Ex 9) Write the Taylor Series for $\sin(r^2)$ and Evaluate:

$$\iiint_R \frac{\sin(x^2+y^2)}{2\pi\sqrt{x^2+y^2}}\,dV$$

with an error of no more than 0.001; $R: 0 \le z \le 1$, $0 \le x^2+y^2 \le 1$

{ Thas Final 2010 }

- we know $Sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

- replace x with r^2

- $Sin(r^2) = \sum_{n=0}^{\infty} \frac{(-1)^n r^{4n+2}}{(2n+1)!}$

- $x^2+y^2=r^2$, $0 \leq z \leq 1$ and $0 \leq r \leq 1$

$$\iiint_R \frac{Sin(r^2)}{2\pi\sqrt{r^2}}\, dV$$

↳

$$\int_0^{2\pi}\int_0^1\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n r^{4n+2}}{(2n+1)!} \cdot r\, dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n r^{4n+2}}{2\pi(2n+1)!}\, dz\, dr\, d\theta$$

Find first few terms

$$\sum_{n=0}^{\infty} \frac{(-1)^n r^{4n+2}}{2\pi(2n+1)!} = \frac{r^2}{2\pi} - \frac{r^6}{12\pi} + \frac{r^{10}}{240\pi}$$

. Find $\iiint$ for these terms

$$= \int_0^{2\pi}\int_0^1\int_0^1 \frac{r^2}{2\pi}\,dz\,dr\,d\theta - \int_0^{2\pi}\int_0^1\int_0^1 \frac{r^6}{12\pi}\,dz\,dr\,d\theta + \int_0^{2\pi}\int_0^1\int_0^1 \frac{r^{10}}{240\pi}\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \frac{r^2}{2\pi}\,dr\,d\theta - \int_0^{2\pi}\int_0^1 \frac{r^6}{12\pi}\,dr\,d\theta + \int_0^{2\pi}\int_0^1 \frac{r^{10}}{240\pi}\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^3}{6\pi}\Big|_0^1 d\theta - \int_0^{2\pi} \frac{r^7}{84\pi}\Big|_0^1 + \int_0^{2\pi} \frac{r^{11}}{2640\pi}\Big|_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{1}{6\pi}\,d\theta - \int_0^{2\pi} \frac{1}{84\pi}\,d\theta + \int_0^{2\pi} \frac{1}{2640\pi}\,d\theta$$

$$= \frac{1}{3} - \frac{1}{42} + \frac{1}{1320}$$