T_x = 1750 / 2 = 875$$

$$I = 2\tan^{-1}(875/550) = 115°41'43.5''$$

$$I_x = 180° - I = 64°18'16.5''$$

$$R_x = \frac{875}{\tan(64°18'16.5'')} = 1392.04 \text{ ft}$$

24.25 After a backsight on the PC with $0°00'$ set on the instrument, what is the deflection angle to the following circular curve points?

***(a)** Setup at curve midpoint, deflection to the PT. ***I*/2**

(b) Instrument at curve midpoint, deflection to the 3/4 point. **3/8*I***

(c) Setup at 1/4 point of curve, deflection to 3/4 point. **3/8*I***

24.26 In surveying a construction alignment, why should the *I* angle be measured by repetition?

To account for possible instrumental errors, to increase the precision of the observation, and to check for possible mistakes.

24.27 A highway curve (arc definition) to the right, having R = 500 m and I = 18°30′, will be laid out by coordinates with a total station instrument setup at the PI. The PI station is 3 + 855.200 m, and its coordinates are X = 75,428.863 m and Y = 36,007.434 m. The azimuth (from north) of the back tangent proceeding toward the PI is 48°17′12″. To orient the total station, a backsight will be made on a POT on the back tangent. Compute lengths and azimuths necessary to stake the curve at 30-m stations.

Station	δ_a	**Total** δ_a	**Chord**	**Chord Azimuth**
3+935.213	0°17'55"	9°15'00"	160.743	57°32'12"
3+930.000	1°43'08"	8°57'05"	155.595	57°14'17"
3+900.000	1°43'08"	7°13'57"	125.895	55°31'09"
3+870.000	1°43'08"	5°30'49"	96.082	53°48'01"
3+840.000	1°43'08"	3°47'41"	66.182	52°04'53"
3+810.000	1°43'08"	2°04'33"	36.222	50°21'45"
3+780.000	0°21'25"	0°21'25"	6.23	48°38'37"
3+773.770				48°17'12"

24.28 In Problem 24.27, compute the XY coordinates at 30-m stations.

Station	Azimuth	Chord	X	Y
3+935.213	57°32'12"	160.743	75,503.701	36,039.530
3+930.000	57°14'17"	155.595	75,498.921	36,037.450
3+900.000	55°31'09"	125.895	75,471.854	36,024.523
3+870.000	53°48'01"	96.082	75,445.611	36,009.996
3+840.000	52°04'53"	66.182	75,420.287	35,993.922
3+810.000	50°21'45"	36.222	75,395.971	35,976.357
3+780.000	48°38'37"	6.230	75,372.753	35,957.366
3+773.770	48°17'12"		75,368.077	35,953.250

24.29 A exercise track must consist of two semicircles and two tangents, and be exactly 1500 ft along its centerline. The two tangent sections are 200 ft each. Calculate L, R, and D_a for the curves.

Curves = 1500 − 400 = 1100 ft.

R = 1100/(2π) = **143.24 ft**

D_a = 5729.58/143.24 = **40°00′**

L = 143.24(40*π/180) = **100.00 ft**

24.30 Make the computations necessary to lay out the curve of Problem 24.8 by the tangent offset method. Approximately half the curve is to be laid out from the PC and the other half from the PT.

R 2292.01

Station	Chord	δ	TD	TO
61+63.67	63.67	0°47'45"	63.66	0.88
61+00.00	163.64	2°2'45"	163.54	5.84
60+00.00	263.54	3°17'45"	263.11	15.15
59+00.00	363.31	4°32'45"	362.17	28.79
58+00.00	236.24	2°57'15"	235.93	12.18
57+00.00	136.32	1°42'15"	136.26	4.05
56+00.00	36.33	0°27'15"	36.33	0.29
55+63.67				

What sight distance is available if there is an obstruction on a radial line through the PI inside the curves in Problems 24.31 and 24.32?

***24.31** For Problem 24.7, obstacle 10 ft from curve.

By Equation 24.24: $C = \sqrt{8(10)1521.13} = 349$ ft

24.32 For Problem 24.12, obstacle 10 m from curve.

By Equation 24.24: $C = \sqrt{8(10)400} = 179$ m

24.33 If the misclosure for the curve of Problem 24.7, computed as described in Section 24.8, is 0.12 ft, what is the field layout precision?

Precision = 0.12/[2(327.95) + 646.02], or 1:10,800

24.34 Assume that a 150-ft entry spiral will be used with the curve of Problem 24.7. Compute and tabulate curve notes to stake out the alignment from the TS to ST at full stations using a total station and the deflection-angle, total chord method.

```
                Spiral Angle: 2°49'30"
                Spiral Throw: 0.62
         Spiral Long Tangent: 100.01
        Spiral Short Tangent: 50.01
               Spiral Length: 150.00
    Spiral Long Chord Length: 149.98

   Exit spiral notes for layout from ST to CS
           with tangent as backsight.

          Station |      Chord |       Defl. Angle |
       ==============================================
ST       33+57.14 |                                |
         33+00.00 |      57.14 |          0°08'12" |
CS       32+07.14 |      92.86 |          0°56'30" |
```

Station	Chord	Defl. Increment	Defl. Angle
32+07.14	7.14	0°08'04"	9°20'30"
32+00.00	99.98	1°53'00"	9°12'26"
31+00.00	99.98	1°53'00"	7°19'26"
30+00.00	99.98	1°53'00"	5°26'26"
29+00.00	99.98	1°53'00"	3°33'26"
28+00.00	88.87	1°40'26"	1°40'26"
27+11.12			

	Station	Chord	Defl. Angle
SC	27+11.12	11.12	0°56'30"
	27+00.00	99.99	0°48'26"
	26+00.00	38.88	0°03'48"
TS	25+61.12		

24.35 Same as Problem 24.34, except use a 300-ft spiral for the curve of Problem 24.8.

```
                Spiral Angle: 3°44'59"
                Spiral Throw: 1.64
         Spiral Long Tangent: 200.04
        Spiral Short Tangent: 100.04
               Spiral Length: 300.00
    Spiral Long Chord Length: 299.94
```

Exit spiral notes for layout from ST to CS
with tangent as backsight.

	Station	Chord	Defl. Angle
ST	63+13.52		
	63+00.00	13.52	0°00'09"
	62+00.00	100.00	0°10'44"
	61+00.00	100.00	0°37'59"
CS	60+13.52	86.47	1°15'00"

Horizontal Curve Notes -- Chord Definition

```
Intersection Angle = 15°00'00" (Back to Forward Tangent)
Circular Curve Intersection Angle = 7°30'02"
              Degree of Curvature = 2°30'00"
                           Radius = 2,292.01
            Circular Curve Length = 300.05
         Tangent Distance (TS-PI) = 451.94
        Circular Curve Long Chord = 299.83
            Long Chord (TS - ST) = 896.15
                         External = 26.38
  Circular Curve Tangent Distance = 150.24

                  PI Stationing = 58+65.42
                  63+13.52 Back = 63+17.36 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
60+13.52	13.53	0°10'09"	3°45'01"
60+00.00	100.00	1°15'00"	3°34'52"
59+00.00	100.00	1°15'00"	2°19'52"
58+00.00	86.52	1°04'54"	1°04'52"
57+13.48			

	Station	Chord	Defl. Angle
SC	57+13.48	13.48	1°14'59"
	57+00.00	100.00	1°08'24"
	56+00.00	100.00	0°28'59"
	55+00.00	86.52	0°06'14"
TS	54+13.48		

24.36 Same as Problem 24.34, except for the curve of Problem 24.9, with a 50-m entry spiral using stationing of 30 m and a total station instrument.

```
                Spiral Angle: 1°47'26"
                Spiral Throw: 0.130
         Spiral Long Tangent: 33.335
        Spiral Short Tangent: 16.668
               Spiral Length: 50.000
    Spiral Long Chord Length: 49.998
```

Exit spiral notes for layout from ST to CS
with tangent as backsight.

	Station	Chord	Defl. Angle
ST	3+390.070		
	3+390.000	0.070	0°00'00"
	3+360.000	30.000	0°12'57"
CS	3+340.070	19.929	0°35'49"

Defining Curve Parameters

```
   Intersection Angle = 12°00'00" (Back to Forward Tangent)
Circular Curve Intersection Angle = 8°25'08"
              Degree of Curvature = 2°10'59"
                           Radius = 800.000
            Circular Curve Length = 117.552
          Tangent Distance (TS-PI) = 109.096
        Circular Curve Long Chord = 117.446
            Long Chord (TS - ST) = 216.997
                         External = 4.930
   Circular Curve Tangent Distance = 58.882

                     PI Stationing = 3+281.615
              3+390.070 Back = 3+390.711 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
3+340.070	10.070	0°21'38"	4°12'34"
3+330.000	29.998	1°04'27"	3°50'56"

```
      3+300.000 |     29.998 |          1°04'27" |          2°46'29" |
      3+270.000 |     29.998 |          1°04'27" |          1°42'01" |
      3+240.000 |     17.481 |          0°37'34" |          0°37'34" |
      3+222.519
======================================================================

                 Station |      Chord |       Defl. Angle |
              =============================================
 SC            3+222.519 |     12.519 |          0°35'49" |
               3+210.000 |     30.000 |          0°20'07" |
               3+180.000 |      7.481 |          0°00'48" |
 TS            3+172.519 |                                |
```

24.37 Compute the area bounded by the two arcs and tangent in Problem 24.24.

<u>306,460 ft²</u>

Area of parallelogram; = 1,699,285

Area of Sector R = 305,414

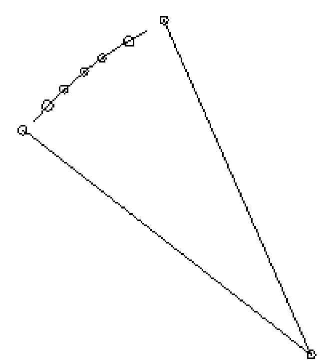
Area of Sector R_x = 1,087,408

Area = 1699285 − 305,414 - 1,087,408 = 306,460

24.38 In an as-built survey, the *XY* coordinates in meters of three points on the centerline of a highway curve are determined to be *A*: (295.338, 419.340); *B*: (312.183, 433.927); *C*: (326.969, 445.072). What are the radius, and coordinates for the center of the curve in meters?

```
Center of Circle at: X = 500.670
                     Y = 199.244
With A Radius of   : 301.004
```

24.39 In Problem 24.38, if the (*x, y*) coordinates in meters of two points on the centerline of the tangents are (262.066, 384.915) and (378.361, 476.370), what are the coordinates of the PC, PT, and the curve parameters *L*, *T*, and *I*?



PC @ (**<u>281.476,405.538</u>**) PT @ (**<u>349.006, 459.247</u>**)

PC and PT can be reversed.

I = **<u>16°28′51″</u>** *L* = **<u>86.582</u>** m *T* = **<u>43.592</u>** m

Angles: T1–O–PC: 5°22′30″; PT–O–T2: 6°26′30″

Azimuths: O–PC: 313°15′49″; O–PT: 329°44′51″

Distances: O–T1: 302.333 m; T1–PC: 28.321 m

O–T2: 302.916 m; T2–PT: 33.984

24.40 Write a computational program to calculate circular curve notes for layout with a total station by deflection angles and total chords from a set up at the PC.

Independent

24.41 Develop a computational program to calculate the coordinates of the stations on a circular curve.

Independent

25 VERTICAL CURVES

Asterisks (*) indicate problems that have answers given in Appendix G.

25.1 Why are vertical curves needed on the grade lines for highways and railroads?

From Section 25.1, paragraph 1: Curves are needed to provide smooth transitions between straight segments (tangents) of grade lines for highways and railroads.

25.2 What is meant by the “rate of grade change” on vertical curves and why is it important?

From Section 25.3, paragraph 4: "The rate of change of grade, r, for an equal-tangent parabolic curve equals the total grade change from BVC to EVC divided by length *L* (in stations for the English system, or 1/10th stations for metric units), over which the change occurs"

Tabulate station elevations for an equal-tangent parabolic curve for the data given in Problems 25.3 through 25.8. Check by second differences.

25.3 A +1.55% grade meets a −2.50% grade at station 44+25 and elevation 682.34 ft, 800-ft curve, stakeout at half stations.

```
BVC Station = 40+25.00
BVC Elevation = 676.14
```

Station	x (Sta)	g1*x	r/2*x*x	Elevation
48+25.00	8.00	12.40	-16.20	672.34
48+00.00	7.75	12.01	-15.20	672.95
47+50.00	7.25	11.24	-13.30	674.07
47+00.00	6.75	10.46	-11.53	675.07
46+50.00	6.25	9.69	-9.89	675.94
46+00.00	5.75	8.91	-8.37	676.68
45+50.00	5.25	8.14	-6.98	677.30
45+00.00	4.75	7.36	-5.71	677.79
44+50.00	4.25	6.59	-4.57	678.16
44+00.00	3.75	5.81	-3.56	678.39
43+50.00	3.25	5.04	-2.67	678.50
43+00.00	2.75	4.26	-1.91	678.49
42+50.00	2.25	3.49	-1.28	678.35
42+00.00	1.75	2.71	-0.78	678.08
41+50.00	1.25	1.94	-0.40	677.68
41+00.00	0.75	1.16	-0.14	677.16
40+50.00	0.25	0.39	-0.02	676.51
40+25.00	0	0	0	676.14

```
Maximum elevation = 678.51 @ station 43+31.17
```

25.4 A −2.50% grade meets a +2.50% grade at station 4 + 200 and elevation 293.585 m, 300-m curve, stakeout at 30-m increments.

BVC Station = 4+50.000
BVC Elevation = 297.335

Station	x (Sta)	g1*x	r/2*x*x	Elevation
4+350.000	3.000	-7.500	7.500	297.335
4+320.000	2.700	-6.750	6.075	296.660
4+290.000	2.400	-6.000	4.800	296.135
4+260.000	2.100	-5.250	3.675	295.760
4+230.000	1.800	-4.500	2.700	295.535
4+200.000	1.500	-3.750	1.875	295.460
4+170.000	1.200	-3.000	1.200	295.535
4+140.000	0.900	-2.250	0.675	295.760
4+110.000	0.600	-1.500	0.300	296.135
4+80.000	0.300	-0.750	0.075	296.660
4+50.000	0.000	-0.000	0.000	297.335

Minimum elevation = 295.460 @ station 4+200.000

25.5 A 375-ft curve, grades of $g_1 = -2.60\%$ and $g_2 = +0.90\%$, VPI at station 36 + 40, and elevation 605.35 ft, stakeout at full stations.

