

Math 201 Exam 1 (Fall 2010)

- Please write your name and section number on your booklet
- Please answer each problem on the indicated page(s) of the booklet. Any part of your answer that is not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.

Problem 1 (answer on pages 1 and 2 of the booklet) (7 points each)

Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence

(i) $a_n = \frac{n^{2 \ln 3} (\sqrt[n]{n})}{(9 \ln n - n) n!}$
 (ii) $b_n = \left(\frac{n+3}{n+4}\right)^n$
 (iii) $c_n = \sqrt[n]{n+(-1)^n}$
 (iv) $d_n = n(4^{1/n} - 1)$

Problem 2 (answer on pages 3 and 4 of the booklet). (7 points each)

Which of the following series converge, and which diverge?

Find the sum of the series when possible.

(i) $\sum_{n=0}^{\infty} \left(\frac{(-1)^n}{3^n} + \frac{3^n}{4^{n-1}} \right)$
 (ii) $\sum_{n=1}^{\infty} n \left(\frac{1}{n} - \ln \left(1 + \frac{1}{n} \right) \right)$
 (iii) $\sum_{n=1}^{\infty} \frac{(-1)^n}{0.1^n (e^{2n} + 1)}$
 (iv) $\sum_{n=1}^{\infty} \left(1 - \frac{\ln 3}{n} \right)^{n^2}$

Problem 3 (answer on page 5 of the booklet)

Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n (x-2)^n}{3^n \ln n}$$

(22 points)

For what values of x does the series converge absolutely? Conditionally?

Problem 4 (answer on page 6 of the booklet)

Consider the function $f(x) = e^{-x^2}$

(i) Find the power series expansion for $f(x)$ about the point $x = 0$. (5 points)

(ii) Use the alternating series estimation theorem to approximate $\int_0^{0.1} e^{-x^2} dx$ with an error of magnitude less than 10^{-5} (6 points)

Problem 5 (answer on last page of the booklet)

Consider the function $f(x) = (1+x)^{1/2}$

(i) Find the Taylor polynomials $p_1(x)$ and $p_2(x)$ generated by $f(x)$ at $x = 0$ (5 points)

(ii) Approximate $\sqrt{0.9}$ by $p_2(x)$ and estimate the resulting error using Taylor's Theorem (6 points)