Math 201–Exam 2 (Fall 15) Prof. B. Shayya and Dr. H. Yamani

- Please write your section number on your booklet.
- Please place your student ID card on the desk in front of you.
- Please answer each problem on the indicated page(s) of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.

Problem 1 (answer on pages 1 and 2 of the booklet.)

(a) (8 pts) Does

$$\lim_{(x,y)\to(0,0)} \frac{x\,y^4}{3x^8+y^4}$$

exist? Why or why not?

(b) (8 pts) What about

$$\lim_{(x,y)\to(0,0)} \frac{x^6\,y}{3x^8+y^4}\,?$$

(c) (8 pts) What about

$$\lim_{(x,y)\to(0,0)} \frac{x\,y}{3x+y}\,?$$

Problem 2 (answer on page 3 of the booklet.)

Suppose $f(x,y,z)$ is a differentiable function of three variables such that

$$f(1,2,3)=f(1,1,-4)=4, \qquad \nabla f(1,1,-4)=\mathbf{i}+\mathbf{j}+\mathbf{k}, \qquad \nabla f(8,3,-4)=3\mathbf{i}-\mathbf{j}+\mathbf{k}.$$

Let

$$x=5r+3s, \qquad y=2r+s, \qquad z=-2(r^2+s^2), \quad \text{and} \quad w=f(x,y,z).$$

(i) (12 pts) Find $\partial w/\partial r$ and $\partial w/\partial s$ at the point $(r,s)=(1,1)$.

(ii) (12 pts) Estimate $f(1.01, 1.02, -3.98)$.

Problem 3 (answer on page 4 of the booklet.)

Consider the function $f(x)=e^x$.

(a) (12 pts) Use Taylor's theorem to find a power series expansion for $f(x)$ about the point $x=0$.

(b) (12 pts) Find the Taylor polynomial $p_3(x)$ generated by f at $x=0$. Then use Taylor's theorem to estimate the error resulting from the approximation $e^{-0.1}\approx p_3(-0.1)$. Conclude that $e^{0.1}<10/9$.

Problem 4 (answer on pages 5 and 6 of the booklet.)

Let $\mathcal{C}$ be the two-sided cone $z^2=x^2+y^2$.

(i) (15 pts) Find the tangent plane and normal line of $\mathcal{C}$ at the point $(3,4,5)$.

(ii) (10 pts) Let S be the set of all points (x,y,z) in $\mathcal{C}$ such that the normal line of $\mathcal{C}$ at (x,y,z) is perpendicular to the vector $\mathbf{v}=\mathbf{i}+\mathbf{j}+\sqrt{2}\,\mathbf{k}$. Prove that S lies on a line L.

(iii) (3 pts) Find parametric equations for the line L from part (ii).