BVC Station = 34+52.500
BVC Elevation = 610.225

Station	x (Sta)	g1*x	r/2*x*x	Elevation
38+27.500	3.750	-9.750	6.563	607.038
38+00.000	3.475	-9.035	5.635	606.825
37+00.000	2.475	-6.435	2.859	606.649
36+00.000	1.475	-3.835	1.015	607.405
35+00.000	0.475	-1.235	0.105	609.095
34+52.500	0	0	0	610.225

Minimum elevation = 606.604 @ station 37+31.071

25.6 A 450-ft curve, grades of $g_1 = -4.00\%$ and $g_2 = -3.00\%$, VPI at station 66 + 50, and elevation 560.00 ft, stakeout at full stations.

BVC Station = 63+70.000
BVC Elevation = 571.200

Station	x (Sta)	g1*x	r/2*x*x	Elevation
69+30.000	5.600	-22.400	2.800	551.600
69+00.000	5.300	-21.200	2.508	552.508
68+00.000	4.300	-17.200	1.651	555.651
67+00.000	3.300	-13.200	0.972	558.972
66+00.000	2.300	-9.200	0.472	562.472
65+00.000	1.300	-5.200	0.151	566.151
64+00.000	0.300	-1.200	0.008	570.008
63+70.000	0	0	0	571.200

==

25.7 A 150-m curve, g_1 = +3.00%, g_1 = −2.00%, VPI station = 2 + 175, VPI elevation = 157.830 m, stakeout at 30-m increments.

BVC Station = 2+100.000
BVC Elevation = 155.580

Station	x (Sta)	g1*x	r/2*x*x	Elevation
2+250.000	1.500	4.500	-3.750	156.330
2+220.000	1.200	3.600	-2.400	156.780
2+190.000	0.900	2.700	-1.350	156.930
2+160.000	0.600	1.800	-0.600	156.780
2+130.000	0.300	0.900	-0.150	156.330
2+100.000	0.000	0.000	-0.000	155.580

Maximum elevation = 156.930 @ station 2+190.000

25.8 A 200-ft curve, g_1 = −1.50%, g_2 = +2.50%, VPI station = 46 + 00, VPI elevation = 895.00 ft, stakeout at quarter stations.

BVC Station = 45+00.000
BVC Elevation = 896.500

Station	x (Sta)	g1*x	r/2*x*x	Elevation
47+00.000	2.000	-3.000	4.000	897.500
46+75.000	1.750	-2.625	3.063	896.938
46+50.000	1.500	-2.250	2.250	896.500
46+25.000	1.250	-1.875	1.563	896.188
46+00.000	1.000	-1.500	1.000	896.000
45+75.000	0.750	-1.125	0.563	895.938
45+50.000	0.500	-0.750	0.250	896.000
45+25.000	0.250	-0.375	0.063	896.188
45+00.000	0.000	-0.000	0.000	896.500

Minimum elevation = 895.938 @ station 45+75.000

25.9 An 90-m curve, $g_1 = -1.50\%$, $g_2 = +0.75\%$, VPI station = 6 + 280, VPI elevation = 550.600 m, stakeout at 10-m increments.

BVC Station = 6+235.000
BVC Elevation = 551.275

Station	x (Sta)	g1*x	r/2*x*x	Elevation
6+325.000	0.900	-1.350	1.012	550.938
6+320.000	0.850	-1.275	0.903	550.903
6+310.000	0.750	-1.125	0.703	550.853
6+300.000	0.650	-0.975	0.528	550.828
6+290.000	0.550	-0.825	0.378	550.828
6+280.000	0.450	-0.675	0.253	550.853
6+270.000	0.350	-0.525	0.153	550.903
6+260.000	0.250	-0.375	0.078	550.978
6+250.000	0.150	-0.225	0.028	551.078
6+240.000	0.050	-0.075	0.003	551.203
6+235.000	0	0	0	551.275

Minimum elevation = 550.825 @ station 6+295.000

Field conditions require a highway curve to pass through a fixed point. Compute a suitable equal-tangent vertical curve and full-station elevations for Problems 25.10 through 25.12.

***25.10** Grades of $g_1 = -2.50\%$ and $g_2 = +1.00\%$, VPI elevation 750.00 ft at station 30 + 00. Fixed elevation 753.00 ft at station 30 + 00.

L = 685.714 ft   reduced Equation (25.3): $0.4375L = 3$

BVC Station = 26+57.143
BVC Elevation = 758.571

Station	x (Sta)	g1*x	r/2*x*x	Elevation
33+42.857	6.857	-17.143	12.000	753.429
33+00.000	6.429	-16.071	10.547	753.047
32+00.000	5.429	-13.571	7.521	752.521
31+00.000	4.429	-11.071	5.005	752.505
30+00.000	**3.429**	**-8.571**	**3.000**	**753.000**
29+00.000	2.429	-6.071	1.505	754.005
28+00.000	1.429	-3.571	0.521	755.521
27+00.000	0.429	-1.071	0.047	757.547
26+57.143	0	0	0	758.571

Minimum elevation = 752.449 @ station 31+46.939

25.11 Grades of $g_1 = -2.50\%$ and $g_2 = +1.50\%$, VPI elevation 2560.00 ft at station 315 + 00 Fixed elevation 2567.00 ft at station 314 + 00.

L = 1268.46 ft

$2567 = 2560 + 2.5(L/2) - 2.5(L/2 - 1) + (1.5 + 2.5)/(2L)\ (L/2 - 1)^2$

$L^2 - 13L + 4 = 0$

BVC Station = 308+65.77
BVC Elevation = 2575.86

Station	x (Sta)	g1*x	r/2*x*x	Elevation
321+34.23	12.68	-31.71	25.37	2,569.51
321+00.00	12.34	-30.86	24.02	2,569.02
320+00.00	11.34	-28.36	20.28	2,567.78
319+00.00	10.34	-25.86	16.87	2,566.87
318+00.00	9.34	-23.36	13.76	2,566.26
317+00.00	8.34	-20.86	10.97	2,565.97
316+00.00	7.34	-18.36	8.50	2,566.00
315+00.00	6.34	-15.86	6.34	2,566.34
314+00.00	**5.34**	**-13.36**	**4.50**	**2,567.00**
313+00.00	4.34	-10.86	2.97	2,567.97
312+00.00	3.34	-8.36	1.76	2,569.26
311+00.00	2.34	-5.86	0.87	2,570.87
310+00.00	1.34	-3.36	0.28	2,572.78
309+00.00	0.34	-0.86	0.02	2,575.02
308+65.77	0	0	0	2,575.86

Minimum elevation = 2,565.95 @ station 316+58.56

25.12 Grades of $g_1 = +5.00\%$ and $g_2 = +1.50\%$ VPI station 6+300 and elevation 185.920 m. Fixed elevation 185.610 m at station 6+400. (Use 100-m stationing)

L = 761.163 m

$185.610 = 185.920 - 5(L/2) + 5(L/2 + 1) + (1.50 - 5)/(2L)(L/2 + 1)^2$

$-0.4375L^2 + 3.56L - 1.75$

BVC Station = 5+919.418
BVC Elevation = 166.891

Station	x (Sta)	g1*x	r/2*x*x	Elevation
6+680.582	7.612	38.058	-13.320	191.629
6+600.000	6.806	34.029	-10.649	190.271
6+500.000	5.806	29.029	-7.750	188.170
6+400.000	4.806	24.029	-5.310	185.610
6+300.000	3.806	19.029	-3.330	182.590
6+200.000	2.806	14.029	-1.810	179.110
6+100.000	1.806	9.029	-0.750	175.170
6+000.000	0.806	4.029	-0.149	170.771
5+919.418	0	0	0	166.891

25.13 A −1.10% grade meets a +0.90% grade at station 36 + 00 and elevation 800.00 ft. The +0.90% grade then joins a +1.50% grade at station 39 + 00. Compute and tabulate the notes for an equal-tangent vertical curve, at half-stations, that passes through the midpoint of the 0.90% grade.

Midpoint of curve = 37 + 50

Elevation at 37+50 =800.00 + 0.9(1.5) = 801.35 ft

Setup simultaneous equations to solve for station of VPI and elevation

Along g_1: $\Delta e_1 = -1.1x$ so $x = -e_1/(1.1)$

Along g_3: $\Delta e_2 = 1.5(3 - x)$

Along g_2: $\Delta e_1 + e_2 = 0.9(3)$

So $\Delta e_2 = 1.5(3 + \Delta e_1/1.1)$ and substitute this into last equation to solve for Δe_1.

Solve equation for Δe_1 as −0.7615, so $x = 0.7615/1.1 = 0.69231$ or 69.23 ft

VPI at 36+69.23 with elevation of 799.238

Setup equation as

$$799.238 - 801.35 + 1.1\left(\frac{L}{2}\right) - 1.1\cdot\left(\frac{L}{2} + 0.8077\right) + \left(\frac{1.5 + 1.1}{2\cdot L}\right)\cdot\left(\frac{L}{2} + 0.8077\right)^2$$

L = 552.948 ft

```
BVC Station = 33+92.756
BVC Elevation = 802.279
```

Station	x (Sta)	g1*x	r/2*x*x	Elevation
39+45.704	5.529	-6.082	7.188	803.385
39+00.000	5.072	-5.580	6.049	802.749
38+50.000	4.572	-5.030	4.915	802.165
38+00.000	4.072	-4.480	3.899	801.699
37+50.000	**3.572**	**-3.930**	**3.000**	**801.350**
37+00.000	3.072	-3.380	2.219	801.119
36+50.000	2.572	-2.830	1.556	801.005
36+00.000	2.072	-2.280	1.010	801.009
35+50.000	1.572	-1.730	0.581	801.131
35+00.000	1.072	-1.180	0.270	801.370
34+50.000	0.572	-0.630	0.077	801.727
34+00.000	0.072	-0.080	0.001	802.201
33+92.756	0	0	0	802.279

```
Minimum elevation = 800.993 @ station 36+26.696
```

25.14 When is it advantageous to use an unequal-tangent vertical curve instead of an equal-tangent one?

From Section 26.8, paragraph 1: They are used to enable the vertical curve to closely fit the ground conditions, which is used to minimize excessive cut or fill quantities.

Compute and tabulate full-station elevations for an unequal-tangent vertical curve to fit the requirements in Problems 25.15 through 25.18.

25.15 A +4.00% grade meets a −2.00% grade at station 60+00 and elevation 1086.00 ft. Length of first curve 500 ft, second curve 400 ft.

BVC Station = 55+00.000
BVC Elevation = 1066.000

Station	x (Sta)	g1*x	r/2*x²	Elevation
64+00.000	4.000	5.333	-6.667	1,078.000
63+00.000	3.000	4.000	-3.750	1,079.583
62+00.000	2.000	2.667	-1.667	1,080.333
61+00.000	1.000	1.333	-0.417	1,080.250
60+00.000	0.000	0.000	-0.000	1,079.333
CVC				
60+00.000	5.000	20.000	-6.667	1,079.333
59+00.000	4.000	16.000	-4.267	1,077.733
58+00.000	3.000	12.000	-2.400	1,075.600
57+00.000	2.000	8.000	-1.067	1,072.933
55+00.000	0	0	0	1,066.000

25.16 Grade g_1 = +1.25%, g_1 = +3.50%, VPI at station 62+00 and elevation 650.00 ft, L_1 = 600 ft and L_2 = 600 ft.

BVC Station = 56+00.000
BVC Elevation = 642.500

Station	x (Sta)	g1*x	r/2*x²	Elevation
68+00.000	6.000	14.250	3.375	671.000
67+00.000	5.000	11.875	2.344	667.594
66+00.000	4.000	9.500	1.500	664.375
65+00.000	3.000	7.125	0.844	661.344
64+00.000	2.000	4.750	0.375	658.500
63+00.000	1.000	2.375	0.094	655.844
62+00.000	0.000	0.000	0.000	653.375
CVC				
62+00.000	6.000	7.500	3.375	653.375
61+00.000	5.000	6.250	2.344	651.094
60+00.000	4.000	5.000	1.500	649.000
59+00.000	3.000	3.750	0.844	647.094
58+00.000	2.000	2.500	0.375	645.375
56+00.000	0	0	0	642.500

25.17 Grades g_1 of +4.00% and g_2 of −2.00% meet at the VPI at station 4+300 and elevation 154.960 m. Lengths of curves are 100 m and 200 m. (Use 30-m stationing.)

```
BVC Station = 4+200.000
BVC Elevation = 150.960
```

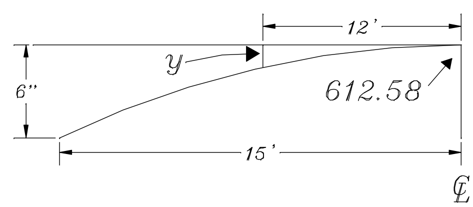
Station	x (Sta)	g1*x	r/2*x²	Elevation
4+500.000	2.000	-0.000	-2.000	150.960
4+470.000	1.700	-0.000	-1.445	151.515
4+440.000	1.400	-0.000	-0.980	151.980
4+410.000	1.100	-0.000	-0.605	152.355
4+380.000	0.800	-0.000	-0.320	152.640
4+350.000	0.500	-0.000	-0.125	152.835
4+320.000	0.200	-0.000	-0.020	152.940
CVC				
4+300.000	1.000	4.000	-2.000	152.960
4+260.000	0.600	2.400	-0.720	152.640
4+230.000	0.300	1.200	-0.180	151.980
4+200.000	0	0	0	150.960

25.18 A −1.80% grade meets a +3.00% grade at station 95 + 00 and elevation 320.64 ft. Length of first curve is 300 ft, of second curve, 200 ft.

```
BVC Station = 92+00.000
BVC Elevation = 326.040
```

Station	x (Sta)	g1*x	r/2*x²	Elevation
97+00.000	2.000	0.240	2.880	326.640
96+00.000	1.000	0.120	0.720	324.360
95+00.000	0.000	0.000	0.000	323.520
CVC				
95+00.000	3.000	-5.400	2.880	323.520
94+00.000	2.000	-3.600	1.280	323.720
93+00.000	1.000	-1.800	0.320	324.560
92+00.000	0	0	0	326.040

***25.19** A manhole is 12 ft from the centerline of a 30-ft wide street that has a 6-in. parabolic crown. The street center at the station of the manhole is at elevation 612.58 ft. What is the elevation of the manhole cover?



Elev = **612.26 ft**

$$y = \left(\frac{12}{15}\right)^2 6 = 3.84\,\text{in.} = \ 0.32\text{ ft}$$

25.20 A 60-ft wide street has an average parabolic crown from the center to each edge of 1/4 in./ft. How much does the surface drop from the street center to a point 6 ft from the edge?

5.63 in. = 0.47 ft

For 30 ft, drop is 30(1/4) = 7.5 in.

Drop at 4 ft is $\left(\frac{30-4}{30}\right)^2 7.5$

25.21 Determine the station and elevation at the high point of the curve in Problem 25.3.

43 + 31.17 @ 678.51 ft

25.22 Calculate the station and elevation at the low point of the curve in Problem 25.4.

4 + 200 @ 295.460 m

25.23 Compute the station and elevation at the low point of the curve of Problem 25.5.

37 + 31.07 @ 606.60 ft

25.24 What are the station and elevation of the high point of the curve of Problem 25.7?

2+190 @ 156.930 m

25.25 What additional factor must be considered in the design of crest vertical curves that is not of concern in sag curves?

