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AMERICAN UNIV. OF BEIRUT

QUIZ-I-MATH.201(Thomas-12th-Sec.10.1-10.8) **Oct. 3,2013**

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NAME:.. I.D... ....

SECTION: 26 (2:00F) 27 (4:00F)

Question	1	2	3	4	5	BONUS		QUIZ I GRADE
Max.	15	35	16	18	16	4		
Gr.	9	26	3	12	13	0		64

+3

1. (*15Points*) Find the limits of the following sequences whose general term is:

a) $a_n = \frac{2^{\sin n}}{\sqrt{n}}$

$-1 < \sin n < +1$ (Sandwich theorem).

$\lim a_n = \lim \frac{2^{\sin n}}{\sqrt{n}} = \frac{2^{(\frac{\sqrt{3}}{2})}}{\infty} = 0$

$\frac{1}{2} < 2^{\sin n} < 2^1$; $\frac{1}{2\sqrt{n}} < \frac{2^{\sin n}}{\sqrt{n}} < \frac{2}{\sqrt{n}}$

$\frac{1}{2\sqrt{n}} \to 0$, $\frac{2}{\sqrt{n}} \to 0$

So by sandwich theorem $a_n \to 0$.

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b) $b_n = \frac{3^n + 25^n}{8 \cdot 5^{2n}}$

$\lim \frac{3^n + 25^n}{8,5^{2n}}$

$= \lim \frac{3^n}{8,5^{2n}} + \lim \frac{25^n}{8,5^{2n}} = \lim \frac{1}{\frac{(8,5)^{2n}}{3^n}} + \lim$

$\lim \left(\frac{3}{8,5}\right)^n \cdot \frac{1}{8,5^n} = 0$ similary $\lim \frac{25^n}{8,5^{2n}} = 0$

$\left(\frac{3}{8,5}\right)^n \to 0$, $\frac{1}{8,5^n} \to 0$

then $b_n \to 0$

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c) $c_n = \frac{(3n)!}{(n!)^3}$

we know that $\frac{x^n}{n!} \to 0$

$\lim \frac{(3n)!}{(n!)^3} = \frac{3 \cdot (3n)! / 3}{(n!)^3}$

$\lim \frac{(3n)!}{(n!)(n!)(n!)}$.

0

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2. (35 Points)Determine the divergence or convergence of the following series.(Give the name of the test used in each case and justify your response).

a) $\sum_{n=10}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}-2)}$ $a_n = \sum \frac{1}{\sqrt{n}(\sqrt{n}-2)}$ let $b_n = \sum \frac{1}{\sqrt{n}}$

$\frac{\sqrt{n}}{\sqrt{n}(\sqrt{n}-2)} = \frac{1}{\sqrt{n}-2} \to 0$ (Test fails).

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• let $b_n = \sum_{n=10} \frac{1}{n}$; $\sum \frac{1}{n}$ is a divergent p-series $p=1$.

$\Rightarrow \frac{1}{n} < \frac{1}{\sqrt{n}(\sqrt{n}-2)}$ so by D.C.T the given series diverges.

$a_n =$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{3-(1+\frac{1}{n})^n} \right)^n$ $\sqrt[n]{a_n} = \frac{1}{3-(1+\frac{1}{n})^n}$

$\lim \frac{1}{3-(1+\frac{1}{n})^n} \to \frac{1}{3-e^1} > 1$ thus the given series diverges by root test.

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a_n c) $\sum_{n=1}^{\infty} \ln(\frac{2n+1}{n})$, let $b_n = \sum \frac{1}{n}$ which is a divergent p-serie

$\frac{a_n}{b_n} = \frac{\ln(2+\frac{1}{n})}{\frac{1}{n}}$; apply l'hopital $\lim \frac{(1/n)'}{(1/n)'(2+\frac{1}{n})} = \lim \frac{1}{2+0} = \frac{1}{2}$

$\frac{1}{2} = c$ then by L.C.T the series diverges

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d) $\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{\ln(n+3)}$; $u_n = \frac{1}{\ln(n+3)}$.

