Math 201 Final Examination (Time: 2.5 hours) Feb 1, 2006

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Name

I.D Section Number

Part 1: (60/100 points:) 15 problems with 4 points each

Be Careful: Many problems have 2 parts (with 2 points each)

Circle your multiple choice answers & fill in the blanks (as in problem 1)

Penalty: – 1/2

1) Let $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$ be the Fourier series of $f(x) = x$ on $-\pi < x < \pi$.

____(i) Fill in the blank: $b_3 = \ldots\int_{\ldots}^{\ldots}\ldots\ldots\ldots\ldots dx$.

____(ii) A) $b_3 = 0$ B) $b_3 = 2/5$ C) $b_3 = -2/5$ D) $b_3 = 1/5$ E) $b_3 = -1/5$

(F) None of the above

2) The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n \ln n + 5}$ is

____(i) X) Convergent Y) Divergent

____(ii) A) Absolutely convergent B) Conditionally convergent C) Divergent

Reminder: Answer both parts

3) The series $\sum_{n=1}^{\infty} (-1)^n (\frac{n}{n+2})^n$ is

A) Absolutely convergent B) Conditionally convergent C) Divergent

4) The series $\sum_{n=1}^{\infty}(-1)^n\frac{(2n!)}{(n!)^2}$ is

A) Absolutely convergent B) Conditionally convergent C) Divergent

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5) Consider the power series $\sum_{n=1}^{\infty}(-1)^n\frac{(x-1)^n}{\sqrt{n}}$

___(i) Fill in the blank: The domain of convergence WITHOUT ENDPOINTS

___(ii) The (full) domain of convergence is

A) $0 < x < 2$ B) $0 \le x < 2$ C) $0 \le x \le 2$

D) $0 < x \le 2$ E) x=1 F) None of the above

Reminder: Answer both parts

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6) Consider the power series $\sum_{n=1}^{\infty} n^{2n}x^{(n^2)}$

___(i) Fill in the blank: The domain of convergence WITHOUT ENDPOINTS

___(ii) The (full) domain of convergence is

A) $-1 < x < 1$ B) $-1 \le x < 1$ C) $-1 \le x < 1$

D) $-2 < x \le 2$ E) $x = 0$ F) None of the above

7) Let $f(x) = x^7e^{-x^2}$. Then $f^{(107)}(0) =$

A) $\dfrac{50!}{107!}$ B) $\dfrac{107!}{50!}$ C) $-\dfrac{50!}{107!}$ D) $-\dfrac{107!}{50!}$ E) None of the above

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8a) The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^7}$ is A) Convergent B) Divergent

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8b) The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{7^{\ln n}}$ is A) Convergent B) Divergent

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9) Consider the paraboloid $x^2 + y^2 - 4z = -2$ and the sphere $x^2 + y^2 + z^2 = 3$.
Then the ***tangent planes*** to both surfaces at the intersection point (1, 1, 1) are

A) parallel B) perpendicular C) neither perpendicular nor parallel.

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10) Given that $F(x, y, z) = 100$. If the components of ∇F are never zero, then
$\dfrac{\partial z}{\partial x}.\dfrac{\partial x}{\partial y}.\dfrac{\partial y}{\partial z}$ & $\dfrac{\partial z}{\partial x}.\dfrac{\partial x}{\partial y}$ are A) 1 & $-\dfrac{\partial z}{\partial y}$ resp. B) -1 & $-\dfrac{\partial z}{\partial y}$ resp.

C) 1 & $\dfrac{\partial z}{\partial y}$ resp. D) -1 & $\dfrac{\partial z}{\partial y}$ resp. E) None of the above

11) The value of the double integral $\int_0^2 \int_{y/2}^1 6ye^{x^3}\,dxdy$ is

A) 9 (e-1) B) 4(e-1) C) 16(e-1) D) 25 (e-1) E) None of the above

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12) The critical point (1, 1) of the function $f(x,y) = x^5 + y^5 - 5xy + 1$ is

A) Local Maximum B) Local Minimum C) Saddle point

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13) The function $f(x,y)$ at the point $p(-1, 2)$ has the following directional derivavtives.
$(D_v f)(p) = 20$ & $(D_w f)(p) = 4$ where $\mathbf{v} = \mathbf{4i} + \mathbf{3j}$ & $\mathbf{w} = 7070\,\mathbf{j}$.

(i) Fill in the blank: The gradient vector of $f(x,y)$

(Hint: Is $D_w f = f_y$?)

(ii) The **minimum** possible directional derivative of $f(x,y)$ at p is

A) $-\sqrt{13}$ B) $-\sqrt{15}$ C) $-\sqrt{17}$ D) $-\sqrt{20}$ E) None of the above

Reminder: Answer both parts

14) Let $P = f(t^5, t^3, t^2)$ where $f(u,v,w)$ is a differentiable function.
Suppose $f(1,1,1) = 2$ & { $f_u(1,1,1) = 3$, and $f_v(1,1,1) = -5$ and $f_w(1,1,1) = 10$.}.
Then at $t = 1$, $dP/dt =$

A) 20 B) 30 C) 40 D) 50 E) None of the above

15a) $\lim_{(x,y \to (0,0)} xy\ (\sin \frac{1}{x^2})(\cos \frac{1}{y^2})$ A) 0 B) does not exist

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15b) $\lim_{(x,y \to (0,0)} \frac{xy^6}{x^6 + y^4}$ A) 0 B) does not exist

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Part 2 : (40 points) Subjective

16) (6 pts). Set up **<u>but do not evaluate</u>** the triple integral(s) in **<u>cylinderical</u>** coordinates that represent the volume of the region bounded by the paraboloids
$z = x^2 + y^2$ and $z = 18 - (x^2 + y^2)$

17) **(10 pts)** Set up **but do not evaluate** the triple integral(s) in **cylinderical & spherical** coordinates that represent the volume of the region above z=0 bounded by the right cylinder $x^2 + y^2 = 9$ and the sphere $x^2 + y^2 + z^2 = 25$

cylinderical :

spherical

$$\oint_C y^2 dx - xy dy = -3/2$$

where C is the square with vertices (0,0), (0,1), (1, 0), (1,1) (traced once and counter clockwise)

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19) (3 pts)(i) Show that $(2xy^5 + x)dx + 5x^2y^4$ dy is an exact differential

(5 pts) (ii) Use part (i) to find (if possible) $\oint_C (2xy^5 + x)dx + 5x^2y^4\ dy$ where C is a very complicated curve from A(1,1) to B(2,5)

20) (3 pts) Find the **Jacobian** $\partial(x,y)/\partial r,q)$ for the transformation $x = r^2 q \quad y = \dfrac{r}{q}$

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21) (5 pts) Find the absolute maximum & absolute minimum of the function

$$T(x,y,z) = 4x^2 + 3y^2 - 4y + 5z^2 + 20$$

on the ellipsoid $x^2 + y^2 + 5z^2 = 9$.

(Eliminate $5z^2$ OR use Lagrange method)