From Section 25.11: **Sight distance**

***25.26** Compute the sight distance available in Problem 25.3. (Assume h_1 = 3.50 ft and h_2 = 4.25 ft.)

781.61 ft

25.27 Similar to Problem 25.26, except h_2 = 2.00 ft.

652.94 ft

25.28 Similar to Problem 25.26, except for the data of Problem 25.7, where h_1 = 1.0 m and h_2 = 0.5 m.

132.232 m

25.29 In determining sight distances on vertical curves, how does the designer determine whether the cars or objects are on the curve or tangent?

Try either formula in Section 25.11 and compare the derived sight distance with the length of the curve. If the derived sight distance does not fit on the curve, then use the other formula.

What is the minimum length of a vertical curve to provide a required sight distance for the conditions given in Problems 25.30 through 25.32?

***25.30** Grades of +3.00 ft and −2.50%, sight distance 600 ft, h_1 = 3.50 ft and h_2 = 1.25 ft.

1108.2 ft

25.31 A crest curve with grades of +4.50% and −3.00% sight distance 500 ft, h_1 = 4.25 ft and h_2 = 1.00 ft.

1000.2 ft

25.32 Sight distance of 200 m, grades of +1.00% and −2.25%, h_1 = 1.1 m and h_2 = 0.3 m.

255 m

***25.33** A backsight of 6.85 ft is taken on a benchmark whose elevation is 567.50 ft. What rod reading is needed at that HI to set a blue top at grade elevation of 572.55 ft?

1.80 ft = 567.50 + 6.85 − 572.55

25.34 A backsight of 6.92 ft is taken on a benchmark whose elevation is 867.50 ft. A foresight of 3.64 ft and a backsight of 7.04 ft are then taken in turn on TP_1 to establish a HI. What rod reading will be necessary to set a blue top at a grade elevation of 872.06 ft?

5.76 ft = 867.50 + 6.92 − 3.64 + 7.04 − 872.06

25.35 Develop a computational program that performs the vertical-curve computations.

Independent project

26 VOLUMES

Asterisks $^{(*)}$ indicate problems that have answers given in Appendix G.

26.1 Why must cut and fill volumes be totaled separately?

From Section 26.2, last paragraph and Section 26.9, paragraph 3: To balance cuts and fills volumes so that materials is kept on site as much as possible and since contractors are generally paid for cuts only.

26.2 Prepare a table of end areas versus depths of fill from 0 to 20 ft by increments of 4 ft for level sections, a 36-ft wide level roadbed, and side slopes of 1-1/2:1.

Area = $1.5h^2 + 36h$

Fill Depth (ft)	**End Area (ft^2)**
0	0
4	168
8	384
12	648
16	960
20	1320

26.3 Similar to Problem 26.2, except use side slopes of 2-1/2:1.

Area = $2.5h^2 + 36h$

Fill Depth (ft)	**End Area (ft^2)**
0	0
4	184
8	448
12	792
16	1216
20	1720

Draw the cross sections and compute V_e for the data given in Problems 26.4 through 26.7.

***26.4** Two level sections 75 ft apart with center heights 4.8 and 7.2 ft in fill, base width 30 ft, side slopes 2:1.

<u>708 yd^3 = 19,116 ft^3</u>

End areas = 190.1 and 319.7 ft^2

26.5 Two level sections of 40-m stations with center heights of 2.04 and 2.53 m. in cut, base width 15 m, side slopes 3:1.

2005 m^3

End areas = 43.1 and 57.2 m^2

26.6 The end area at station $36+00$ is $265\ \text{ft}^2$. Notes giving distance from centerline and cut ordinates for station $36+60$ are C 4.8/17.2; C 5.9/0; C 6.8/20.2. Base is 20 ft.

481.5 yd^3 = 13,000 ft^3

x	y	−	+
−10	0		0
−17.2	4.8	−48	0
0	5.9	−101.48	119.18
20.2	6.8	0	68
10	0	0	0
−10	0	0	
		−149.48	187.18

End Area @ 36 + 00 = 168.3 ft^2

26.7 An irrigation ditch with b = 12 ft and side slopes of 2:1. Notes giving distances from centerline and cut ordinates for stations $52+00$ and $53+00$ are C 2.4/10.8; C 3.0; C 3.7/13.4; and C 3.1/14.2; C 3.8; C 4.1/14.2.

261 yd^3 = 7048 ft^3

x	y	-	+
-6	0		0
-2.4	10.8	-64.8	0
0	3	-7.2	40.2
13.4	3.1	0	18.6
6	0	0	0
-6	0	0	
		-72	58.8

x	y	-	+
-6	0		0
-14.2	3.1	-18.6	0
0	3.8	-54.0	54.0
14.2	4.1	0	24.6
6	0	0	0
-6	0	0	
		-72.6	78.6

End areas: 52 + 00 = 65.4 ft^2 and 53 + 00 = 75.6 ft^2

26.8 Why is a roadway in cut normally wider than the same roadway in fill?

From Section 26.4, paragraph 2: The roadway is usually wider in cut than on fills to provide for drainage ditches.

***26.9** For the data tabulated, calculate the volume of excavation in cubic yards between stations $10+00$ and $15+00$.

6168.5 yd^3

Station	Cut End Area (ft^2)	Volume (yd^3)
10 + 00	263	
11 + 00	358	1150.0
12 + 00	446	1488.9
13 + 00	402	1570.4
14 + 00	274	1251.9
15 + 00	108	707.4

26.10 For the data listed, tabulate cut, fill, and cumulative volumes in cubic yards between stations $10+00$ and $20+00$. Use an expansion factor of 1.30 for fills.

	End Area (ft^2)		Volumes (yd^3)			
Station	Cut	Fill	Cut	Fill	1.30 Fill	Cumulative
10 + 00	0					
11 + 00	168		311.1			311.1
12 + 00	348		955.6			1266.7
13 + 00	371		1331.5			2598.1
14 + 00	146		957.4			3555.6
14 + 60	0	0	162.2			3717.8
15 + 00		142		105.2	136.7	3581.0
16 + 00		238		703.7	914.8	2666.2
17 + 00		305		1005.6	1307.2	1359.0
18 + 00		247		1022.2	1328.9	30.1
19 + 00		138		713.0	926.9	−896.7
20 + 00		106		451.9	587.4	−1484.1

26.11 Calculate the section areas in Problem 26.4 by the coordinate method.

Areas: **190.1 ft^2** and **319.7 ft^2**

x	y	−	+
−15	0		0
−24.6	4.8	−72.0	118.1
24.6	4.8	−118.1	72.0
15	0	0	0
−15	0	0	
		−190.1	190.1

x	y	−	+
−15	0		0
−29.4	7.2	−108.0	211.7
29.4	7.2	−211.7	108.0
15	0	0	0
−15	0	0	
		−319.7	319.7

26.12 Compute the section areas in Problem 26.5 by the coordinate method.

Areas: **43.1 m³** and **57.2 m³**

x	y	−	+
−7.5	0		0
−13.62	2.04	−15.3	27.8
13.62	2.04	−27.8	15.3
7.5	0	0	0
−7.5	0	0	
		−43.1	43.1

x	y	−	+
−7.5	0		0
−15.09	2.53	−19.0	38.2
15.09	2.53	−38.2	19.0
7.5	0	0	0
−7.5	0	0	
		−57.2	57.2

26.13 Determine the section areas in Problem 26.7 by the coordinate method.

See solution to problem 26.7

***26.14** Compute C_P and V_P for Problem 26.4. Is C_P significant?

C_p = 3 yd³, V_p = 705 yd³; represents on 0.3% of volume, so not significant.

By Equation (26.4): $C_p = \frac{75}{12(27)}(4.8-7.2)(39.6-44.4) = 2.7 \text{ yd}^3$

26.15 Calculate C_P and V_P for Problem 26.7. Would C_P be significant in rock cut?

C_p = 1 yd³, V_p = 240 yd³; represents on 0.4% of volume, so not significant.

By Equation (26.4): $C_p = \frac{100}{12(27)}(3.0-3.8)(36.2-40.4) = 1.0 \text{ yd}^3$

26.16 From the following excerpt of field notes, plot the cross section on graph paper and superimpose on it a design template for a 30-ft wide level roadbed with fill slopes of 2-1/2:1 and a subgrade elevation at centerline of 970.30 ft. Determine the end area graphically by counting squares.

HI = 968.31 ft						
20 + 00 Lt	$\frac{5.2}{50}$	$\frac{4.8}{22}$	$\frac{6.6}{0}$	$\frac{5.9}{12}$	$\frac{7.0}{30}$	$\frac{8.1}{50}$

399 ft²

26.17 For the data of Problem 26.16, determine the end area by plotting the points in a CAD package, and listing the area.

399 ft²

26.18 For the data of Problem 26.16, calculate slope intercepts, and determine the end area by the coordinate method.

399 ft^2

```
From WolfPack: Station: 46+00  Roadbed elevation:    970.300
-32.34    -22.00      0.00     12.00     30.00     38.67     15.00    -15.00
963.36    963.51    961.71    962.41    961.31    960.83    970.30    970.30
Fill End Area = 398.8
```

26.19 From the following excerpt of field notes, plot the cross section on graph paper and superimpose on it a design template for a 40-ft wide level roadbed with cut slopes of 3:1 and a subgrade elevation of 1239.50 ft. Determine the end area graphically by counting squares.

HI = 1254.80 ft						
46 + 00 Lt	$\frac{8.0}{60}$	$\frac{7.9}{27}$	$\frac{5.5}{10}$	$\frac{4.9}{0}$	$\frac{6.6}{24}$	$\frac{7.5}{60}$

579.5 ft^2

26.20 For the data of Problem 26.19, calculate slope intercepts and determine the end area by the coordinate method.

579.5 ft^2

From WOLFPACK

```
Station: 46+00  Roadbed elevation:   1239.500
 -42.06    -27.00    -10.00      0.00     24.00     44.56     20.00    -20.00
1246.85   1246.90   1249.30   1249.90   1248.20   1247.69   1239.50   1239.50
Cut End Area = 579.5
```

***26.21** Complete the following notes and compute V_e and V_P. The roadbed is level, the base is 30 ft.

Station 89 + 00	$\frac{C3.1}{24.3}$	$\frac{C4.9}{0}$	$\frac{C4.3}{27.9}$
Station 88 + 00	$\frac{C6.4}{34.2}$	$\frac{C3.6}{0}$	$\frac{C5.7}{32.1}$

V_e = **728.7 yd^3 = 19,674 ft^3**; V_p = **734.4 yd^3**

89 + 00

x	y	-	+
0	4.9		136.7
27.9	4.3	0	64.5
15	0	0	0.0
-15	0	0	0.0
-24.3	3.1	-46.5	0.0
0	4.9	-119.07	
		-165.57	201.2

88 + 00

x	y	-	+
0	3.6		115.6
32.1	5.7	0.0	85.5
15	0	0.0	0.0
-15	0	0.0	0.0
-34.2	6.4	-96.0	0.0
0	3.6	-123.1	
		-219.1	201.1

Area of 88 + 00 = 183.4 ft^2 Area of 89 + 00 = 210.1 ft^2

$$C_P = \frac{100}{12(27)}(3.6-4.9)(66.3-52.2) = -5.7 \text{ yd}^3$$

26.22 Similar to Problem 26.21, except the base is 24 ft.

V_e = 674.5 yd^3; V_p = 680.2 yd^3

89 + 00

x	y	−	+
0	4.9		136.7
27.9	4.3	0	51.6
12	0	0	0.0
−12	0	0	0.0
−24.3	3.1	−37.2	0.0
0	4.9	−119.07	
		−156.27	188.3

Area 172.3

88 + 00

x	y	−	+
0	3.6		115.6
32.1	5.7	0.0	68.4
12	0	0.0	0.0
−12	0	0.0	0.0
−34.2	6.4	−76.8	0.0
0	3.6	−123.1	
		−199.9	184.0

Area 191.9

Volume 18211.5 674.5

26.23 Calculate V_e and V_P for the following notes. Base is 36 ft.

$$12+90 \quad \frac{C6.4}{43.6} \quad \frac{C3.6}{0} \quad \frac{C5.7}{40.8}$$

$$12+30 \quad \frac{C3.1}{30.4} \quad \frac{C4.9}{0} \quad \frac{C4.3}{35.2}$$

V_e = 202.5 yd^3; V_p = 207.0 yd^3

12 + 90

x	y	−	+
0	3.6		146.9
40.8	5.7	0	102.6
18	0	0	0.0
−18	0	0	0.0
43.6	6.4	−115.2	0.0
0	3.6	156.96	
		41.76	249.5

Area 103.9

12 + 30

x	y	−	+
0	4.9		172.5
35.2	4.3	0.0	77.4
18	0	0.0	0.0
−18	0	0.0	0.0
30.4	3.1	−55.8	0.0
0	4.9	149.0	
		93.2	249.9

Area 78.4

Volume 5466.6 202.5

$$C_P = \frac{60}{12(27)}(4.9-3.6)(65.6-84.4) = -4.5 \text{ yd}^3$$

26.24 Calculate V_e, C_P, and V_P for the following notes. The base in fill is 20 ft and base in cut is 30 ft.

46 + 00	$\frac{C3.4}{20.1}$	$\frac{C2.0}{0}$	$\frac{C0.0}{6.0}$	$\frac{F2.0}{13.0}$
45 + 00	$\frac{C2.2}{18.3}$	$\frac{0.0}{0}$	$\frac{F3.0}{14.5}$	

Cut: **V_e = 401.1 yd^3; C_p = 1.1 yd^3; V_p = 400 yd^3**

Fill: **V_e = 35.2 yd^3; C_p = −1.0 yd^3; V_p = 36.2 yd^3**

46 + 00 cut

x	y	−	+
0	2		12.0
6	0	0	0.0
−15	0	0	0.0
−20.1	3.4	−51	0.0
0	2	−40.2	
		−91.2	12.0

45 + 00 cut

x	y	−	+
0	0		0.0
−15	0	0.0	0.0
−18.3	22	−330.0	0.0
0	0	0.0	0.0
		−330.0	0.0

Cut area	51.6	Cut area	165
Fill area	4	Fill area	15

46 + 00 fill

6	0		0.0
13	2	12	20.0
10	0	0	0.0
6	0	0	
		12	20.0

45 + 00 fill

0	0		0
14.5	3	0	30
10	0	0	0
0	0	0	
		0	30

	ft^3	yd^3
Cut Volume	10830	401.1
Fill Volume	950	35.2

Cut: $C_P = \frac{100}{12(27)}(0-2.0)(18.3-20.1) = 1.1 \text{ yd}^3$

Fill: $C_P = \frac{100}{12(27)}(0-2.0)(14.5-13) = -1.0 \text{ yd}^3$

For Problems 26.25 and 26.26, compute the reservoir capacity (in acre-ft) between highest and lowest contours for areas on a topographic map.

