Fall 2015-2016	Quiz 2	Date: October 29
Prof. M. Egeileh	Math 201 - Sections 29 to 32	Duration: 1 hour

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Problem 1 *(answer on page 1 of the booklet)*
Find the domain and range of the function $f(x, y, z) = \dfrac{7}{5 - 2x^2 - 3y^2 - 4z^2}$.
Decide if the domain of f is an open region, a closed region, or neither. Also, decide if the domain of f is bounded or unbounded. *(12 pts)*

Problem 2 *(answer on page 2 of the booklet)*
Find the tangent line to the curve $y = x^3 + \sin x + \cos x + e^{xy}$ at the point $(0, 2)$. *(16 pts)*

Problem 3 *(answer on page 3 of the booklet)*
A mountain climber's oxygen mask is leaking. If the mountain is represented by the surface $z = 5 - x^2 - 2y^2$ and the climber is at $(\frac{1}{2}, -\frac{1}{2}, \frac{17}{4})$, in what direction should the climber turn to descend most rapidly? Find also the instantaneous rate of change of the climber's altitude in that direction (at the point $(\frac{1}{2}, -\frac{1}{2})$). *(10 pts)*

Problem 4 *(answer on page 4 of the booklet)*
Let $f(x, y)$ be a function of two variables, and (x_0, y_0) an interior point of the domain of f. Prove that if f is differentiable at (x_0, y_0), then f is continuous at (x_0, y_0). *(16 pts)*

Problem 5 *(answer on pages 5 and 6 of the booklet)*
For each of the following limits, say if it exists or no, justifying your answer. *(8+8+8 pts)*

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4y^7}{x + y^2}$ b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{\sqrt{x}}x^4y^7}{x + y^2}$ c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{xy} - 1}{\ln(1 + y - x)}$

Problem 6 *(answer on last page of the booklet)*
Suppose $f(x, y)$ is a differentiable function of two variables such that

$\vec{\nabla} f(3, -3) = 6\vec{i} - 2\vec{j}$, $\quad \vec{\nabla} f(2, 1) = \vec{i} + \vec{j}$,

$f(9, -9) = 5$, $\quad f(2, 1) = 4$.

Suppose also that the points $(3, -3)$ and $(9, -9)$ belong to the same level curve of f.

Let $x = r^2 - s^2$, $\quad y = s^2 - r^2$, $\quad$ and $\quad w = f(x, y)$.

1. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ at the point $(r, s) = (2, 1)$, then estimate $w(2.01, 0.98)$. *(12 pts)*
2. Suppose that the restriction of f to the line passing through $(9, -9)$ and directed by $\vec{u} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ has a local maximum at $(9, -9)$. Prove that $\vec{u}$ is a direction of zero change for f at $(9, -9)$. *(10 pts)*