**Given:**

$k = 9\times10^{9}\ \text{N.m}^2/\text{C}^2$

$\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2$

$g = 9.81\ \text{m/s}^2$

Mass of proton: $1.67\times10^{-27}$ kg

Mass of electron: $9.1\times10^{-31}$ kg

Charge of electron: $-1.6\times10^{-19}$ C

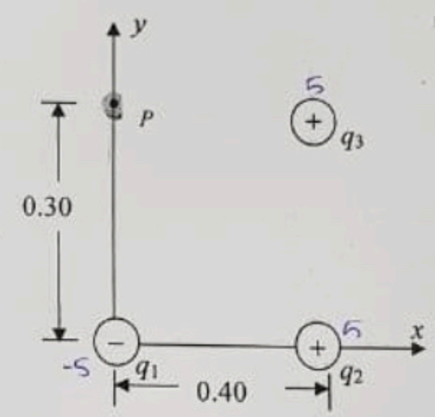
Charge of proton: $1.6\times10^{-19}$ C

$$\int\frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2(a^2+x^2)^{1/2}} \quad \text{and} \quad \int\frac{x\,dx}{(a^2+x^2)^{3/2}} = -\frac{1}{\sqrt{(a^2+x^2)}}$$

30

**Problem 1:** **(30 Points) The Electric Field of Multiple Point Charges**

A charge $q_1 = -5.0$ nC is located at the origin. A charge $q_2 = +5.0$ nC is located on the $x$-axis, 0.40 m from the origin as shown in the figure to the right. A charge $q_3 = +5.0$ nC is placed at the point (0.4, 0.3). Find the electric field, in component and magnitude and direction (angle) form, at the point $P$ (0, 0.30) m. Follow these steps:




1. Find the electric field $\vec{E}_1$ in components form due to $q_1$ at the point $P$.
2. Find the electric field $\vec{E}_2$ in components form due to $q_2$ at the point $P$.
3. Find the electric field $\vec{E}_3$ in components form due to $q_3$ at the point $P$.
4. Find the resultant field $\vec{E}$ in components form at the point $P$.
5. Draw on the diagram the resultant field $\vec{E}$ at the point $P$.
6. Find the magnitude of $\vec{E}$ and the angle that $\vec{E}$ makes with the positive $x$-direction.

-486,36
-392,03
578,388

$1.\ \vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2}\hat{j} = \frac{1}{4\pi\varepsilon_0}\frac{(5\times10^{-9})}{(0,3)^2}\hat{j} = -500\hat{j}$ N/C ✓ 5

$2.\ \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2}(\cos\theta\,\hat{i} \cdot \sin\theta\,\hat{j})$  $\tan\theta = \frac{r_1}{r_2} = \frac{0,3}{0,4}$

$\theta = \tan^{-1}\left(\frac{0,3}{0,4}\right) = 36,86°$

$= \frac{1}{4\pi\varepsilon_0}\frac{5\times10^{-9}}{(\sqrt{0,4^2+0,3^2})^2}(\cos 36,86° + \sin 36,86)$

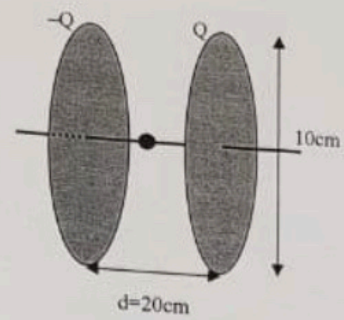
$= -180\cos 36,86\,\hat{i} + 180\sin 36,86\,\hat{j}$

$= -144,01\,\hat{i} + 107,97\,\hat{j}$ N/C ✓ 5

**Problem 3:** **(20 Points) Charged Disks**

Two 10cm diameter charged disks face each other d=20cm apart. The left disk is charged to $-Q = -40nC$ and the right disk is charged to $Q = +40nC$. Follow these steps:

1. What is the electric field strength and direction at the midpoint between the disks?
2. What is the electric force on a $-1nC$ charge placed at the midpoint?