***26.25**

Elevation (ft)	860	870	880	890	900	910
Area (ft^2)	1370	1660	2293	2950	3550	4850

3.1136 ac-ft

Contour	Area	Volume (ft^3)
860	1370	
870	1660	0.34780
880	2293	0.45374
890	2950	0.60181
900	3550	0.74610
910	4850	0.96419
		3.11364

26.26

Elevation (ft)	1015	1020	1025	1030	1035	1040
Area (ft^2)	1815	2097	2391	2246	2363	2649

2.6001 ac-ft

Contour	Area	Volume (ft^3)
1015	1815	
1020	2097	0.44904
1025	2391	0.51515
1030	2246	0.53225
1035	2363	0.52904
1040	2649	0.57530
		2.60078

26.27 State two situations where prismoidal corrections are most significant.

From Section 26.8, paragraph 1: When paying for expensive cuts, such as in rock.

26.28 Write a computer program to calculate slope intercepts and end areas by the coordinate method, given cross-section notes and roadbed design information. Use the program to calculate the slope intercepts for the data of Problem 26.16.

Individual project.

***26.29** Distances (ft) from the left bank, corresponding depths (ft), and velocities (ft/sec), respectively, are given for a river discharge measurement. What is the volume in ft^3/sec? 0, 1.0, 0; 10, 2.3, 1.30; 20, 3.0, 1.54; 30, 2.7, 1.90; 40, 2.4, 1.95; 50, 3.0, 1.60; 60, 3.1, 1.70; 74, 3.0, 1.70; 80, 2.8, 1.54; 90, 3.3, 1.24; 100, 2.0, 0.58; 108, 2.2, 0.28; 116, 1.5, 0.

<u>419.3 ft³/s</u>

Distance	Depth	Area	Velocity	V_{avg}	Discharge
0	1.0		0.00		
10	2.3	16.5	1.30	0.65	10.73
20	3.0	26.5	1.54	1.42	37.63
30	2.7	28.5	1.90	1.72	49.02
40	2.4	25.5	1.95	1.93	49.09
50	3.0	27.0	1.60	1.78	47.93
60	3.1	30.5	1.70	1.65	50.33
70	3.0	30.5	1.70	1.70	51.85
80	2.8	29.0	1.54	1.62	46.98
90	3.3	30.5	1.24	1.39	42.40
100	2.0	26.5	0.58	0.91	24.12
108	2.2	16.8	0.28	0.43	7.22
116	1.5	14.8	0.00	0.14	2.07
					419.3

26.30 Prepare a computational program that computes the volumes in Problem 26.9.

26.31 Prepare a computational program that computes the end-areas in Problem 26.20.

27 PHOTOGRAMMETRY

27.1 Describe the difference between vertical, low oblique, and high oblique aerial photos.

From Section 27.4, Paragraph 1:
Aerial photographs exposed with single-lens frame cameras are classified as vertical (taken with the camera axis aimed vertically downward, or as nearly vertical as possible) and oblique (made with the camera axis intentionally inclined at an angle between the horizontal and vertical). Oblique photographs are further classified as high if the horizon shows on the picture, and low if it does not.

27.2 Discuss the advantages of softcopy stereoplotters over optical stereoplotters.

From Section 27.14.4, Paragraph 5:
Softcopy photogrammetry systems are efficient, as well as versatile. Not only are they capable of producing maps, cross sections, digital elevation models, and other digital topographic files, but they can also be employed for a variety of image interpretation problems and they can support the production of mosaics and orthophotos (see Section 27.15). Also, digital maps produced by softcopy systems are created in a computer environment and are therefore in formats compatible for CADD applications and for in the databases of Geographic Information Systems. Softcopy systems have the added advantage that their major item of hardware is a computer rather than an expensive single-purpose stereoplotter, so it can be used for many other tasks in addition to stereoplotting.

27.3 Define the terms (a) metric photogrammetry and (b) interpretative photogrammetry.

(a) From Section 27.1, Paragraph 4:
Metrical photogrammetry is accomplished in different ways depending upon project requirements and the type of equipment available. Simple analyses and computations can be made by making measurements on paper prints of aerial photos using engineer's scales, and assuming that the photos are "truly vertical," i.e., the camera axis coincided with a plumb line at the time of photography. These methods produce results of lower order, but they are suitable for a variety of applications. Other more advanced techniques, including analog, analytical, and softcopy methods, do not assume vertical photos and provide more accurate determinations of the spatial locations of objects. The analog procedure relies on precise optical and mechanical devices to create models of the terrain that can be measured and mapped. The analytical method is based upon precise measurements of the photographic positions of the images of objects of interest, followed by a mathematical solution for their locations. Softcopy instruments utilize digital images in computerized procedures that are highly automated.

(b) From Section 27.1, Paragraph 2:

Interpretative photogrammetry involves recognizing objects from their photographic images and judging their significance. Critical factors considered in identifying objects are the shapes, sizes, patterns, shadows, tones, and textures of their images. This area of photogrammetry was traditionally called photographic interpretation because initially it relied on aerial photos. More recently, other sensing and imaging devices such as multispectral scanners, thermal scanners, radiometers, and side-looking airborne radar have been developed which aid greatly in interpretation. These instruments sense energy in wavelengths beyond those which the human eye can see, or standard photographic films can record. They are often carried in aircraft as remote as satellites; hence the term, remote sensing, is now generally applied to the interpretative area of photogrammetry.

27.4 Describe briefly how a digital camera operates.

From Section 27.3, Paragraph 6:
A new type of camera is now used for obtaining images in digital form. Instead of film, these cameras employ an array of solid state detectors, which are placed in the focal plane. The most common type of detector is the charge-coupled device (CCD). The array is composed of tiny detectors arranged in contiguous rows and columns, as shown in Figure 27.3. Each detector senses the energy received from its corresponding ground scene and this constitutes one “picture element” (pixel) within the overall image. The principle of operation of CCDs is fundamentally quite simple. At any specific pixel location, the CCD element is exposed to incident light energy which builds up an electric charge proportional to the intensity of the incoming light. The electric charge is amplified, converted from analog to digital form and stored in a file together with its row and column location within the array. Currently, the sizes of the individual CCD elements being manufactured are in the range of from about 5 to 15 micrometers square with arrays consisting of from 500 rows and columns (250,000 pixels) for inexpensive cameras, to more than 4,000 rows and columns. Obviously, significant storage and data handling capabilities are necessary in acquiring and processing digital images.

27.5 The distance between two points on a vertical photograph is ab and the corresponding ground distance is AB. For the following data, compute the average photographic scale along the line ab.

(a)* ab = 2.41 in.; AB = 4820 ft. 4820 / 2.41 = 1/2000 in./ft.

(b) ab = 5.29 in.; AB = 13,218 ft. 13,218 / 5.29 = 1/2500 in./ft.

(c) ab = 107.389 mm; AB = 536.943 m. 536.943 / 107.89 = 1:5000

27.6 On a vertical photograph of flat terrain, section corners appear a distance d apart. If the camera focal length is f compute flying height above average ground in feet for the following data:

(a) d = 1.85 in.; f = 3 ½ in. f = 3.5 * (5280 / 1.85) = 10,000 ft.

(b) d = 82.184 mm; f = 153.20 mm f = 153.20 * (5280 / 82.184) = 9,800 ft.

27.7 On a vertical photograph of flat terrain, the scaled distance between two points is ab. Find the average photographic scale along ab if the measured length between the same line is AB on a map plotted at a scale of S_{map} for the following data.

(a) ab = 1.47 in.; AB = 3.55 in.; S_{map} = 1:6000
AB = (3.55 / 12) 6000 = 1775 ft.
S = 1775 / 1.47 = 1 in./ 1207 ft.

(b) ab = 41.53 mm; AB = 6.23 mm; S_{map} = 1:20,000
AB = (0.00623) 20,000 = 124.6 m
S = 124.6 / 0.04153 = 1:3000

27.8 What are the average scales of vertical photographs for the following data, given flying height above sea level, H, camera focal length, f, and average ground elevation h?

*(a) H = 7300 ft.; f = 152.4 mm; h = 1250 ft.

$$S = \frac{f}{H - h_a} = \frac{6}{7300 - 1250} = 1 \text{ in. / 1008.3 ft.}$$

(b) H = 6980 ft.; f = 6.000 in.; h = 1004 ft.
S = 6 / (6980 – 1004) = 1 in. / 996 ft.

(c) H = 2610 m.; f = 152.4 mm; h = 324 m.
S = 0.1524 / (2610 – 324) = 1:15000

27.9 The length of a football field from goal post to goal post scales 49.15 mm on a vertical photograph. Find the approximate dimensions (in meters) of a large rectangular building that also appears on this photo and whose sides measure 20.5 mm by 6.8 mm. (Hint: Football goal post are 120 yards apart.)

120 yds = 360 ft = 109.728 m.
0.04915 / 109.728 = 1:2232.5
0.0205 * 2232.5 = 45.767 m.
0.0068 * 2232.5 = 15.181 m.

***27.10** Compute the area in acres of a triangular parcel of land whose sides measure 48.78 mm, 84.05 mm, and 69.36 mm on a vertical photograph taken from 6050 ft above average ground with a 152.4 mm focal length camera.

S = [(152.4 / 25.4) / 12] / 6050 = 1:12,100
a = 0.04878 * 12,100 = 590.238 m.
b = 0.08405 * 12,100 = 1017.005 m.
c = 0.06936 * 12,100 = 839.256 m.
s = 2446.499 / 2 = 1223.250
area = sqrt[s(s-a)(s-b)(s-c)] = 247,638 m^2 = 24.76 ha.

27.11 Calculate the flight height above average terrain that is required to obtain vertical photographs at an average scale of S if the camera focal length is f for the following data:

(a) S = 1:8000; f = 152.4 mm
H = 0.5 (8000) = 16,000 ft.

(b) S = 1:6000; f = 88.9 mm
H = 0.0889 (6000) = 533.4 m.

27.12 Determine the horizontal distance between two points A and B whose elevations above datum are h_A = 1560 ft. and h_B = 1425 ft. and whose images a and b on a vertical photograph have photo coordinates x_a = 2.95 in., y_a = 2.32 in., x_b = -1.64 in., and y_b = -2.66 in. The camera focal length was 152.4 mm and the flying height above datum 7500 ft.

S_A = 6/(7500 – 1560) = 1 in. / 990 ft
X_A = 2.95(990) = 2920.50 ft. Y_A = 2.32 (990) = 2296.80 ft.
S_B = 6 / (7500 – 1425) = 1 in. / 1012.50 ft.
X_B = -1.64 (1012.50) = −1660.50 ft. Y_B = −2.66 (1012.50) = −2693.25 ft.
Distance = $\sqrt{(-1660.5-2920.50)^2+(-2693.25-2296.80)^2} = 6773.93\text{ ft.}$

27.13* Similar to Problem 27.12, except that the camera focal length was 3-1/2 in., the flying height above datum 4075 ft, and elevations h_A and h_b 983 ft and 1079 ft, respectively. Photo coordinates of images a and b were x_a = 108.81 mm., y_a = −73.73 mm., x_b = −87.05 mm., and y_b = 52.14 mm.

S_A = 3.5 / (4075 – 983) = 1 in. / 883.4 ft.
X_A = (108.81 / 25.4) (883.4) = 3784.48 ft.
Y_A = (-73.73 / 25.4) (883.4) = -2564.38 ft.
S_B = 3.5 / (4075 - 1079) = 1 in. / 856 ft.
X_B = (-87.05 / 25.4) (856) = -2933.65 ft.
Y_B = (52.14 / 25.4) (856) = 1757.16 ft.
Distance = 7988 ft.

27.14 On the photograph of Problem 27.12, the image c of a third point C appears. Its elevation h_C = 1365 ft. and its photo coordinates are x_c = 2.96 in. and y_c = -3.02 in. Compute the horizontal angles in triangle ABC.

S_C = 6 / (7500 – 1365) = 1 in. / 1022.50 ft.
X_C = 2.96 (1022.50) = 3026.60 ft. Y_C = -3.02 (1022.50) = -3087.95 ft.

$$Az_{AB} = \tan^{-1}\left(\frac{-1660.50-2920.50}{-2693.25-2296.90}\right)+180° = 222°33'10''$$

AZ_{BC} = 94°48’19” AZ_{CA} = 358°52’16”
Angle A = 43°40’54” Angle B = 52°15’39” Angle C = 84°03.27”

27.15 On the photograph of Problem 27.12, the image d of a third point D appears. Its elevation is h_D = 1195 ft. and its photo coordinates are x_d = 56.86 m. and y_d = 63.12 mm. Calculate the area, in acres, of triangle ABD.

S_D = 6 / (7500 – 1195) = 1 in. / 1050.8 ft.
X_D = (56.86 / 25.4) (1050.80) = 2352.30 ft.
Y_D = (63.12 / 25.4) (1050.80) = 2611.28 ft.

$Dist._{AB}$ = 6773.93 ft. $Dist._{BC}$ = 4703.69 ft. $Dist._{DA}$ = 649.42 ft.
s = 12127.04 / 2 = 6063.52
area = sqrt [s(s-a)(s-b)(s-c)] = 129.3 ac.

27.16 Determine the height of a radio tower, which appears on a vertical photograph for the following conditions of flying height above the tower base H, distance on the photograph from principal point to tower base r_d and distance from principal point to toer top r_t

*(a) H = 2425 ft.; r_b = 3.18 in.; r_t = 3.34 in.

$$d = \frac{rh}{H} \qquad h = \frac{dH}{r} \qquad h = \frac{(3.34-3.18)2425}{3.34} = 116.17\text{ ft.}$$

(b) H = 6600 ft.; r_b =96.83 mm; r_t = 98.07 mm.

$$h = \frac{(98.07-96.83)6600}{98.07} = 83.45\text{ ft.}$$

27.17 On a vertical photograph, images a and b of ground points A and B have photographic coordinates x_a = 3.27 in., y_a = 2.28 in., x_b = -1.95 in. and y_b = -2.50 in. The horizontal distance between A and B is 5283 ft, and the elevations of A and B above datum are 646 ft and 756 ft, respectively. Using Equation (27.9), calculate the flying height above datum for a camera having a focal length of 152.4 mm.

$$L^2 = \left[\frac{(H-h_b)x_b-(H-h_a)x_a}{f}\right]^2+\left[\frac{(H-h_b)y_b-(H-h_a)y_a}{f}\right]^2$$

$$5283^2 = \left[\frac{-1.95(H-756)-3.27(H-646)}{6}\right]^2+\left[\frac{-2.50(H-756)-2.28(H-646)}{6}\right]^2$$

$$0 = 1.39127H^2 - 1933.15446H - 27{,}238{,}622.1967$$

H = 5173.06 ft.

