AMERICAN UNIVERSITY OF BEIRUT

Mathematics Department

Math 218 - Quiz I

Fall 2005-2006

Name:................................ ID:...............................

Section 1	Section 2	Section 3
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Time: 60 min

I- **(20 points)** For what values of m does the following system have:

$$\begin{cases} x + mz = 0 \\ x + y = 1 \\ x + my - 2z = -1 \end{cases}$$

i- no solutions.

ii- infinitely many solutions.

iii- a unique solution.

/5

/10

$$\left(\begin{array}{ccc|c} 1 & 0 & m & 0 \\ 1 & 1 & 0 & 1 \\ 1 & m & -2 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & m & 0 \\ 0 & 1 & -m & 1 \\ 0 & m & -2-m & -1 \end{array}\right) \longleftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & m & 0 \\ 0 & 1 & -m & 1 \\ 0 & 0 & m^2-2-m & -(m+1) \end{array}\right)$$

$(m+1)(m-2)$

$m = -1$ infinitely many

$m = (+2)$ No sol

$m \neq (-1, 2)$ One sol.

II- Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 2 & 4 \\ 1 & 2 & 1 \end{pmatrix}$$

a- **(10 points)** Find the inverse of the matrix A.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 2 & 4 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} R_2-2R_1 \\ R_3-R_1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -2 & 4 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_1-2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & -1 & 1 & 0 \\ 0 & 1 & -2 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$R_2+2R_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -\frac{1}{2} & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$R_1-4R_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & -4 \\ 0 & 1 & 0 & -1 & -\frac{1}{2} & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 3 & 1 & -4 \\ -1 & -\frac{1}{2} & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

b- **(5 points)** Use part (a) to solve the following system

$$\begin{cases} x + 2y = 2 \\ 2x + 2y + 4z = -1 \\ x + 2y + z = 2 \end{cases}$$

$$X = A^{-1} b = \begin{bmatrix} 3 & 1 & -4 \\ -1 & -\frac{1}{2} & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}.$$

$$= \begin{bmatrix} 6 - 1 - 8. \\ -2 + \frac{1}{2} + 4 \\ -2 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ \frac{5}{2} \\ 0 \end{bmatrix}$$

III- a- **(5 points)** Let A be a square matrix such that $A^3 + 4AA^T - 2A - 7I = 0$. Show that A is invertible and find A^{-1}.

$$A(A^2 + 4A^T - 2I) = 7I$$

$$A\left(\frac{1}{7}(A^2 + 4A^T - 2I)\right) = I.$$

$$A^{-1} = \frac{1}{7}(A^2 + 4A^T - 2I).$$

b- **(5 points)** Let A, B and C be square matrices of the same size such that $B = A^{-1}CA$. Show that $B^2 = A^{-1}C^2A$ and deduce B^n.

$$B = A^{-1}CA.$$

$$B^2 = A^{-1}CAA^{-1}CA.$$

$$= A^{-1}C^2A.$$

$$B^n = A^{-1}C^nA.$$

IV- (10 points) Find the values of k for which the matrix is **not** invertible.

$$A = \begin{pmatrix} 1 & 3 & 5 & 1 \\ 0 & 3-k & 5 & 2 \\ 3 & 9 & 10 & k \\ 0 & 0 & 0 & -k \end{pmatrix}$$

$$R_3 - 3R_1 \begin{pmatrix} 1 & 3 & 5 & 1 \\ 0 & 3-k & 5 & 2 \\ 0 & 0 & -5 & k-3 \\ 0 & 0 & 0 & -k \end{pmatrix}$$

$$|A| = (3-k)(-k)(-5)$$

$k = 3$

$k = 0$

✓

V- Let $A = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 1 & 1 \\ 3 & 5 & 2 \end{pmatrix}$.

a- **(10 points)** Find $\det(A)$

$= -3 + 3(2) = 3$.

b- **(20 points)** Find the following

i- $\det(A^4)$

3^4

ii- $\det((2A)^{-3})$ $\quad (2A)^{-3} = \frac{1}{8} A^{-3}$.

$\cancel{\frac{1}{8} |A|^{-3}} = \frac{1}{8^3} \frac{1}{|A|^3} = \frac{1}{8^3} \cdot \frac{1}{3^3}$

iii- $\det(A^T A^{-1})$ $= |A| . |A^{-1}| = 3 \frac{1}{3} = 1$

iv- $\det(\mathrm{adj}(A))$ $=$

$A . \mathrm{adj}(A) = |A|^n I \qquad |A| |\mathrm{adj}| = |A|^n$

$|\mathrm{adj} A| = \frac{3^3}{|A|} = 3^2$.

$|\mathrm{adj}| = |A|^n / |A| = |A|^{n-1}$

VI- a- **(10 points)** Answer **TRUE** or **FALSE** without proof.

i- $(\det(A))^2 = \det(A^2)$ T

ii- If $\det(A) = 0$ the system $AX = b$ has infinitely many solutions. F

iii- Let A and B be two matrices of the same size, if AB is invertible then A and B are both invertible. T

iv- If A is invertible then the system $AX = 2X$ has exactly one solution. F

v- $\mathrm{tr}(A^T)=\mathrm{tr}(A)$ T

b- **(5 points)** Justify your answer if **TRUE** or give a counter example if **FALSE** for part **(iii) ONLY**.

$|AB| = |A| \cdot |B| \neq 0$

$\Rightarrow |A| \neq 0$

$|B| \neq 0$

GOOD LUCK!!