$u_n > 0$; $u_{n+1} < u_n$; $u_n \to 0$

sin $\frac{1}{\ln(n+3)} \to 0$. so by Alternating series Test the given series converges.

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$a_n = $ e) $\sum_{n=1}^{\infty} \frac{1\cdot 4\cdot 7\cdot \ldots \cdot(3n-2)}{(n!)5^n} = \sum \frac{n(3n-2)}{(n!)5^n}$

$\frac{a_{n+1}}{a_n} = \frac{(n+1)(3(n+1)-2)(n!)5^n}{n(3n-2)((n+1)(n)!\cdot 5^{n+1}}$

$= \frac{(3(n+1)-2)\cdot 5}{n(3n-2)} \to \frac{5}{n} = 0$

$0 < 1 \Rightarrow$ the series converges by Ratio Test

$a_n = $ f) $\sum_{n=1}^{\infty} \frac{1\cdot 4\cdot 7\cdot \ldots \cdot(3n-2)(n!)}{(n!)\cdot 3^n}$ $\quad a_n = \sum \frac{n(3n-2)}{3^n}$

$\sqrt[n]{a_n} = \frac{(n(3n-2))^{\frac{1}{n}}}{\sqrt[n]{3^n}} \to \frac{1}{3}$

$\frac{1}{3} < 1 \Rightarrow$ the series converges by root test

g) $\sum_{n=2}^{\infty} \frac{(\ln n)^{100}}{n^{1.2}}$ $\quad a_n = \sum \frac{(\ln n)^{100}}{n^{1.2}}$

$= \sum \left(\frac{\ln n}{n^{\frac{1.2}{100}}}\right)^{100} = \sum \left(\frac{\ln n}{n^{0.012}}\right)^{100}$

~~let $b_n = \sum \frac{\ln n}{n^{0.012}}$ which is~~

let $b_n = \sum \frac{1}{n^{1.1}}$ which is a conv. p-series.

$\frac{a_n}{b_n} = \lim \frac{(\ln n)^{100} \cdot n^{1.1}}{n^{1.2}} = \lim \frac{(\ln n)^{100}}{n^{0.1}} = \lim \left(\frac{\ln n}{n^{\frac{0.1}{100}}}\right)^{100}$

but $\frac{\ln n}{n^{\frac{0.1}{100}}} \to 0$ then $\lim \left(\frac{\ln n}{n^{\frac{0.1}{100}}}\right)^{100} \to 0$ so by L.C.T the series converges

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3. (8+4+4=16 Points)You are given the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2)^n(n!)}$ whose sum is to be approximated with the absolute error not exceeding 0.0001. Answer the following questions:

a) What is the minimum number of terms needed?

if the error is less than $0.0001 = \frac{1}{10^4}$ then the sum of the term needed must be less than the term giving us $\frac{1}{10^4}$

so $n = ?$;

$|L - S_n| \ error| < \frac{1}{10^4}$ so

then $(2)^n.(n!) < 10^4$

b) Is the result an overestimation or an underestimation of the actual sum? Justify.

since $\frac{1}{10^4} > 0$ thus the result is an Over estimation

c) What is the numerical value of your estimate?

$\frac{(-1)^{n+1}}{(2^n)(n!)}$ must converges to a

$$\sum \frac{(-1)^{n+1}}{2^n(n!)} = \frac{1}{2} - \frac{1}{2^2.(2)!}$$

$$= \frac{1}{2} - \frac{1}{8} + \frac{1}{54} + \dots +$$

$$\frac{1}{2} + x = \frac{1}{10^4}$$

$x = -4$

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4. (<u>*12+6=18 Points*</u>) **Solve the following two independent parts:**

a) Find the sum the series: $\sum_{n=2}^{\infty}\{(0.3^n+\tan^{-1}(\frac{n+1}{2})-\tan^{-1}(\frac{n}{2})\}$

the series is $\sum (0.3)^n + \sum \tan^{-1}(\frac{n+1}{2}) - \tan^{-1}(\frac{n}{2})$.