27.18 Similar to Problem 27.17, except x_a = -52.53 mm, y_a = 69.67 mm, x_b = 26.30 mm, y_b = -59.29 mm line length AB = 4706 ft. and elevations of points A and B are 925 and 875 ft, respectively.

$$4706^2 = \left[\frac{26.30(H-875)+52.53(H-925)}{152.4}\right]^2+\left[\frac{-59.29(H-875)-69.67(H-925)}{152.4}\right]^2$$

$$0 = 0.9835997H^2 - 1777.81255H - 21{,}343{,}099.036$$

H = 5648 ft.

***27.19** An air base of 3205 ft exists for a pair of overlapping vertical photographs taken at a flying height of 5500 ft above MSL with a camera having a focal length of 152.4 mm. Photo coordinates of points A and B on the left photograph are x_a = 40.50 mm, y_a = 42.80 mm, x_b = 23.59 mm, and y_b = -59.15 mm. The x photo coordinates on the right photograph are x_{1a} -60.68 mm and x_{1b}-70.29 mm. Using the parallax equations, calculate horizontal length AB.

$$p_a = x_a - x_{1a} = 40.50-(-60.68) = 101.18\text{ mm.}$$

$$p_b = x_b - x_{1b} = 23.59-(-70.29) = 93.88\text{ mm.}$$

$$X_A = \frac{B}{p_a}x_a = \frac{3205}{101.18}40.50 = 1282.89\text{ ft.}$$

$$X_B = \frac{B}{p_b}x_b = \frac{3205}{93.88}23.59 = 805.35\text{ ft.}$$

$$Y_A = \frac{B}{p_a}y_a = \frac{3205}{101.18}42.80 = 1355.74\text{ ft.}$$

$$Y_B = \frac{B}{p_a}y_b = \frac{3205}{93.88}(-59.15) = -2019.34\text{ ft.}$$

$$\text{Distance}_{AB} = \sqrt{(805.35-1282.89)^2+(-2019.34-1355.74)^2} = 3408.70\text{ ft.}$$

27.20 Similar to Problem 27.19, except the air base is 6940 ft, the flying height above mean sea level is 12,520 ft, the x and y photo coordinates on the left photo are x_a = 37.98 mm., y_a = 50.45 mm., x_b = 24.60 mm., and y_b = -42.89 mm, and the x photo coordinates on the right photo are x_{1a} = -52.17 mm and x_{1b} = -63.88 mm.

$$p_a = x_a - x_{1a} = 37.98 - (-52.17) = 90.15 \text{ mm.}$$

$$p_b = x_b - x_{1b} = 24.60 - (-63.88) = 88.48 \text{ mm.}$$

$$X_A = \frac{B}{p_a} x_a = \frac{6940}{90.15} 37.98 = 2923.81 \text{ ft.}$$

$$X_B = \frac{B}{p_b} x_b = \frac{6940}{88.48} 24.60 = 1929.52 \text{ ft.}$$

$$Y_A = \frac{B}{p_a} y_a = \frac{6940}{90.15} 50.45 = 3883.78 \text{ ft.}$$

$$Y_B = \frac{B}{p_a} y_b = \frac{6940}{88.48} (-42.89) = -3364.11 \text{ ft.}$$

$$\text{Distance}_{AB} = \sqrt{(1929.52 - 2923.81)^2 + (-3364.11 - 3883.78)^2} = 1121.90 \text{ ft.}$$

27.21 Calculate the elevations of points A and B in Problem 27.19.

$$h_A = H - \frac{Bf}{p_a} = 5500 - \frac{3205(152.4)}{101.18} = 672.54 \text{ ft.}$$

$$h_B = H - \frac{Bf}{p_b} = 5500 - \frac{3205(152.4)}{93.88} = 297.17 \text{ ft.}$$

27.22 Compute the elevations of points A and B in Problem 27.20.

$$h_A = H - \frac{Bf}{p_a} = 12{,}520 - \frac{6940(152.4)}{90.15} = 787.82 \text{ ft.}$$

$$h_B = H - \frac{Bf}{p_b} = 12{,}520 - \frac{6940(152.4)}{88.48} = 566.38 \text{ ft.}$$

27.23 List and briefly describe the four different categories of stereoscopic plotting instruments.

From Section 27.14:
Stereoplotters can be classified into four different categories: (1) optical projection, (2) mechanical projection, (3) analytical, and (4) digital or “softcopy” systems.

27.24 Name the three stages in stereoplotter orientation, and briefly explain the objectives of each.

From Section 27.14, Paragraphs 5-7:
Interior orientation ensures that the projected light rays are geometrically correct, i.e., angles and of Figure 27.14(b), (i.e., the angles between the projected light rays and the axis of the projector lens), must be identical to corresponding angles and respectively, in Figure 27.14(a), (i.e., the angles between the incoming light rays and the camera axis). Preparing the diapositives to exacting specifications, and centering them carefully in the projectors accomplish this.

After the diapositives have been placed in the projectors and the lights turned on, corresponding light rays will not intersect to form a clear model because of tilts in the photographs and unequal flying heights. To achieve intersections of corresponding light rays, the projectors are moved linearly along the X, Y, and Z-axes and also rotated about these axes until they duplicate the relative tilts and flying heights that existed when the photographs were taken. This process is called relative orientation, and when accomplished, parallactic angle of Figure 27.14(b) for each corresponding pair of light rays will be identical to its corresponding parallactic angle of Figure 27.14(a), and a perfect three-dimensional model will be formed.

The model is brought to required scale by making the rays of at least two, but preferably three, ground control points intersect at their positions plotted on a manuscript map prepared at the desired scale. It is leveled by adjusting the projectors so the counter reads the correct elevations for each of a minimum of three, but preferably four, corner ground control points when the floating mark is set on them. Absolute orientation is a term applied to the processes of scaling and leveling the model.

27.25 What advantages does a softcopy plotter have over an analytical plotter?

From Section 27.14.4, Paragraph 5:
Softcopy photogrammetry systems are efficient, as well as versatile. Not only are they capable of producing maps, cross sections, digital elevation models, and other digital topographic files, but they can also be employed for a variety of image interpretation problems and they can support the production of mosaics and orthophotos (see Section 27.15). Also, digital maps produced by softcopy systems are created in a computer environment and are therefore in formats compatible for CADD applications and for in the databases of Geographic Information Systems. Softcopy systems have the added advantage that their major item of hardware is a computer rather than an expensive single-purpose stereoplotter, so it can be used for many other tasks in addition to stereoplotting.

27.26 What kind of images do softcopy stereoplotters require? Describe two different ways they can be obtained.

From Section 27.14.4, Paragraph 1:
These systems utilize digital or "softcopy" images. The images can be acquired by using a digital camera of the type described in Section 27.3, but more often they are obtained by scanning the negatives of aerial photos taken with film cameras.

27.27 Compare an orthophoto with a conventional line and symbol map.
From Section 27.15, Paragraph 4:
Orthophotos combine the advantages of both aerial photos and line maps. Like photos, they show features by their actual images rather than as lines and symbols, thus making them more easily interpreted and understood. Like maps, orthophotos show the features in their true planimetric positions. Therefore true distances, angles, and areas can be scaled directly from them.

27.28 Discuss the advantages of orthophotos as compared to maps.
From Section 27.15, Paragraph 4:
Orthophotos combine the advantages of both aerial photos and line maps. Like photos, they show features by their actual images rather than as lines and symbols, thus making them more easily interpreted and understood. Like maps, orthophotos show the features in their true planimetric positions. Therefore true distances, angles, and areas can be scaled directly from them.

Aerial photography is to be taken of a tract of land that is X mi square. Flying height will be H ft above average terrain, and the camera has focal length f. If the focal plane opening is 9 X 9 in. and minimum sidelap is 30 percent, how many flight lines will be needed to cover the tract for the data given in Problems 27.29 and 27.30?

***27.29** X = 8; H = 4000; f = 152.4 mm
S = 6 / 4000 = 1 in. / 666.67 ft.
d_S = 9 * S (1 – 0.3) = 4200 ft.
# of Flight Lines = [8 (5280)]/4200 = 10.05 + 1 = 11 so choose 12

27.30 X = 30; H = 10,000; f = 6 in.
S = 6 / 10,000 = 1 in. / 1666.67 ft.
d_S = 9 * 1666.67 (1 -0.3) = 10500ft.
# of Flight Lines = [30 (5280)] / 10500 = 15.08 + 1 = 16 so choose 17

Aerial photography was taken at a flying height H ft above average terrain. If the camera focal plane dimensions are 9 X 9 in. the focal length is f and the spacing between adjacent flight lines is X ft, what is the percent sidelap for the data given in Problems 27.31 and 27.32?

***27.31** H = 4500; f = 152.4 mm; X = 4700
S = 6 / 4500 = 1 in. / 750 ft.
4700 = 9 * 750x   x = 0.696 = 69.6 = 30.4% sidelap

27.32 H = 6800; f = 88.9; X = 13,500
S = 3.5 / 6800 = 1 in. / 1942.857 ft.
13,500 = 9 * 1942.857x   x = 0.772 = 22.8% sidelap

Photographs at a scale of S are required to cover an area X mi square. The camera has a focal length f and focal plane dimensions of 9 X 9 in. If endlap is 60% and sidelap 30%, how many photos will be required to cover the area for the data given in Problems 27.33 and 27.34?

27.33 S = 1:6000; X = 6; f = 152.4 mm
de = (9 / 12)(6000)(1 – 0.6) = 1800 ft.
ds = (9 / 12)(6000)(1 – 0.3) = 3150 ft.
# of Flight Lines = 6(5280) / 3150 = 10.057 +1 = 11 so choose 12
# of Photos per Line = 6(5280) / 1800 + 2 + 2 + 1 = 22.6 = 23
Total # of photos = 12 * 23 = 276 photos

27.34 S = 1:14,400; X = 40; f = 89.0 mm
de = (0.089 * 3.280833333)(14,400)(1 – 0.6) = 1682 ft.
ds = (0.089 * 3.280833333)(14,400)(1 – 0.3) = 2943 ft.
# of Flight Lines = 40(5280) / 2943 = 71.76 +1 = 73
# of Photos per Line = 40(5280) / 1682 + 2 + 2 + 1 = 130.6 = 131
Total # of photos = 73 * 131 = 9563 photos

27.35 Describe a system that employs GPS and which can reduce or eliminate ground control surveys in photogrammetry?

From Section 27.16, Paragraph 3:
Currently GPS is being used for real-time positioning of the camera at the instant each photograph is exposed. The kinematic GPS surveying procedure is being employed (see Chapter 15), which requires two GPS receivers. One unit is stationed at a ground control point; the other is placed within the aircraft carrying the camera. The integer ambiguity problem is resolved using on-the-fly techniques (see Section 15.2). During the flight, camera positions are continuously determined at time intervals of a few seconds using the GPS units and precise timing of each photo exposure is also recorded. From this information, the precise location of each exposure station, in the ground coordinate system, can be calculated. Many projects have been completed using these methods and they have produced highly accurate results, especially when

supplemented with only a few ground control points. It is now possible to complete photogrammetric projects with only a few ground photo control points used for checking purposes.

27.36 To what wavelengths of electromagnetic energy is the human eye sensitive? What wavelengths produce the colors blue, green, and red?

From Section 27.20, Paragraph 4:
Within the wavelengths of visible light, the human eye is able to distinguish different colors. The primary colors (blue, green, and red) consist of wavelengths in the ranges of 0.4–0.5, 0.5–0.6, and respectively. All other hues are combinations of the primary colors. To the human eye, an object appears a certain color because it reflects energy of wavelengths producing that color. If an object reflects all wavelengths of energy in the visible range, it will appear white, and if it absorbs all wavelengths, it will be black. If an object absorbs all green and red energy but reflects blue, that object will appear blue.

27.37 Discuss the uses and advantages of satellite imagery.
From Section 27.20:
Satellite imagery is unique because it affords a practical means of monitoring our entire planet on a regular basis. Images of this type have been applied for land-use mapping; measuring and monitoring various agricultural crops; mapping soils; detecting diseased crops and trees; locating forest fires; studying wildlife; mapping the effects of natural disasters such as tornadoes, floods, and earthquakes; analyzing population growth and distribution; determining the locations and extent of oil spills; monitoring water quality and detecting the presence of pollutants; and accomplishing numerous other tasks over large areas for the benefit of humankind.

Problems 27.38 through 27.42 involve using WolfPack with images 5 and 6 on the CD that accompany this book. The ground coordinates of the paneled points are listed in the file "ground.crd." The coordinates of the fiducials are listed in the file "camera.fid." To do these problems, digitize the eight fiducials and paneled points 21002, 4, 41, GYM, WIL1A, WIL1B, and RD on both images. After digitizing the points, perform an interior orientation to compute photo coordinates for the points on images 5 and 6. The focal length of the camera is 153.742 mm.

Responses will be affected by the quality of the observations. Approximate values for the problem are supplied herewith.

27.38 Using photo coordinates for points 4 and GYM on image 5, determine the scale of the photo.

Photo Coordinates:	4: (–1.187, 70.338) mm
	GYM:(78.889, –14.642) mm
Ground Coordinates:	4: (745143.093, 128206.079) m.
	GYM: (745413.425, 127875.820) m.
ab = 116.771 mm	AB = 426.791 m.

S = 0.116771 / 426.791 = 1:3655

27.39 Using photo coordinates for points 4 and GYM on image 5, determine the flying height of the camera at the time of exposure.

H = 938 m. or -220 m.

27.40 Using photo coordinates for points 4 and GYM on image 5 and 6, determine the ground coordinates of points WIL1A and WIL1B using Eq. (27.12) and Eq. (27.13).

Distance = 423.354 m.

27.41 Using the exterior orientation option in WolfPack, determine the exterior orientation elements for image 5.

Responses will be affected by the quality of the observations.

27.42 Using the exterior orientation option in WolfPack, determine the exterior orientation elements for image 6.

Responses will be affected by the quality of the observations.

28 INTRODUCTION TO GEOGRAPHIC INFORMATION SYSTEMS

28.1 Describe the concept of layers in a geographic information system.

From Section 28.1, Paragraph 4:
A generalized concept of how data of different types or "layers" are collected and overlaid in a GIS is illustrated in Figure 28.1. In that figure, maps A through G represent some of the different layers of spatially related information that can be digitally recorded and incorporated into a GIS database, and include parcels of different land ownership A, zoning B, floodplains C, wetlands D, land cover E, and soil types F. Map G is the geodetic reference framework, consisting of the network of survey control points in the area. Note that these control points are found in each of the other layers thereby providing the means for spatially locating all data in a common reference system. Thus composite maps that merge two or more different data sets can be accurately created. For example in Figure 28.1, bottom map H is the composite of all layers

28.2 Discuss the role of a geographic reference framework in a GIS.

From Section 28.1, Paragraph 4:
It allows the user to relate information on different layers of the GIS

28.3 List the fundamental components of a GIS.

From Section 28.1, Paragraph 2:
A more detailed definition (Hanigan, 1988) describes a GIS as "any information management system that can:

1. Collect, store, and retrieve information based on its spatial location;
2. Identify locations within a targeted environment that meet specific criteria;
3. Explore relationships among data sets within that environment;
4. Analyze the related data spatially as an aid to making decisions about that environment;
5. Facilitate selecting and passing data to application-specific analytical models capable of assessing the impact of alternatives on the chosen environment; and
6. Display the selected environment both graphically and numerically either before or after analysis."