1) $dE = \frac{dq}{4\pi\varepsilon_0 r^2} \cdot \frac{z}{(z^2+R^2)^{1/2}}$

$= \frac{dq\, z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} = \frac{z}{4\pi\varepsilon_0} \int dq \qquad Q_i = \eta \Delta A_i = \eta 2\pi r_i\, dr$

$= \frac{z Q_i}{4\pi\varepsilon_0 (z^2+r_i^2)^{3/2}} = \frac{z \eta 2\pi r_i\, dr}{4\pi\varepsilon_0 (z^2+r_i^2)^{3/2}}$

$E = \frac{z \eta 2\pi}{4\pi\varepsilon_0} \int_0^R \frac{r_i\, dr}{z^2+r_i^2} = \frac{z\eta}{2\varepsilon_0} \left[\frac{-1}{\sqrt{z^2+r^2}}\right]_0^R$

$= \frac{z\eta}{2\varepsilon_0}\left(\frac{-1}{\sqrt{z^2+R^2}} - \frac{-1}{\sqrt{z^2+0}}\right) = \frac{z\eta}{2\varepsilon_0}\left[\frac{-1}{\sqrt{z^2+R^2}} + \frac{1}{z}\right]$

$E = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$

$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{40\times10^{-9}}{\pi(10\times10^{-2})^2} = 1{,}273\times10^{-6}$

$E = \frac{1{,}273\times10^{-6}}{2\times 8{,}85\times10^{-12}}\left[~~1~~ - \frac{(10\times10^{-2})}{\sqrt{(10\times10^{-2})^2+(5\times10^{-2})^2}}\right]$

$= 7592{,}89\ N/C = 7{,}59\times10^{3}\ N/C$

rightward

71980, 40395, 5.9...

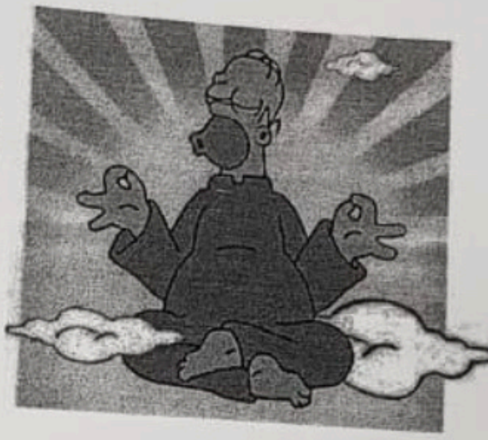
18

2) $\vec{F} = q\vec{E}$

$= (-1\times10^{-9})\times 7592{,}89 = 7{,}59\times10^{-6}$

**Problem 4 : (15 points) Levitation**

Homer wants to cheat while meditating so he can levitate without effort. He wants to use the earth's electric field near the surface which is typically 100 N/C, downward in order to create an electric force to counter his weight of 900 N. He is able to do so by acquiring a negative charge. You need to help Homer calculate the needed charge as well as the needed number of electrons to create this charge.



$F = QE$

$F = n e^- E$

$n = \frac{F}{e^- E}$

$= 5,5 \times 10^{18}$ electron

q = ?

10

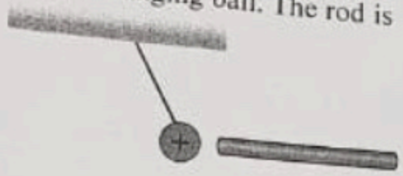
**Problem 5 : (10 points) Multiple Choice Questions**

Multiple choice questions (2 points each). **Fill in the following table with capital letters (A, B, C, D, or E), nothing else will be graded!**

| Question | Answer |
|---|---|
| 1 | B |
| 2 | C |
| 3 | A |
| 4 | C |
| 5 | A |

4

1. A rod attracts a positively charged hanging ball. The rod is



   A. Positive
   B. Negative
   C. Neutral
   D. Either A or C
   E. Either B or C

2. Two positive point charges $Q$ and $2Q$ are separated by a distance $R$. If the charge $Q$ experiences a force of magnitude $F$ when the separation is $R$, what is the magnitude of the force on the charge $2Q$ when the separation is $2R$?
   A. $F$
   B. $F/4$
   C. $4F$
   D. $2F$
   E. None of the above

3. At a distance $D$ from an infinitely long uniform line of charge, with linear charge density $\rho_L$, the electric field strength is 1000 N/C. For the field strength to be 2000 N/C, the distance from the line would have to be:
   A. $2D$
   B. $D$
   C. $D/2$
   D. $D/4$
   E. None of the above

4. Two electrons are passing 27.0 mm apart. What is the electric repulsive force that they exert on each other?
   A. 1.2 N
   B. $3.2 \times 10^{-27}$ N
   C. $3.2 \times 10^{-25}$ N
   D. $1.2 \times 10^{10}$ N
   E. None of the above

5. You have two positively charged particles and one negatively charged particle, all of which have the same magnitude of charge. Is it possible to place the three particles so that the net force on one of the **positively charged** particles due to the other two particles is zero?
   A. True
   B. False