$\sum (0.3)^n$ is a geometric series of $S = \frac{\cancel{a}}{1-\cancel{r}} = \frac{0.09}{1-\frac{3}{10}}$

$= \frac{9}{70}$

• $\sum \tan^{-1}(\frac{n+1}{2})$; $\tan^{-1}(\frac{n+1}{2}) \not\to 0$ then the series $\sum \tan^{-1}(\frac{n+1}{2})$ diverges by nth term test.

• The series $\sum \tan^{-1}(\frac{n}{2})$; $\tan^{-1}(\frac{n}{2})$ also diverges by nth term test so we can't have the sum

so the ~~and~~ sum of this series is $\frac{9}{70}$

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b) Write down the first three terms of the product:

$(\sum_{n=0}^{\infty}\frac{x^n}{(n!)^2})\cdot(\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{(n+1)^2})$

$a_n = \sum_{n=0}^{\infty} \frac{x^n}{(n!)^2} = \left(1 + x + \frac{x^2}{(2!)^2} + \cancel{x}\right)$

$b_n = \sum \frac{(-1)^n x^n}{(n+1)^2} = \left(1 + -\frac{x}{2^2} + \frac{x^2}{3^2}\right)$

$a_n \cdot b_n = 1 + x\left(-\frac{1}{2^2}+1\right) + x^2\left(\frac{1}{3^2} - (1)\frac{1}{2^2} + \frac{1}{(2!)^2}\right)$

6

13+

5. (<u>*10+3+3=16 Points*</u>) Find the value(s) of x for which the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n \cdot (4)^n}$

a) Converges absolutely

$a_n = \sum \frac{(x+2)^n}{n.(4)^n}$ $\sqrt[n]{|a_n|} = \frac{|x+2|}{|n^{\frac{1}{n}}. 4|} \longrightarrow \frac{|x+2|}{4} = \rho$

$\frac{|x+2|}{4} < 1$; $-4 < x+2 < 4$; $-6 < x < 2$.

so for x between -6 and 2 the series converges absolutely.

b) Converges conditionally

There are no values for where the series converges conditionally.

c) Diverges

so the ~~test~~ the series diverges for

$x \in]-\infty, -6] \cup [2, +\infty[$

• For $x = -6$; $\sum \frac{(-4)^n}{n.(4)^n}$; $\sum \frac{|4^n|}{n.4^n} = \sum \frac{1}{n}$ which diverge

$\sqrt[n]{\cancel{\frac{|4^n|}{n.4^n}}} = \frac{4}{4.n^{\frac{1}{n}}} \longrightarrow \cancel{1}$

• For $x = 2$. $\sum \frac{(4)^n}{n.(4)^n} = \cancel{\frac{1}{n}} = \sum \frac{1}{n}$.

$\sum \frac{1}{n}$ is a divergent p-series.

<u>BONUS QUESTIONS</u>: *Two points or none for each question:*

a) Determine the value(s) of x for which the series $\sum_{n=1}^{\infty} \frac{(\lfloor x \rfloor)^n}{n}$ converges.

$a_n = \frac{(\lfloor x \rfloor)^n}{n}$ let $\sqrt[n]{|a_n|} = \frac{\lfloor x \rfloor}{n^{\frac{1}{n}}}$

$\lim \frac{\lfloor x \rfloor}{n^{\frac{1}{n}}} = x$ so $-1 < \lfloor x \rfloor < 1$

but $\lfloor x \rfloor = -1$ then $a_n = \sum \frac{(-1)^n}{n}$

$\sum \left|\frac{1}{n}\right|$ is a divergent series so the series will not converge.

b) Find the value of the summation : $\sum_{n=1}^{\infty} n^2 \cdot (0.3)^n$

$\sum n^2 . (0.3)^n = \sum \frac{n^2}{(3)^n}$

$= \underbrace{\sum n^2}_{\text{divergent series}} \cdot \sum \frac{1}{(3)^n}$

the $\sum \frac{1}{(3)^n} = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$

$\sum n^2 \Rightarrow \infty$