28.4 List the fields within surveying and mapping that are fundamental to the development and implementation of GISs.

From Section 28.1, Paragraph 7:
Virtually every aspect of surveying, and thus all material presented in the preceding chapters of this book, bear upon GIS development, management, and use.

28.5 Discuss the importance of metadata to a GIS.

From Section 28.8, Paragraph 1:
Once created, data can travel almost instantaneously through a network and be transformed, modified, and used for many different kinds of spatial analyses. It can then be re-transmitted to another user, and then to another, etc.2 It is important that each change made to any data set be documented by updating its associated metadata.

28.6 Name and describe the different simple spatial objects used for representing graphic data in digital form. Which objects are used in raster format representations?

From Section 28.4.1:
The simple spatial objects most commonly used in spatially locating data are illustrated in Figure 28.2 and described as follows:

1. Points define single geometric locations. They are used to locate features such as houses, wells, mines, or bridges [see Figure 28.2(a)]. Their coordinates give the spatial locations of points, commonly in state plane or UTM systems (see Chapter 20).

2. Lines and strings are obtained by connecting points. A line connects two points, and a string is a sequence of two or more connected lines. Lines and strings are used to represent and locate roads, streams, fences, property lines, etc. [see Figure 28.2(b)].

3. Interior areas consist of the continuous space within three or more connected lines or strings that form a closed loop [see Figure 28.2(c)]. For example, interior areas are used to represent and locate the limits of governmental jurisdictions, parcels of land ownership, different types of land cover, or large buildings.

4. Pixels are usually tiny squares that represent the smallest elements into which a digital image is divided [see Figure 28.2(d)]. Continuous arrays of pixels, arranged in rows and columns, are used to enter data from aerial photos, orthophotos, satellite images, etc. Assigning a numerical value to each pixel specifies the distributions of colors or tones throughout the image. Pixel size can be varied, and is usually specified by the number of dots per inch (dpi). As an example, 100 dpi would correspond to squares having dimensions of 1/100 in. on each side. Thus 100 dpi yields 10,000 pixels per square inch.

5. Grid cells are single elements, usually square, within a continuous geographic variable. Similar to pixels, their sizes can be varied, with smaller cells yielding improved resolution. Grid cells may be used to represent slopes, soil types, land cover, water table depths, land values, population density, and so on. The distribution of a given data type within an area is indicated by assigning a numerical value to each cell; for example, showing soil types in an area using the number 2 to represent sand, 5 for loam, and 9 for clay, as illustrated in Figure 28.2(e).

28.7 What are the primary differences between a GIS and LIS?

From Section 28.2, Paragraph 1:
The distinguishing characteristic between the two is that a LIS has its focus directed primarily toward land records data.

28.8 How many pixels are required to convert the following documents to raster form for the conditions given:

(a)* A 384-in. square map scanned at 200 dpi.
$384^2(200)^2 = 5{,}898{,}240{,}000$

(b) A 9-in. square aerial photo scanned at 1200 dpi.
$9^2(1200)^2 = 116{,}640{,}000$

(c) An orthophoto of 11 × 17 in. dimensions scanned at 300 dpi
$11(17)(300)^2 = 16{,}830{,}000$

28.9 Explain how data can be converted from:

(a) Vector to raster format
See Section 28.6.1

(b) Raster to vector format
See Section 28.6.2

28.10 For what types of data is the vector format best suited?

From Section 28.4.2, Paragraph 7:
Examples include aerial photos, orthophotos, and satellite images.

28.11 Discuss the compromising relationships between grid cell size and resolution in raster data representation.

From Section 28.4.2, Paragraph 6:
It is important to note, that as grid resolution increases, so does the volume of data (number of grid cells) required to enter the data.

28.12 Define the term topology and discuss its importance in a GIS.

From Section 28.4.3, Paragraph 1:
Topology is a branch of mathematics that describes how spatial objects are related to each other. The unique sizes, dimensions, and shapes of the individual objects are not addressed by topology. Rather, it is only their relative relationships that are specified
From Section 28.4.3, Last Paragraph:
The relationships expressed through the identifiers for points, lines, and areas of Table 28.1, and the topology in Table 28.2, conceptually yield a “map.” With these types of information available to the computer, the analysis and query processes of a GIS are made possible.

28.13 Develop identifier and topology tables similar to those of Tables 28.1 and 28.2 in the text for the vector representation of (see the following figures):

(a) Problem 28.13(a)

		Line		Area	
Identifier	Coordinates	Identifier	Points	Identifier	Lines
1	x_1,y_1	a	1,2	I	a,g,f,e
2	x_2,y_2	b	2,3	II	b,h,g
3	x_3,y_3	c	3,4	III	c,d,f,h
4	x_4,y_4	d	4,5		
5	x_5,y_5	e	5,1		
		f	5,6		
		g	6,2		
		h	6,3		

Connectivity		Direction			Adjacency		
Nodes	Chains	Chain	From Node	To Node	Chain	Left Polygon	Right Polygon
1-2	a	a	1	2	a	0	I

2-3	b	b	2	3	b	0	II
3-4	c	c	3	4	c	0	III
4-5	d	d	4	5	d	0	III
5-1	e	e	5	1	e	0	I
5-6	f	f	5	6	f	I	III
6-2	g	g	6	2	g	I	II
6-3	h	h	6	3	h	II	III

(b) Problem 28.13(b)

		Line		Area	
Identifier	Coordinates	Identifier	Points	Identifier	Lines
1	x_1,y_1	a	1,2	I	b,c,q,l
2	x_2,y_2	b	3,4	II	d,r,m,q
3	x_3,y_3	c	4,5	III	e,f,n,r
4	x_4,y_4	d	5,6	IV	g,k,l,m,n,o,p
5	x_5,y_5	e	6,7	V	a,o,s,j
6	x_6,y_6	f	7,8	VI	h,i,s,p
7	x_7,y_7	g	8,9		
8	x_8,y_8	h	9,10		
9	x_9,y_9	i	10,11		
10	x_{10},y_{10}	j	11,1		
11	x_{11},y_{11}	k	2,3		
12	x_{12},y_{12}	l	3,13		
13	x_{13},y_{13}	m	13,14		
14	x_{14},y_{14}	n	14,8		
15	x_{15},y_{15}	o	2,12		
16	x_{16},y_{16}	p	12,9		
17	x_{17},y_{17}	q	5,13		
18	x_{19},y_{18}	r	6,14		
19	x_{19},y_{19}	s	11,12		

Connectivity	Direction	Adjacency	Nestedness

Nodes	Chains	Chain	From Node	To Node	Chain	Left Polygon	Right Polygon	Polygon	Nested Node
1,2	a	a	1	2	A	0	V	III	d
3,4	b	b	3	4	B	0	I	IV	b,c
4,5	c	c	4	5	C	0	I	V	a
5,6	d	d	5	6	D	0	II		
6,7	e	e	6	7	E	0	II		
7,8	f	f	7	8	F	0	III		
8,9	g	g	8	9	G	0	IV		
9,10	h	h	9	10	H	0	VI		
10,11	i	i	10	11	I	0	VI		
1,11	j	j	1	11	J	0	V		
2,3	k	k	2	3	K	0	IV		
3,13	l	l	3	13	L	I	IV		
13,14	m	m	13	14	M	II	IV		
8,14	n	n	8	14	N	III	IV		
2,12	o	o	2	12	O	IV	V		
9,12	p	p	9	12	P	IV	VI		
5,13	q	q	5	13	Q	I	II		
6,14	r	r	6	14	R	II	II		
11,12	s	s	11	12	S	V	VI		

28.14 Compile a list of linear features for which the topological relationship of adjacency would be important.

Streets, railroads, rivers, streams, transmission lines, bus routes, mail routes, electric circuits, water mains, and so on

28.15 Prepare a raster (grid cell) representation of the sample map of:

(a) Problem 28.15(a), using a cell size of 0.10-in. square (see accompanying figure).

(b) Problem 28.15(b), using a cell size of 0.20-in. square (see accompanying figure).

28.16 Discuss the advantages and disadvantages of using the following equipment for converting maps and other graphic data to digital form:

(a) tablet digitizers

From Section 28.7.3, Last Paragraph:
Data files generated in this manner can be obtained quickly and relatively inexpensively. Of course the accuracy of the resultant data can be no better than the accuracy of the document being digitized, and its accuracy is further diminished by differential shrinkages or expansions of the paper or materials upon which the document is printed and by inaccuracies in the digitizer and the digitizing process.

(b) scanners.

From Section 28.7.6, Last Paragraph:
Accuracy of the raster file obtained from scanning depends somewhat on the instrument's precision, but pixel size or resolution is generally the major factor. A smaller pixel size will normally yield superior resolution. However there are certain tradeoffs that must be considered. Whereas a large pixel size will result in a coarse representation of the original, it will require less scanning time and computer storage. Conversely, a fine resolution, which generates a precise depiction of the original, requires more scanning time and computer storage. An additional problem is that at very fine resolution, the scanner will record too much "noise," that is, impurities such as specks of dirt. For these reasons and others, this is the least preferred method of capturing data in a GIS.

28.17 Explain the concepts of the following terms in GIS spatial analysis, and give an example illustrating the beneficial application of each: (a) adjacency; and (b) connectivity.

From Section 28.9.2:
Adjacency and connectivity are two important boundary operations that often assist significantly in management and decision-making. An example of adjacency is illustrated in Figure 28.10(d) and relates to a zoning change requested by the owner of parcel A. Before taking action on the request, the jurisdiction's zoning administrators are required to notify all owners of adjacent properties B through H. If the GIS database includes the parcel descriptions with topology and other appropriate attributes, an adjacency analysis will identify the abutting properties and provide the names and addresses of the owners. Connectivity involves analyses of the intersections or connections of linear features. The need to repair a city water main serves as an example to illustrate its value. Suppose that the decision has been made that these repairs will take place between the hours of 1:00 and 4:00 P.M. on a certain date. If infrastructure data are stored within the city's GIS database, all customers connected to this line whose water service will be interrupted by the repairs can be identified and their names and addresses tabulated. The GIS can even print a letter and address labels to facilitate a mailing announcing details of the planned interruption to all affected customers.

28.18 If data were being represented in vector format, what simple spatial objects would be associated with each of the following topological properties?

(a) Connectivity	Points and Lines
(b) Direction	Points and Lines
(c) Adjacency	Interior Areas
(d) Nestedness	Interior Areas and Points

28.19 Prepare a transparency having a 0.10-in grid, overlay it onto Figure 28.4(a), and indicate the grid cells that define the stream. Now convert this raster representation to vector using the method described in Section 28.6.2. Repeat the process using a 0.20-in grid. Compare the two resulting vector representations of the stream and explain any differences.

Suggestion: If you have access to a scanner, scan Figure 28.4(a) and have the students import it into their CAD package. Then set both the grid and snap to 0.1 and 0.2. and have the student trace the image in raster format. They will quickly realize the number of problems that occur when scanning linear features such as streams and edges of features.

28.20 Discuss how spatial and non-spatial data are related in a GIS.

From Section 28.5, Paragraph 2:
In general, spatial data will have related nonspatial attributes and thus some form of linkage must be established between these two different types of information. Usually this is achieved with a common identifier that is stored with both the graphic and the nongraphic data. Identifiers such as a unique parcel identification number, a grid cell label, or the specific mile point along a particular highway may be used.

28.21 What are the actual ground dimensions of a pixel for the following conditions:

(a) A 1:10,000 scale, 9 in. square orthophoto scanned at 500 dpi?

$(9 \times 500) / 10{,}000 = 0.45$ in.

(b)* A 748 in. square, 1:24,000 map, scanned at 200 dpi?

$(748 \times 200) / 24{,}000 = 6.23$ in.

28.22 Describe the following GIS functions, and give two examples where each would be valuable in analysis:

(a) line buffering, and

From Section 28.9.1, Paragraph 2:
Line buffering, illustrated in Figure 28.10(b), creates new polygons along established lines such as streams and roads. To illustrate the use of line buffering, assume that to preserve the natural stream bank and prevent erosion, a zoning commission has set the construction setback distance from a certain stream at D. Line buffering can quickly identify the areas within this zone.

(b) spatial joins

From Section 28.9.3, Paragraph 2:
Having these various data sets available in spatially related layers makes the overlay function possible. Its employment in a GIS can be compared to using a collection of Mylar overlays in traditional mapping. However, much greater efficiency and flexibility are possible when operating in the computer environment of a GIS, and not only can graphic data be overlaid, but attribute information can be combined as well.

28.23 Go to the PASDA web site or a similar web site in your state and download an example of:

(a) An orthophoto
(b) Zoning
(c) Floodplains and wetlands
(d) Soil types

28.24 Compile a list of data layers and attributes that would likely be included in an LIS.

Control to establish a common basis of reference coordinates; political boundaries; U.S. Public Land System, legal descriptions in metes and bounds, block and lot deeds; easements; improvements; parcel ownership, parcel values, hydrography, and so on.

28.25 Compile a list of data layers and attributes that would likely be included in a GIS for:

(a) Selecting the optimum corridor for constructing a new rapid-transit system to connect two major cities

Control; state, county, and municipal boundaries; U.S. Public Land Survey System; legal descriptions and parcels; easements; parcel ownership; parcel values; existing transportation routes; topography; hydrography; existing land use; zoning; soil types; depth to bedrock or underground mines; depth of water table; utilities; and so on.

(b) Choosing the best location for a new airport in a large metropolitan area
All layers in (a) plus building locations with heights; tower locations and heights; and other vertical obstructions; quiet zones; and so on.

(c) Routing a fleet of school buses

Control; state, county, and municipal boundaries; existing transportation routes with data on widths, grades, pavement type/condition, and speed limits; load and height restrictions on highways and bridges; construction activity; detours; and so on.

(d) Selecting the fastest routes for reaching locations of fires from various fire stations in a large city

Control; municipal boundaries; ward boundaries; fire station locations; transportation routes with grades, lighting, and traffic signals; traffic counts; accident records; construction activity; detours; and so on.

28.26 In Section 28.9.3, a flood-warning example is given to illustrate the value of simultaneously applying more than one GIS analytical function. Describe another example.

Answers will vary

28.27 Consult the literature on GISs and, based on your research, describe an example that gives an application of a GIS in:

(a) Natural resource management
(b) Agriculture
(c) Engineering
(d) Forestry

Independent Project

Part II: Sample Introductory Course

Sample Syllabus

15-Week, 3 Credit, Semester Course

Required Text: Ghilani, Charles D. and Paul R. Wolf. 2011. **Elementary Surveying (An Introduction to Geomatics), 13th Ed**. Prentice Hall, Upper Saddle River, NJ.

