Spring 2011	**Final Exam**	**Date:**
K. Yaghi	**Math 201- Sections 1-8**	**Duration: 2 hours**

Problem 1 *(answer on pages* 1 & 2 *of the booklet)*

The two parts of the following problem are independent.

(a) Consider the sequences $a_n = \cos(\frac{\propto}{2^n})$ and $b_n = \sin(\frac{\propto}{2^n})$

(i) Find $\lim_{n\to\infty}(2^n b_n)$ in terms of the constant $\propto$.

(ii) Let $c_n = a_1 \times a_2 \times a_3 \times \dots a_n$. Prove that the sequence c_n converges and find its limit.

(b) Does the series $\sum_{n=1}^{\infty} \frac{1}{(1+n^2)\arctan n}$ converge or diverge?

Problem 2 (*answer on pages* 3 & 4 *of the booklet*)

Let R be the region in plane bounded between the curves $y = e^{x/2}$, $y = e^{x-1}$ and the y-axis. Use the transformation

$$x = u + v \quad and \quad y = e^u$$

to rewrite $\iint_R \frac{x}{y} dA(x, y)$ as an appropriate integral over some region G in the uv plane. Then evaluate the uv integral.

Problem 3 (*answer on pages* 5 & 6 *of the booklet*)

(a) Use Green's theorem to show that if $D \subset \Re^2$ is a bounded region with boundary a positively oriented simple closed curve C, then the area of **D** can be calculated by the formula:

$$Area = \frac{1}{2}\int_C -ydx + xdy$$

(b) Let D be the region lying inside the ellipse $4x^2 + y^2 = 1$ in the xy-plane.

(i) Use part (a) to calculate the area of the ellipse $4x^2 + y^2 = 1$.

(ii) Calculate the flux integral $\oint_D \vec{F}.\vec{n}ds$ directly, where F is the vector field given by $\vec{F} = xy\,\vec{\imath} + y\,\vec{\jmath}$.

(iii) Use Green's theorem to recalculate the flux integral of part (ii).

Problem 4 (*answer on pages* 7 & 8 *of the booklet*)

Consider the vector field $\vec{F} = (2xy + \sin y)\vec{\imath} + (x^2 + x\cos y + 1)\vec{\jmath}$

(a) Show that the vector field F is conservative and find a potential function $f(x, y)$ of F.

(b) Use your answer to part (a) to evaluate the line integral $\oint_C F.dr$, where C is the arc of the parabola $y = x^2$ going from (0, 0) to (2, 4).

Problem 5 *(answer on page* 9 *of the booklet)*

Let D be the region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{3x^2 + 3y^2}$. Set up, but <u>do not evaluate</u>, the iterated triple integral in cylindrical coordinates that gives the volume of D in the order:

(i) $dzdrd\theta$ (ii) $drdzd\theta$

Problem 6 *(answer on page* 10 *of the booklet)*

Let D be the region bounded between the cylinder $x^2 + y^2 = 4$ and the cone $z = \sqrt{x^2 + y^2}$. Set up, but <u>do not evaluate</u>, the iterated triple integral in spherical cooridnates that gives the volume of D in the order:

(i) $d\rho d\emptyset d\theta$ (ii) $d\emptyset d\rho d\theta$

Problem 7 *(answer on page* 11 *of the booklet)*

Find the volume of the tetrahedron cut from the first octant by the plane $2x + y + z = 2$.

Problem 8 *(answer on page* 12 *of the booklet)*

Find the absolute maximum and minimum values of the function $F(x, y, z) = xyz$ on the constraint $x + y + z = 1$. For $x,\ y,\ z \geq 0$.

Problem 9 *(answer on page* 13 *of the booklet)*

If R be the region enclosed by the sphere $x^2 + y^2 + z^2 = 1$. Evaluate $\iiint_R e^{(x^2+y^2+z^2)^{3/2}}\, dv(x, y, z)$.

Problem 10 *(answer on pages* 14 & 15 *of the booklet)*

Consider the function $f(x) = e^{x^2}$

a) Write a power series expansion for $f(x)$ about the point x = 0. Then find the Taylor polynomials p1(x) and p2(x) generated by $f(x)$ about x = 0.

b) In this part we consider the function $g(x) = 2xe^{x^2}$.

   (i) Use part (a) to find a power series expansion of g(x) about x = 0.

   (ii) Use power series expansion of g(x) about the point x = 0 to prove that $\int g(x)\, dx = f(x)$

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.