Performance measures of second-order systems

Settling time (2% criterion) $T_s = 4\tau = \dfrac{4}{\zeta\omega_n}$

Peak time $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Peak value $M_{pt} = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Percent overshoot $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Rise time (10% to 90%) $T_{r1} \cong \dfrac{2.16\zeta + 0.60}{\omega_n}$

Ziegler-Nichols PID Controllers

Table 7.7 Ziegler-Nichols PID Tuning Using Ultimate Gain, K_U, and Oscillation Period, P_U

Ziegler-Nichols PID Controller Gain Tuning Using Closed-loop Concepts

Controller Type	K_P	K_I	K_D
Proportional (P) $G_c(s) = K_P$	$0.5K_U$	–	–
Proportional-plus-integral (PI) $G_c(s) = K_P + \frac{K_I}{s}$	$0.45K_U$	$\frac{0.54K_U}{T_U}$	–
Proportional-plus-integral-plus-derivative (PID) $G_c(s) = K_P + \frac{K_I}{s} + K_D s$	$0.6K_U$	$\frac{1.2K_U}{T_U}$	$\frac{0.6K_U T_U}{8}$

Table 7.8 Ziegler-Nichols PID Tuning Using Reaction Curve Characterized by Time Delay, T_d, and Reaction Rate, R

Ziegler-Nichols PID Controller Gain Tuning Using Open-loop Concepts

Controller Type	K_P	K_I	K_D
Proportional (P) $G_c(s) = K_P$	$\frac{1}{RT_d}$	–	–
Proportional-plus-integral (PI) $G_c(s) = K_P + \frac{K_I}{s}$	$\frac{0.9}{RT_d}$	$\frac{0.27}{RT_d^2}$	–
Proportional-plus-integral-plus-derivative (PID) $G_c(s) = K_P + \frac{K_I}{s} + K_D s$	$\frac{1.2}{RT_d}$	$\frac{0.6}{RT_d^2}$	$\frac{0.6}{R}$

Phase-Lead (or Phase-Lag) design using the root locus method:

$$G_c(s) = \frac{s+z}{s+p}$$

- Root locus method to design Lead network is as follows:
  1. From the performance specifications determine the desired location of the closed loop dominant poles
  2. Verify whether the desired poles can be obtained with the uncompensated system
  3. If a compensator is needed, place the zero of the phase lead network directly below the desired root location (or to the left of the first two real poles of the system $G(s)$)
  4. Determine the angle θ_p of the pole using the root locus phase criterion and then deduce the value of $-p$
  5. Evaluate the total system gain at the desired root location using the magnitude criterion and then calculate the error constant
  6. If the error constant is not satisfactory, repeat the design steps and alter the location of the desired dominant roots as well as the compensator zero and pole

- Root locus method to design Lag network is as follows:
  1. Obtain the root locus of the uncompensated system
  2. Determine the transient performance specifications and locate suitable dominant root locations on the uncompensated root locus that will satisfy the specifications
  3. Calculate the loop gain at the desired root location and thus the uncompensated system error constant
  4. Compare the uncompensated error constant with the desired error constant, and calculate the necessary zero-pole ratio (z/p)
  5. From the known ratio z/p, determine a suitable location of the pole and zero of the compensator (locate them near the origin of the s-plane in comparison to ω_n)

Frequency Response

Resonant frequency ($\zeta \le 0.7$):

$$\omega_r = \omega_n \sqrt{1-2\zeta^2}$$

Maximum magnitude ($\zeta \le 0.7$):

$$M_{p\omega} = |T(\omega_r)| = \left(2\zeta\sqrt{1-\zeta^2}\right)^{-1}$$

Bandwidth:

$$\omega_B = \omega_n(-1.19\zeta + 1.85) \quad \text{for} \quad 0.3 \le \zeta \le 0.8$$

Phase-Lead Design using the Bode Diagram:

$$G_c(j\omega) = \frac{1 + j\omega\alpha\tau}{\alpha(1 + j\omega\tau)}$$

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1} \qquad (10.11)$$

- Design of Phase-Lead network using Bode diagram:
  1. Evaluate the uncompensated system phase margin when the error constants are satisfied
  2. Allowing for a small amount of safety, determine the necessary additional phase lead ϕ_m
  3. Evaluate α from Equation (10.11)
  4. Evaluate $10\log\alpha$ and determine the frequency where the uncompensated magnitude curve is equal to $-10\log\alpha$ dB. This frequency is the new 0-dB crossover frequency and ω_m simultaneously
  5. Calculate the pole $p = \omega_m\sqrt{\alpha}$ and the zero $z = p/\alpha$
  6. Draw the compensated frequency response, check the resulting phase margin, and repeat the steps if necessary. Finally, raise the gain of the amplifier to account for the attenuation ($1/\alpha$).

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Phase-Lag Design using the Bode Diagram:

$$G_c(j\omega) = \frac{1 + j\omega\alpha\tau}{1 + j\omega\tau}$$

- Design of Phase-Lag network using Bode diagram:
  1. Obtain the Bode diagram of the uncompensated system with the gain adjusted for the desired error constant
  2. Determine the phase margin of the uncompensated system and, if it is insufficient, proceed with the following steps
  3. Determine the frequency where the phase margin requirement would be satisfied if the magnitude curve crossed the 0-dB line at this frequency, ω'_c. (Allow for 5^o phase lag from the phase-lag network when determining the new crossover frequency)
  4. Place the zero of the compensator one decade below the new crossover frequency: $\omega_z = \omega'_c/10$, and thus ensure only 5^o of additional phase lag at ω'_c due to the lag network
  5. Measure the necessary attenuation at ω'_c to ensure that the magnitude curve crosses at this frequency
  6. Calculate α by noting that the attenuation introduced by the phase-lag network is $-20 \log \alpha$ at ω'_c
  7. Calculate the pole as $\omega_p = \omega_z/\alpha$

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