Materials:

- Safety vest
- Field book and 3H or 4H pencil
- Computation note pad
- Scientific calculator with 10-digit display
- Engineer's Scale

Grading:

- Homework.... 15%
- Practical exercises20%
- Quizzes (5)..... 5%
- Hour exams (3)30%
- Article reviews (3) 6%
- Portfolio 4%
- Final exam.... 20%

Lecture Schedule

Week	Lecture No.	Subject	Reading	Homework
1	1	Class policies, Introduction	Chapter 1	–
	2	Introduction	Chapter 1	–
2	3	Units Significant Figures	2.1 to 5	2.1, 3, 5, 10, 14
	4	Field Notes	2.6 to 15	–
3	5	Errors - mean, standard deviation, probable error	3.1 to 16	3.3, 6, 11, 16
	6	Error propagation	3.17 to 21	3.19, 21, 27, 30(a)
4	7	Leveling - Theory and Methods	Chapter 4, Part 1	4.1, 4, 13, 16, 18
	8	Leveling - Equipment	Chapter 4, Part 2	4.19, 20, 24, 28
5	–	Exam 1	–	–
5,6	9–10	Leveling - Field Procedures	Chapter 5	5.1, 3, 9, 13, 23
6	11	Taping	Chapter 6,Part 1	6.6, 9, 11, 19, 22

7	12	EDM	Chapter 6, Part 2	6.25, 28(b), 33, 38, 43
	13	Angles, Azimuths, and Bearings	7.1 to 9	7.3, 6, 11, 24, 39
8	14	The Compass and Magnetic Declination	7.10 to 16	7.31, 37
	15	Total Station Instruments	Chapter 8, Part 1	–
9	16	Angle Measurements	Chapter 8, Part 2	8.2, 4, 12, 21, 35
	17	Traversing	Chapter 9	9.5, 10, 12, 13, 24
10	–	Exam 2		
	18	Traverse Computations	Chapter 10	
11	19–20	Traverse Computations	Chapter 10	10.12, 13, 14, 15, 24, 25
12	21–22	COGO – Intersections	11.1 to 6	11.4, 10, 12, 16, 17
13	23–24	COGO – Resection and Coordinate Transformations	11.7 to 11	11.20, 22, 23, 24, 27
14	25	Area by Simple Figures	12. 1 to 12.4	–
	–	Exam 3	–	–
15	26–27	Area by Coordinates	12.4 to 11	12.2, 4, 13, 24, 26

Article Reviews

A short review of journal articles will be due in the following weeks of the course. Possible sources for articles are listed at the end of each chapter in the book. Papers will be graded on completeness of thought, grammar, spelling, and punctuation. All reviews should be word processed and contain the following items.

Citation: See examples of proper citations in the bibliography at the end of each chapter.

Author's thesis: A brief statement or two on the main focus of the article.

Author's argument: A review of the article stating how the author supported the thesis.

Reviewer's opinion:Not all that is written is correct. Write a brief paragraph on why you agree or disagree with the author's thesis and how this article relates to this class.

Week	Subject
2	Problems 1.20 or 21
7	Write an article review on one of articles listed in the bibliography for Chapter 4.
12	Write an article review on one of articles listed in the bibliography for Chapter 8.

Practical Exercises (Refer to the list of Sample Practical Exercises on the following pages.)

Week	Practical Exercise
1 – 2	A
3	B
4	D
5 – 6	E
7	F
8	G
9 – 10	H
11	I
12	J, Problem 11.18
13	J, Problem 11.37
14	K
15	Review for final

Sample Practical Exercises

To fully understand and appreciate the theory discussed in Elementary Surveying, a student should be exposed to a series of practical, hands-on exercises. This section covers a sample set of exercises for your consideration. Some exercises assume that the instructor has assigned a set of traverse stations to the students for leveling, distance and angle observations.

Chapter	Number	Exercise
2	A	Students should read the manual for their survey controller and determine the proper procedure for setting up a project.
3	B	Outdoor lab: Develop a pacing lab. In this lab layout a 100-yard, -meter, line on a level section of ground. Have students pace the line 10 times estimating the length of the last pace. Following this, have the students pace the traverse that will be assigned to them for distance measurement in Chapter 6. Students should develop a report giving the length of their pace and the standard deviation. They should compute the length of the lines of the traverse in feet or meters along with the estimated error in the length. $$E = E_{pace}\sqrt{n}$$ where n is the number of paces.
3	C	Inside lab: Hang a plumb bob from the ceiling of your room. Have the students measure the length of the string from support to the tip of mass center of the bob. Now measure the period of the plumb bob using a stopwatch. Repeat this procedure ten times. Student should develop a report providing the average period (T) of the pendulum, and its standard deviation. They should then compute the “approximate” value for gravity using the formula $$g = \frac{\pi^2 \ell \left(1 + \frac{h}{8\ell}\right)^2}{T^2}$$ where l is the length of the string, h is the height the pendulum falls during a half oscillation. Note the pendulum string is not weightless, nor the pivot frictionless, so do not assume this to be an accurate value for gravity.
4	D	Have the students perform a collimation test of their automatic/digital level following the method discussed in Section 4.15.5. Have the student report on the collimation error in their instrument and discuss how this error will be removed when using the instrument for differential leveling. They should also compute the maximum allowable difference in plus and minus sight distances if this error is to be kept under on-half of their reading. For example 0.005 ft if the minimum reported elevation is to 0.01 ft.

5	E	Using a nearby bench mark as control, the students should run a leveling loop from the bench mark, over their stations, and back to the bench mark meeting Third Order leveling specifications. The report should contain a listing of the final adjusted elevations for each station, discuss any problems encountered in the field, include a copy of the final field notes, and provide the misclosure in the loop. If the exercise for Chapter 4 was performed, then collimation error should be removed from each elevation.
6	F	Using a tape, measure the length of each course in the assigned traverse. The line should be measured twice and a precision computed. The report should contain a copy of the field notes, and discuss any problems encountered.
6	G	Using a EDM, determine the horizontal length of each course in a line. The line should be measured from two stations. The report should contain a copy of the field notes, the average length for each line, and discuss any problems that may have occurred in the field.
8	H	Using a theodolite or total station, the students should close the angular horizon about each of their stations turning each angle two times with each face of the instrument (2DR). Using this information, the students should determine the horizon misclosure, adjust the angles at each station, and then adjust the interior angles of the traverse. The report should contain the original field notes, list the horizon misclosure at each station, adjusted angles, traverse misclosure, and the correct geometric sum of each angle. Students should make sure that all angles are geometrically closed.
10	I	Using the distances observed in Chapter 8 and the angles observed in Chapter 8, and an assigned or assumed azimuth for one course of their traverse, students should perform a compass rule adjustment of the traverse. Using starting coordinates of (1000.000, 5000.00), the report should contain the linear precision, relative precision, the adjusted latitudes and departures, coordinates for each station, and adjusted observations.
11	J	Do problem 11.18, 11.37, 11.38, or 11.39.
12	K	Compute the area of the traverse from the exercise for Chapter 10.

13	L	Do Problem 13.35 and 13.36.
14	M	Do Problem 14.40.
14	N	Perform a rapid static survey of your traverse. Adjust the baselines and the network. Report on the adjusted baseline vector components, the loop closures as discussed in Section 14.5.4.
15	O	Have students perform an kinematic mapping surveying of a local area.
16	P	Do one of the problems from 16.41 to 16.45.
17	Q	Have students collect radial data to map an assigned area around their traverse. If a controller is available, the students should use the codes discussed in Section 17.11 that are appropriate for your software.
18	R	Have students create a map of the data collected in exercise Q.
19	S	Have students create the program for Problem 19.43 or 19.44.
20	T	Have students create the program for Problem 20.47 or 20.48.
21	U	Research the deeds for you school or an assigned parcel and perform a boundary survey. In the report, note the survey procedures used, their closures, found monuments in agreement with the deed, monuments that do not agree, and monuments not found.
22	V	Layout a township at a 1/10th scale following the procedures discussed in Chapter 22.
23	W	Perform a profile level courses for the traverse from Chapter 10 using 25-ft stationing.
24	X	Compute the stakeout notes for a horizontal curve with an intersection angle of 60° and length of 300 ft or 100 m. If you are using English units, use 25-ft stationing. If you are using metric units, use 10-m stationing. Stake the curve in the field using the incremental chord method. If you have a data collector, use WOLFPACK to compute coordinates for the given horizontal curve and stake it out using the controller's stake out functions.

25	Y	After profile-leveling the horizontal curve staked out in the previous exercise, compute a vertical alignment that minimizes excavation.
26	Z	Do Problem 26.31 or 26.32.
27	AA	Have students do either Problem 27.38, 27.39, 27.40, or 27.41.
28	BB	Using a GIS software package and the shape files provided by the NGS at http://www.ngs.noaa.gov/cgi-bin/datasheet.prl develop a GIS that allows the user to find NGS control stations in your county and sort by type and quality.

Sample Quizzes

Quiz 1

1. One acre equal _____________ square feet and ____________ square Gunter's chains.
2. Give the answer of the following problems rounded to the correct number of significant figures:
   a. Sum of 0.0237, 30.05, 254.0 _____________
   b. Product of 31.75 × 4.0 _____________
   c. Quotient of 793.82 ÷ 71 _____________

Quiz 2

1. For the following ten repeated EDM observations what are
   325.686, 325.685, 325.687, 325.681, 325.691, 325.686, 325.681, 325.686, 325.690, and 325.689

   a. Most probable value _____________
   b. Standard error in a single observation _____________
   c. 95 per cent probable error ___________

Quiz 3

1. For the following sequential minus (FS) and plus (BS) sights observed on a closed level circuit, set up the left-side of a standard set of level notes and give an adjusted elevation for BM A.

   BM X (elev = 850.25): 3.87
   TP 1: FS = 3.73, BS = 6.80
   TP 2: FS = 7.04, BS = 6.22
   BM A: FS = 9.16, BS = 3.49
   TP 3: FS = 7.20, BS = 8.65
   TP 4: FS = 6.38, BS = 6.00
   BM X FS = 1.58

Quiz 4

1. A 100-ft tape is calibrated at 68° F, fully supported with 15 lbs of tension and found to be 99.987 ft long. Is this tape is used fully supported with 15 lbs of tension at 86° F to measure a distance that is recorded as 136.48 ft, what is the corrected length of the line?

____

2. If a certain EDM has a centering error of 3 mm and a scalar error of 3 ppm, what is the uncertainty in a observed distance of 1380.25 ft?

____

Quiz 5

1. In 1895 when the magnetic declination was 6°45′ East, line AB had a magnetic bearing of S 7 30 E.
   a. What is the magnetic bearing of AB today if the current magnetic declinations is 2 30 W?

   ____

   b. What are the true bearing and true azimuth of this line?

   ____

Quiz 6

1. If the slope of a line is 0.3258, what is the azimuth of the line? _____________
2. What is the area in square units of a polygon with coordinates at its vertices of (103.45, 214.87), (250.34, 567.98), and (185.02, 386.94) _____________

Sample Exams

Exam 1

(1 point each)
True – False [Fill in the circle indicating whether the statement is true (T) or false (F).]

○ T ○ F 1. The current definition of the meter is 39.37 inches is equivalent to one meter.
○ T ○ F 2. The length of 1429.75 m is equivalent to 4690.72 survey feet to the correct number of significant figures.
○ T ○ F 3. Random errors may be mathematically computed and removed from observations.
○ T ○ F 4. The number 1.0020 has five significant figures.
○ T ○ F 5. A set of precise observations is always accurate.
○ T ○ F 6. National representation of surveying interest is the principal interest of the American Congress on Surveying and Mapping.
○ T ○ F 7. One acre is 43, 560 square feet.
○ T ○ F 8. A Gunter's chain is 100 ft long.
○ T ○ F 9. The correctly round sum of 46.328 + 1.03 + 375.1 is 422.4.
○ T ○ F 10. The National Geodetic Survey is responsible for establishing a network of survey control monuments.
○ T ○ F 11. The arrangement of a field book is a matter of personal preference.
○ T ○ F 12. It is best to only enter a minimum amount of data into a field book.
○ T ○ F 13. A new page should be started in the field book for each new day of work.
○ T ○ F 14. The geoid is an equipotential surface.
○ T ○ F 15. Earth curvature always causes rod readings to be too high.
○ T ○ F 16. Parallax exists when the focal point of the objective lens does not coincide with the focal point of the eyepiece lens.
○ T ○ F 17. The NAVD 88 datum is based on the average elevation of 26 tide gage stations.
○ T ○ F 18. A page check in differential leveling only provides an arithmetic check of the notes.
○ T ○ F 19. Automatic levels guarantee a horizontal line of sight at each setup.
○ T ○ F 20. The statistical term used to express the precision of a data set is called standard deviation.

Problems/Short answers

(5 points)

A. Discuss why the term geomatics is being used to identify the profession of surveying.

(10 points)

B. State the number of significant figures in each of the following values.

______ 0.0024 ______ 7620 ______ 0.0007621 _______ 1050.130 ______ 750.

(10 points)

C. Convert the following observations as indicated.

(a) 164.803 m to U.S. Survey feet ___________

(b) 215.648 grads to degrees-minutes-seconds ___________

(c) 12 ch 7 lks to survey feet ___________

(d) 123,600 sq. ft. to acres ___________

(e) 153 26 14 to radians ___________

(15 points)

D. Compute the most probable value, standard deviation, and 95% probable error for the following set of angle observations.

116°13′46″,116°13′46″,116°13′48″,116°13′44″,116°13′50″

MPV = _____________

σ = ____________

$E_{95\%}$ = _____________

(5 points)

E. Compute the combined Earth curvature and refraction on a 3000-ft site. _____________

(20 points)

F. A differential leveling circuit is starts at bench mark Hydrant (Elevation 430.330 m) and ends at bench mark Post (Elevation 430.002 m). The readings (in meters) list in the order taken are 0.983 on Hydrant, 5.467 and 4.086 pm TP1, 0.952 and 3.905 on Mark; and 2.886 on Post. Use this information to complete the left-side of differential leveling notes.

Exam 2

(1 point each)

True – False [Fill in the circle indicating whether the statement is true (T) or false (F).]

○ T ○ F 1. EDM's are unaffected by refraction.
○ T ○ F 2. The length correction in taping is an example of a random error.
○ T ○ F 3. The velocity of an electromagnetic wave does not change when passing through atmosphere.
○ T ○ F 4. A cut tape is graduated with an extra foot beyond the zero mark.
○ T ○ F 5. The NGS as specifications for "third-class leveling."
○ T ○ F 6. A collimation test checks if the line of sight in a leveling instrument is horizontal.
○ T ○ F 7. When measuring distance with an EDM, the line of sight should never be within 1 m anywhere along its path.
○ T ○ F 8. A rod level will increase both the accuracy and speed in the field.
○ T ○ F 9. In general, humidity is irrelevant when measuring distances with a near-infrared EDM.
○ T ○ F 10. Magnetic declination is the difference between geodetic azimuth and magnetic azimuth.
○ T ○ F 11. A total station is in adjustment if its line-of-sight axis is perpendicular to its vertical axis.
○ T ○ F 12. One-second of arc is about 0.05 ft in 10,000 ft.
○ T ○ F 13. The DIN 18723 standard is based on the observation of a single direction.
○ T ○ F 14. The "principle of reversion" is used when adjusting level bubbles.
○ T ○ F 15. In practice, instruments should always be kept in good adjustment, but used as though they might not be.

(1 point each)

Fill in the blank

The kinds of horizontal angles most commonly observed in surveying are:

(1) ___________________, (2) ________________, and (3) ___________________.

A. Azimuths may be (1) ____________________, (2) ___________________,

(3) ___________________, (4) ___________________, (5) ___________________, and

(6) ___________________.

Short Answer Problems

(10 points)

B. A distance is measured with an EDM having the instrument/reflector offset constant set to 0 mm. The slope distance is reported as 2435.672 m with a zenith angle reading of 93°34′05″. The offset is later determined to be 23 mm. What is the correct horizontal distance for this observation?

____

(10 points)

C. A 100-ft steel tape has a length 99.987 ft when fully supported at a temperature of 68° F and tension of 10-lbs. What is the corrected length of a measured by this fully-supported tape if the recorded length, temperature and tension are 83.05 ft, 33° F, and 25 lbs of tension, respectively.

(10 points)

D. A 867.89 ft distance is measured with an EDM that has a manufacturer’s specified accuracy of 3 mm + 3 ppm. Both the instrument and target miscentering errors are assumed to be ±0.005 ft. What is the uncertainty in this observation?

____

(10 points)

E. The magnetic bearing of a line in 1884 was N 23 15'W . The magnetic declination at this times was 5 12 W. What is the true bearing of this line?

____

(10 points)

F. A zenith angle was measured twice direct giving values of 92 14 26 and 92 14 28 , and twice reversed yielding readings of 267 45 30 both times. What is the mean zenith angle, and the indexing error?

____

(10 points)

G. Discuss the field procedure used to prolong a line of sight.

Exam 3

(1 point each)

True – False [Fill in the circle indicating whether the statement is true (T) or false (F).]

○ T ○ F 1. A closed traverse begins and ends at a station of known coordinates.

○ T ○ F 2. If the azimuth of a line is 272°15′26″, then the bearing of the same line is S92°15′26″W.

○ T ○ F 3. Excepts for deflection angles, surveyors should always turn angles clockwise.

○ T ○ F 4. Angles to the right are clockwise angles with backsights at the "rearward" station and the foresights on the "forward" station.

○ T ○ F 5. To avoid ambiguity, only two reference ties should be used to reference a station.

○ T ○ F 6. Adjustment of angles is dependent on the size of the angle.

○ T ○ F 7. The departure of a course is the change in the x coordinate.

○ T ○ F 8. Open traverses should only be used as a last resort in surveying.

○ T ○ F 9. A single angular mistake can be identified by extending the perpendicular bisector of the linear closure line.

○ T ○ F 10. The intersection of two lines with known lengths always results in two possible solutions.

(5 points)

A. What is the geometric closure on a closed polygon traverse with 18 sides? ______________

(10 points)

B. What is the azimuth of line CD if the azimuth of AB is 105°39′12″, angle ABC is 67°35′08″, and angle BCD is 275°10′15″?

____

(10 points)

C. If line AB has an azimuth of 156°14′34″ and line BC has an azimuth of 41°56′42″, what is angle ABC?

____

(5 points)

D. What is the angular misclosure on a five-sided traverse with observed interior angles of 83°07′23″, 105°23′01″, 124°56′48″, 111°51′31″, and 114°41′27″?

____

(15 points)

E. Fill in the missing parts of the closed traverse table below.

Azimuth	Distance	Departure	Latitude
45°32′15″	415.76		
101°56′35″			−112.85
	644.65	−502.27	
209°23′00″	668.46		

(10 points)

F. Station A has *xy* coordinates (in feet) of (42992.36, 14354.37) and station B has *xy* coordinates of (43476.79, 15110.90). What are the course length and azimuth?

____

____

(10 points)

G. If the sum of the departures in a closed polygon traverse having a total perimeter of 3911.05 ft is 0.22 ft, what is the correction to a course of length 1007.38 ft have a departure of 726.76 ft?

____

(10 points)

H. What is the linear misclosure and relative precision of a traverse of 2169.91ft if the misclosure in departure and latitude are −0.017 ft and −0.086 ft, respectively?

____

____

(10 points)

I. The azimuth of a line in an assumed coordinate system is 242°15′26″. The azimuth of the same line in a datum is 168°38′22″. What is the rotational angle needed to perform a two-dimensional conformal coordinate transformation?

____

(5 points)

J. If the standard error for each angle measurement of a traverse is ±3″, what is the estimated error in the geometric closure in the sum of the angles for a 12-sided traverse?

____

Final Exam

Equation Sheet

$\pm\sqrt{E_a^2 + E_b^2 + E_c^2 + \ldots}$ $\qquad$ $\bar{M} - M$ $\qquad$ $\pm\sqrt{\dfrac{\Sigma v^2}{n-1}}$

$\pm E\sqrt{n}$ $\qquad$ $\left(\dfrac{l - l'}{l'}\right)L$ $\qquad$ $L \cos \alpha$

$0.65 \times 10^{-5}(T_1 - T)L$ $\qquad$ $(P_1 - P)\dfrac{L}{\left(29 \times 10^6\right)A}$ $\qquad$ $-\dfrac{w^2 L_S^3}{24 P_1^2}$

$R\theta$ $\qquad$ $\dfrac{2 \times E_{DIN}}{\sqrt{n}}$ $\qquad$ $\alpha \tan(v)$

$0.574\,M^2 = 0.0206\,F^2$ $\qquad$ $0.0675\,K^2$ $\qquad$ $S \sin \alpha$

$hi + V - r$ $\qquad$ $\dfrac{R_B - r_A - r_B + R_A}{2}$ $\qquad$ $Elev + BS - FS$

$(n-2)180°$ $\qquad$ $L \sin \alpha$ $\qquad$ $L \cos \alpha$

$-\dfrac{(\text{total departure misclosure})}{\text{traverse perimeter}} \times \text{length of } AB$ $\qquad$ $-\dfrac{(\text{total latitude misclosure})}{\text{traverse perimeter}} \times \text{length of } AB$

$X_A + \text{departure } AB$
$Y_A + \text{latitude } AB$ $\qquad$ $\sqrt{(\text{departure } AB)^2 + (\text{latitude } AB)^2}$ $\qquad$ $\tan^{-1}\left(\dfrac{\Delta X}{\Delta Y}\right) + C$

$\sqrt{s(s-a)(s-b)(s-c)}$ $\qquad$ $b\left(\dfrac{h_0}{2} + h_1 + h_2 + \cdots + \dfrac{h_n}{2}\right)$ $\qquad$ $S \times \sigma_S \sqrt{2}$

$$\frac{1}{2}[X_A(Y_E - Y_B) + X_B(Y_A - Y_C) + X_C(Y_B - Y_D) + X_D(Y_C - Y_E) + X_E(Y_D - Y_A)]$$

$\text{meridian distance of } AB + \dfrac{1}{2}\text{departure of } AB + \dfrac{1}{2}\text{departure of } BC$

$\pi R^2 \times (\theta/360°)$ $\qquad$ $\pm\sqrt{A^2 E_b^2 + B^2 E_a^2}$ $\qquad$ $\dfrac{E}{\sqrt{n}}$

Final Exam

(1 point each)
TRUE - FALSE [Fill in the circle indicating whether the statement is true (T) or false (F).]

❍T ❍F 1. The number 768,000 has six significant digits.

❍T ❍F 2. There are 45,360 square feet in one acre.

❍T ❍F 3. Using the survey foot definition, one meter equals 39.37 inches.

❍T ❍F 4. 24 times 360.01 equals 8640 to the correct number of significant digits.

❍T ❍F 5. 78.149 is equal to 78.2 when round to the tenths place.

❍T ❍F 6. Field notes should be discarded when a project is complete.

❍T ❍F 7. Sketches in a field book should be drawn to an accurate scale.

❍T ❍F 8. A zenith angle is measured in the horizontal plane.

❍T ❍F 9. The sensitivity of a level vial is determined by its radius of curvature.

❍T ❍F 10. In leveling, balancing plus sight and minus sight distances corrects for instrument collimation errors.

❍T ❍F 11. Refraction always causes the line of sight to appear to be too high.

❍T ❍F 12. "*Accuracy*" denotes absolute nearness to the truth.

❍T ❍F 13. Bringing the bubble *halfway back* to center compensates for the fact that the vertical axis of a total station is not perpendicular to the axis of the plate bubble.

❍T ❍F 14. Measuring angles both direct and reverse compensates for the fact that the vertical axis in a total station may not be perpendicular to the horizontal axis.

❍T ❍F 15. Under a fixed set of conditions, random errors have the same magnitude and sign.

❍T ❍F 16. Waving the rod during leveling compensates for curvature and refraction.

❍T ❍F 17. A slope distance measured by EDM must be corrected for atmospheric pressure and temperature.

❍T ❍F 18. Three repeated measurements for a distance are 395.28, 395.27, and 395.29 ft., respectively. These measurements are precise when the true value of the measurement is 395.95 ft.

❍T ❍F 19. Vertical lines at all locations are parallel.

❍T ❍F 20. A ten second level vial is more sensitive than a two second vial.

❍T ❍F 20. A prism causes all electronically measured distances to appear to be too long.

❍T ❍F 21. The geoid is an equipotential surface.

❍T ❍F 22. A zenith angle of 88°15′ is equivalent to a vertical angle of 1°45′.

❍T ❍F 23. The sum of the interior angles of a seven-side closed polygon traverse should be 900°.

❍T ❍F 24. The compass rule adjustment is known as an arbitrary adjustment technique.

❍T ❍F 25. Angles of larger magnitude should always receive the largest corrections.

❍T ❍F 26. A precision of 1:5000 means that there can be 0.5 foot of error in every 2500 ft.

❍T ❍F 27. Surveying plats show slope distances recorded between points.

❍T ❍F 28. When the tape is only supported at its ends, the recorded distance is always too long.

❍T ❍F 29. A link traverse is an example of an open traverse.

❍T ❍F 30. 4129.57 m is equal to 2.56599 mi to the correct number of places.

PROBLEMS

The following are six repeated measurements of a taped distance.

429.35 , 429.34, 429.37, 429.32, 429.39, 429.33

{5 points}
A. What is the most probable value of the measurement? (nearest 0.01) MPV = ______________

{5 points}
B. What is the standard deviation? (nearest 0.001) σ = _____________

The following questions apply to measurements using a steel tape which was calibrated to be 99.890-ft long when fully supported at 68°F and 10-lbs. pull. Its cross-sectional area was 0.0050 square inches.

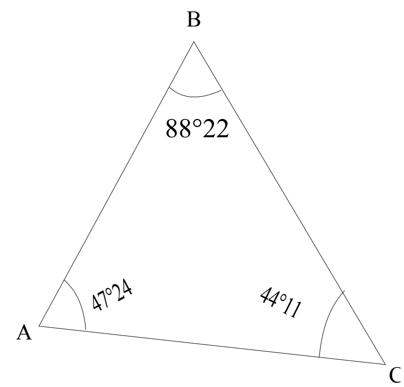
{5 points}
C. A line AB was measured on flat ground with the tape fully supported using 10 lbs. of pull and recorded to be 275.20-ft. long when the temperature was 43°F. What is its corrected horizontal length? (nearest 0.001 ft)

L = __________________

{5 points}
D. A horizontal distance *DE* exactly 275.20 ft. is required to be laid out according to a blue print. If this distance will be laid out on flat ground using the tape fully supported with a 20-lb pull, and the temperature is 85°F what distance must be measured using this tape. (nearest 0.01-ft)

L = ________________

{10 points}
E. For the measured angles given on the figure below, and assuming the fixed azimuth of the line AB is 47°45′, calculate the adjusted angles and azimuths of lines BC and CA and show a check. (nearest 1′)



∠A = ______°______'

∠B = ______°______'

∠C = ______°______'

Az_{BC} = ______°______'

Az_{CA} = ______°______'

{10 points}
F. What is the area of the five-sided parcel below to the nearest square foot? ... nearest 0.001 acre?

Station	X (ft)	Y (ft)
A	0.00	591.64
B	125.66	847.60
C	716.31	294.07
D	523.62	0.00
E	517.55	202.97

AREA = ________________ ft^2

AREA = ________________ acres

(10 points)
G. The coordinates of A and B are (23451.23, 10034.56) and (22678.93, 12387.43), respectively. What are the azimuth and distance of the line AB. (nearest 1", nearest 0.01 ft)

AB = ________________ ft

Az_{AB} = ________°________'________"

(10 points)

H Two measurements that presented unusual difficulty in the field were omitted in the survey of a boundary, as shown in the following field notes. Compute the missing distance and azimuth for line *EA*. (nearest 1″; nearest 0.01 ft)

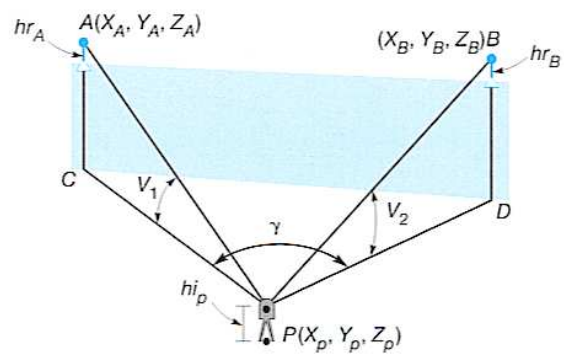
Line	Distance	Azimuth	Departure	Latitude
AB	671.07	88°00′00″		
BC	436.45	91°00′00″		
CD	510.67	206°00′00″		
DE	778.05	318°00′00″		
EA				

EA = ____________________ ft

Az_{EA} = ________°________'________"

(10 points)

I In the figure to the right, the *X*, *Y*, and *Z* coordinates (in feet) of station *A* are 3860.83, 4819.98, and 154.06, respectively, and those at *B* are 6865.48, 5007.21, and 135.69, respectively. Determine the three-dimensional of a total station at point *P* base upon the following observations.



$v_1 = 0°29′06″$ $PA = 2423.67$ ft
$v_2 = -0°03′04″$ $PB = 2413.00$ ft
$\gamma = 76°59′20″$
$hr_A = 5.20$ ft $hr_B = 5.20$ ft $hi_P = 5.20$ ft